Chapter 7: Acceleration and Gravity

7.1 The Principle of Equivalence

We saw in the special theory of relativity that the laws of physics must be the same in all inertial reference systems. *But what is so special about an inertial reference system? The inertial reference frames are, in a sense, playing the same role as Newton's absolute space.* That is, absolute space has been abolished only to replace it by absolute inertial reference frames. Shouldn't the laws of physics be the same in all coordinate systems, whether inertial or noninertial? The inertial frame should not be such a privileged frame. But clearly, accelerations can be easily detected, whereas constant velocities cannot. How can this very obvious difference be reconciled? That is, we must show that even all accelerated motions are relative. How can this be done?

Let us consider the very simple case of a mass m on the floor of a rocket ship that is at rest in a uniform gravitational field on the surface of the earth, as depicted in figure 7.1(a). The force acting on the mass is its weight w, which we write as

$$F = w = mg \tag{7.1}$$

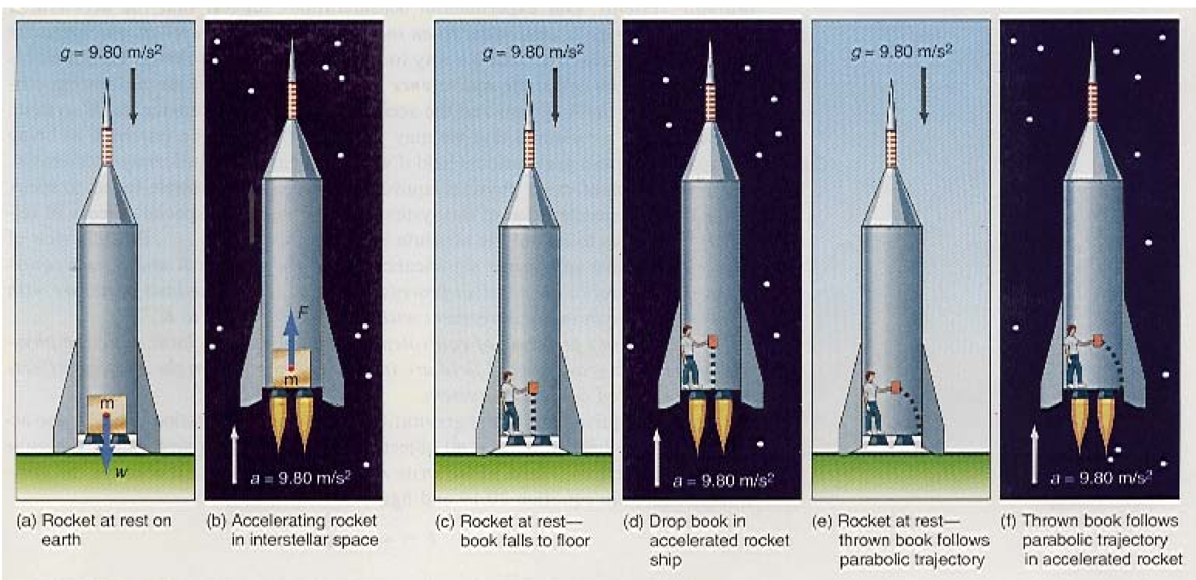

Figure 7.1 An accelerated frame of reference is equivalent to an inertial frame of reference plus gravity.

Let us now consider the case of the same rocket ship in interstellar space far removed from all gravitational fields. Let the rocket ship now accelerate upward, as in figure 7.1(b), with an acceleration a that is numerically equal to the acceleration due to gravity g, that is, $a = g = 9.80$ m/s^2. The mass m that is sitting on the floor of the rocket now experiences the force, given by Newton's second law as

$$F = ma = mg = w \tag{7.2}$$

That is, the mass m sitting on the floor of the accelerated rocket experiences the same force as the mass m sitting on the floor of the rocket ship when it is at rest in the uniform gravitational field of the earth. Therefore, there seems to be some relation between accelerations and gravity.

Let us experiment a little further in the rocket ship at rest by holding a book out in front of us and then dropping it, as in figure 7.1(c). The book falls to the floor and if we measured the acceleration we would, of course, find it to be the acceleration due to gravity, $g = 9.80$ m/s^2. Now let us take the same book in the accelerated rocket ship and again drop it, as in figure 7.1(d). An inertial observer outside the rocket would see the book stay in one place but would see the floor accelerating upward toward the book at the rate of $a = 9.80$ m/s^2. The astronaut in the accelerated rocket ship sees the book fall to the floor with the acceleration of 9.80 m/s^2 just as the astronaut at rest on the earth observed.

The astronaut in the rocket at rest on the earth now throws the book across the room of the rocket ship. He observes that the book follows the familiar parabolic trajectory of the projectile that we studied in our College Physics course and that is again shown in figure 7.1(e). Similarly, the astronaut in the accelerated rocket also throws the book across the room. An outside inertial observer would observe the book moving across the room in a straight line and would also see the floor accelerating upward toward the book. The accelerated astronaut would simply see the book following the familiar parabolic trajectory it followed on earth, figure 7.1(f).

Hence, the same results are obtained in the accelerated rocket ship as are found in the rocket ship at rest in the gravitational field of the earth. Thus, *the effects of gravity can be either created or eliminated by the proper choice of coordinate systems.* Our experimental considerations suggest that *the accelerated frame of reference is equivalent to an inertial frame of reference in which gravity is present.* Einstein, thus found a way to make accelerations relative. He stated his results in what he called the ***equivalence principle.*** Calling the inertial system containing gravity the K system and the accelerated frame of reference the K' system, Einstein said, "we assume that we may just as well regard the system K as being in a space free from gravitational field if we then regard K as uniformly accelerated. This assumption of exact physical equivalence makes it impossible for us to speak of the absolute acceleration of the system, just as the usual (special) theory of relativity forbids us to talk of the absolute velocity of a system… But this view of ours will not have any deeper significance unless *the systems K and K' are equivalent with respect to all physical processes, that is, unless the laws of nature with respect to K are in entire agreement with those with respect to K'*"[1]

Einstein's principle of equivalence is stated as: on a local scale the physical effects of a gravitational field are indistinguishable from the physical effects of an accelerated coordinate system.

The equivalence of the gravitational field and acceleration "fields" also accounts for the observation that all objects, regardless of their size, fall at the same rate in a gravitational field. If we write m_g for the mass that experiences the gravitational force in equation 7.1 and figure 7.1(a), then

$$F = w = m_g g$$

And if we write m_i for the inertial mass that resists the motion of the rocket in figure 7.1(b) and equation 7.2, then

$$F = m_i a = m_i g$$

Since we have already seen that the two forces are equal, by the equivalence principle, it follows that

$$m_g = m_i \tag{7.3}$$

That is, the gravitational mass is in fact equal to the inertial mass. Thus, the equivalence principle implies the equality of inertial and gravitational mass and this is the reason why all objects of any size fall at the same rate in a gravitational field.

As a final example of the equivalence of a gravitational field and an acceleration let us consider an observer in a closed room, such as a nonrotating space station in interstellar space, far removed from all gravitating matter. This space station is truly an inertial coordinate system. Let

[1]"On the Influence of Gravitation on the Propagation of Light," from A. Einstein, *Annalen der Physik* 35, 1911, in *The Principle of Relativity,* Dover Publishing Co.

the observer place a book in front of him and then release it, as shown in figure 7.2(a). Since there