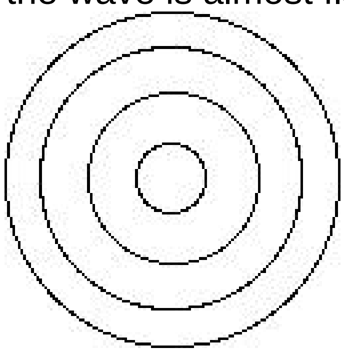
Two derivations of Snells Law

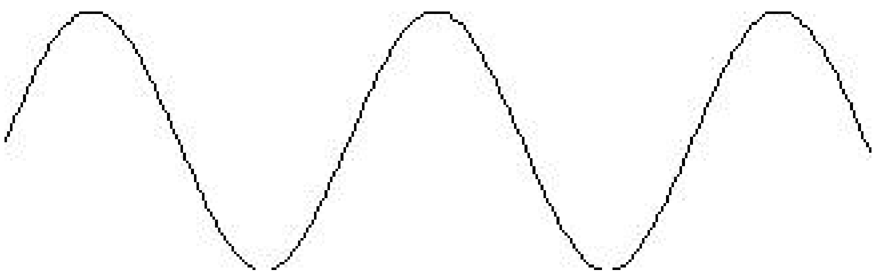
In Phys 220 we are representing the light as rays - straight lines.

We know that light is actually a wave

We can think of this rather like a water wave - these are the ripples set up by a dripping faucet - expanding out into a pond. Very far from the faucet the wave is almost flat



This flat part is called the wave front. If we were to cut perpendicular to the wave front, we would see the wave



So we have three ways of reprsenting the wave:

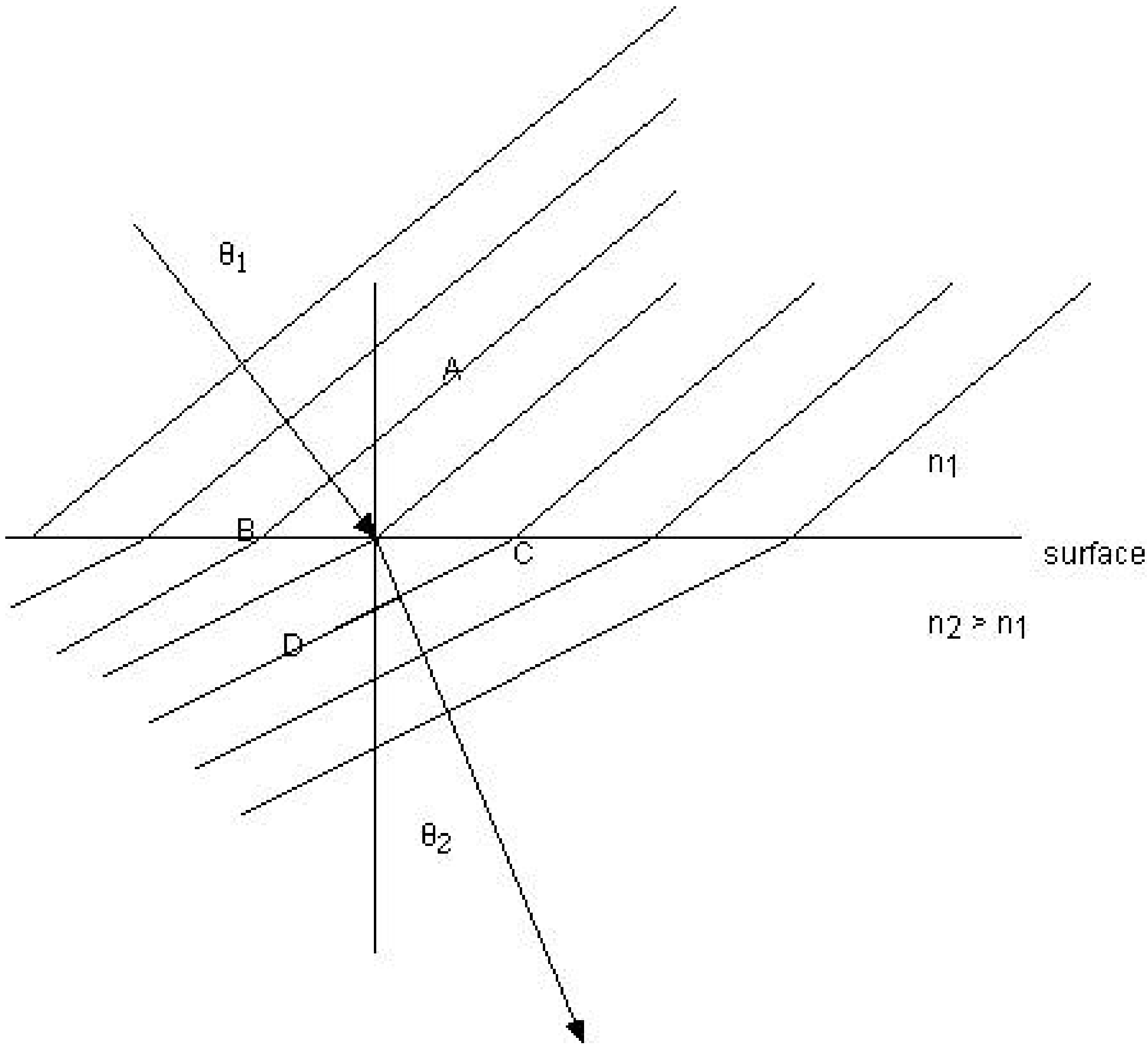
a wave front - a line tangent to the crests

a ray - a line perpendicular to the wave front

the wave

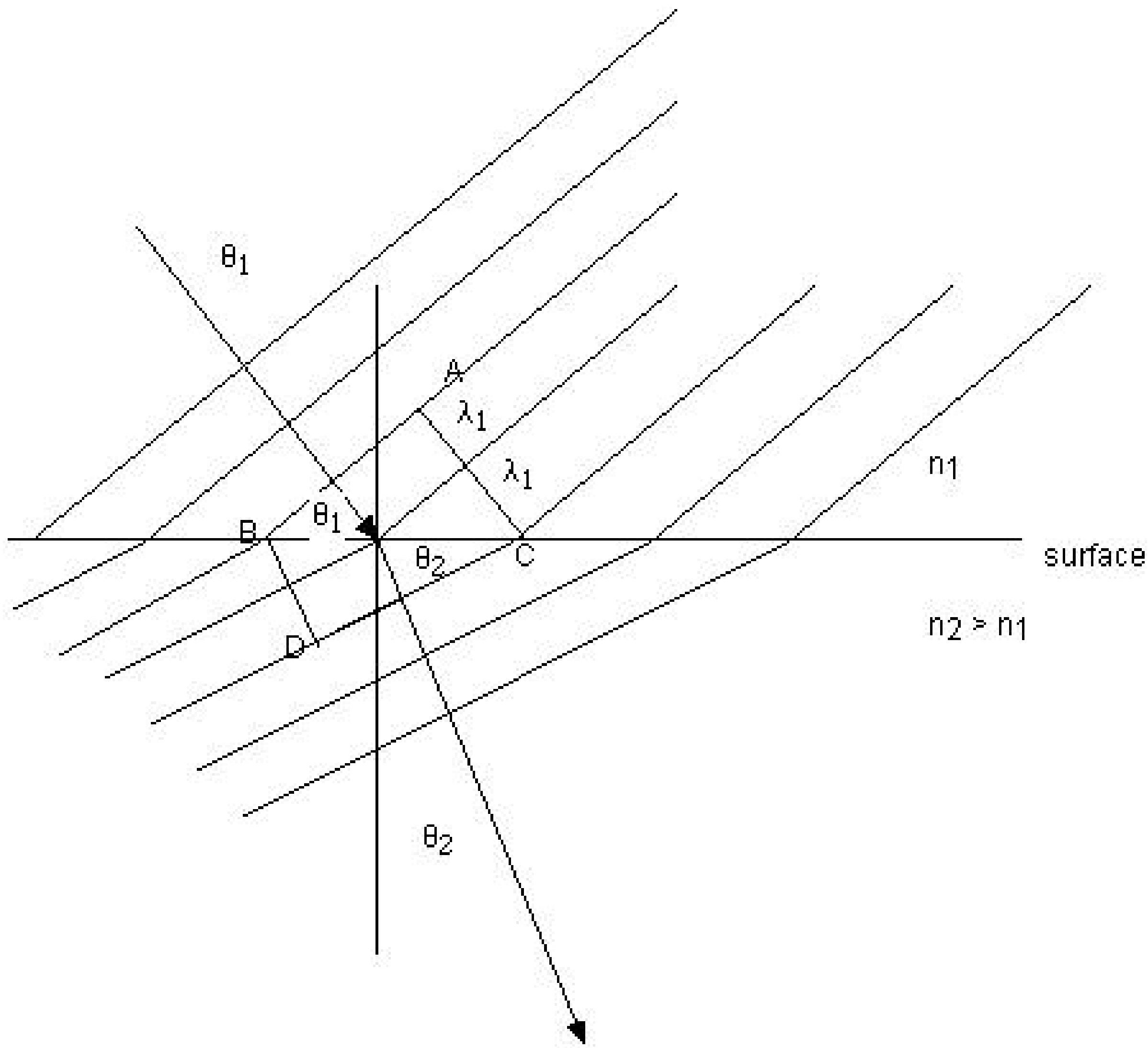
Now suppose we go very far from the source - the wave fronts would be parallel to each other and separated from each other by one wavelength. Now suppose we had some light moving in a medium with an index of refraction n_1 and it falls on the surface of another medium with an index of refraction n_2 where $n_2 > n_1$. This means that the speed of light in medium 2 is LESS than in medium 1.

Draw refraction diagram showing first rays, then wave fronts



Since the frequency of the wave does not change when it hits the surface but the speed decreases ($c_{medium} = c_{vacuum} / n_{medium}$) it must mean that the wavelength of the light in medium 2 (λ_2) is less than the wavelength in medium 1(λ_1) Now put in the letters A. B, C, D

Lets look at the triangle ABC in the figure,



The side AC = BC sin θ_1

But it is also equal to 2 λ_1

or

2 λ_1 = BC sin θ_1

Now triangle BDC The side BD = BC sin θ_2

But it is also equal to 2 λ_2

or

$2 \lambda_2 = BC \sin \theta_2$

we can divide one equation by the other and get

$\lambda_1/\lambda_2 = \sin(\theta_1)/ \sin(\theta_2)$

Since

$\lambda_1 = c_1 T$ (T = period, which is in common)

$\lambda_2 = c_2 T = c_{vac}/n_2 * T$

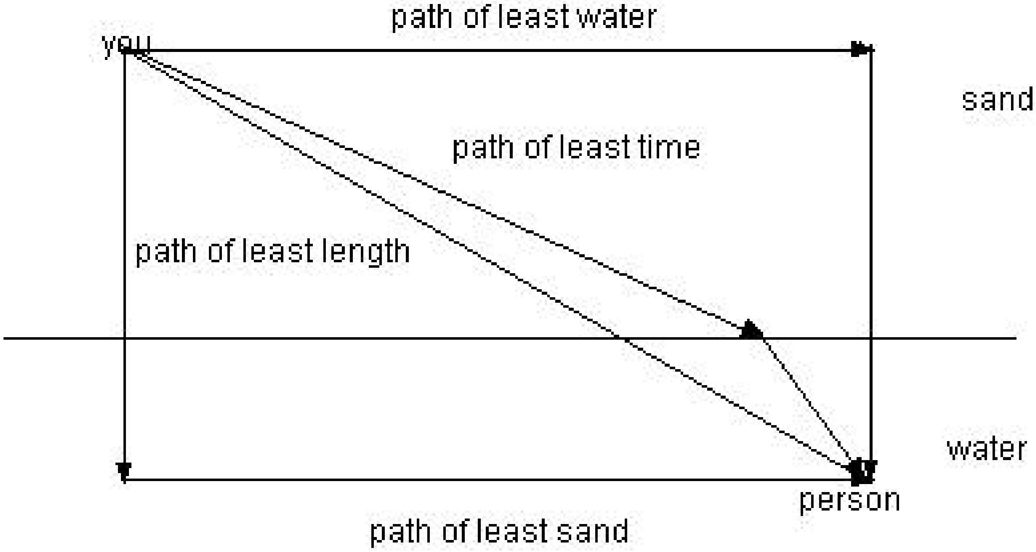
$(c_{vac}/n_1 * T)/(c_{vac}/n_2 * T) = \sin(\theta_1)/ \sin(\theta_2)$

$n_1 \sin(\theta_1) = n_2 \sin(\theta_2)$

which is Snells Law

Another approach

Now lets say you are a life guard and there is someone drowning the the ocean You can run on the sandy beach at lets say 10 miles/hour and swim in the water at 5 miles/hour. What path do you want to take to save the person

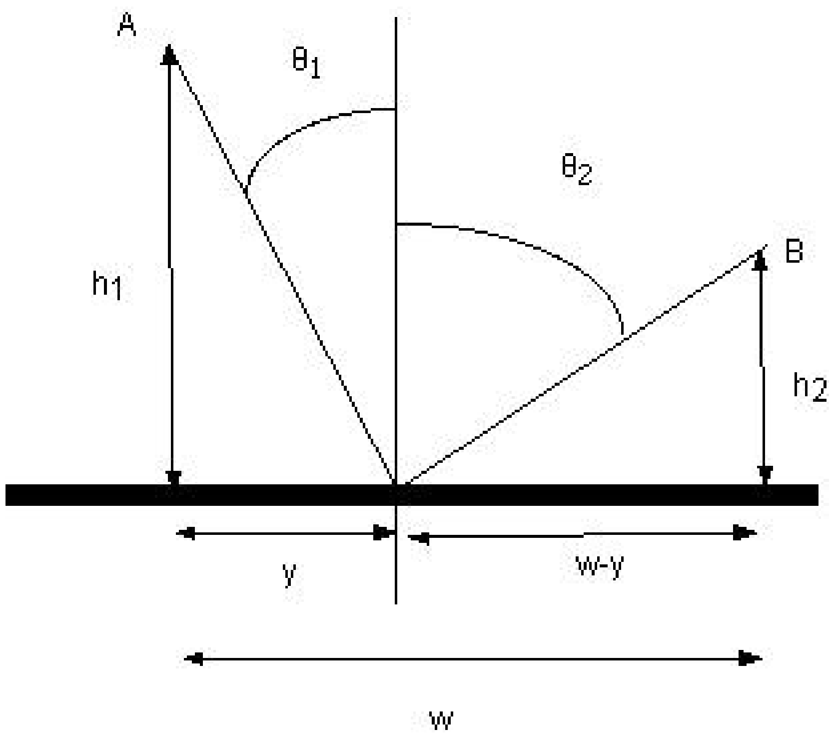


You want the path of least time

It turns out the same is true for light, as first determined by Fermat:

Light travels the path between two points that takes the least possible TIME.

Law of Reflection



$t = t_1 + t_2 = d_1/c + d_2/c$

$= (h_1^2 + y^2)^{1/2} / c + (h_2^2 + (w-y)^2)^{1/2} /c$

now we want the min t so we differentiate it with respect to y and set that equal to zero

$u =(h_1^2 + y^2)$

$t = u^{1/2}$

$dt/dy = du/dy * dt/du$

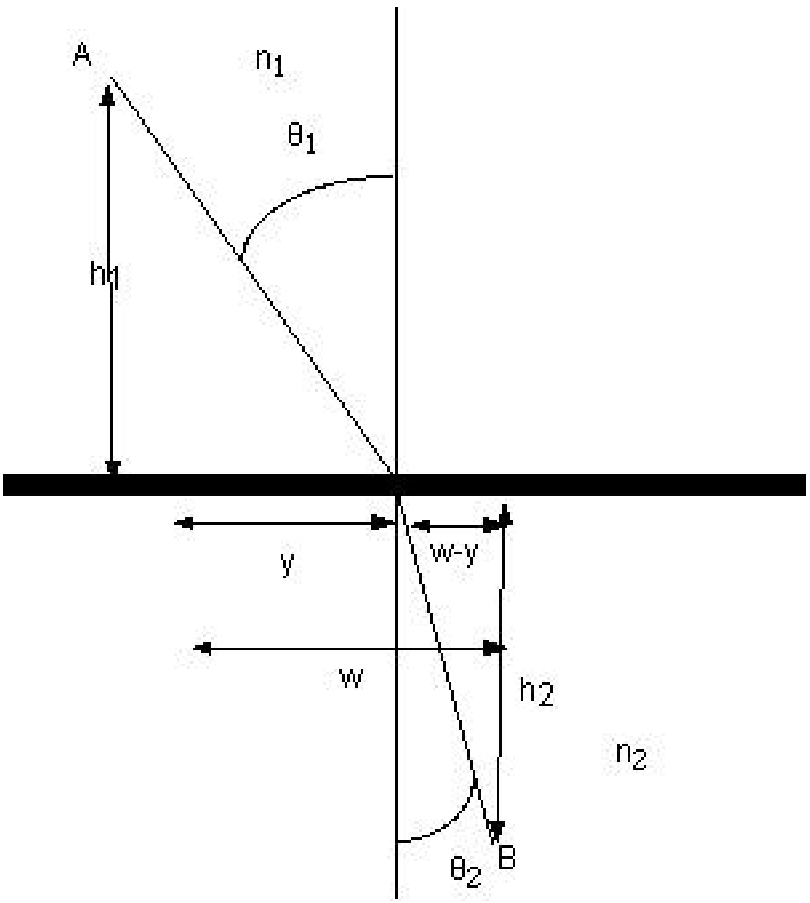
$dt/dy = 0 = y/(h_1^2 + y^2)^{1/2} - (w-y)/ (h_2^2 + (w-y)^2)^{1/2}$

or

$y/(h_1^2 + y^2)^{1/2} = (w-y)/ (h_2^2 + (w-y)^2)^{1/2}$

but that is just the sin (θ_1) = sin(θ_2) or $\theta_1 = \theta_2$, the law of reflection

The law of refraction is almost the same



now the total time is

$t = n_1(h_1^2 + y^2)^{1/2} / c + n_2(h_2^2 + (w-y)^2)^{1/2} /c$

and once again finding dt/dy and setting it equal to zero we get

$n_1\, y_{1/}(h_1^2 + y^2)^{1/2} = n_2\, (w-y)/(h_2^2 + (w-y)^2)^{1/2}$

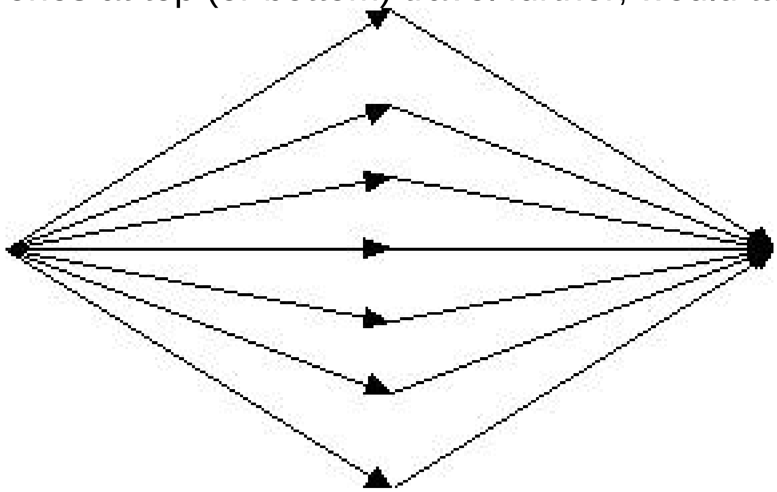
which is

$n_1 \sin (\theta_1) = n_2 \sin (\theta_2)$

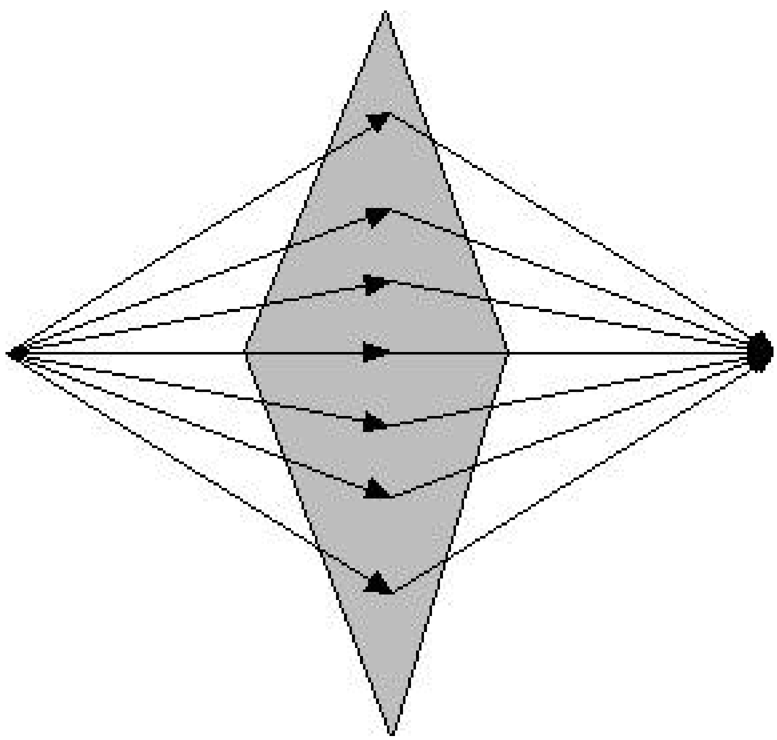
which is Snell's Law

How does a lens work?

ones at top (or bottom) travel further, would take longer



slow down the ones in the center



Fermats Principle raises a rather interesting question.

When we were thinking about the lifeguard, the life guard knew where he wanted to go - to the drowning swimmer.

But light doesn't think something like "I'm here at the bulb, and I want to go over to that wall, so the fastest path must be to bounce off the mirror."

In other words, how does the light know which path to take?

If you read Richard Feynmann's book "QED" you'll find out and the answer is even stranger than the result. It turns out that the light takes ALL POSSIBLE PATHS.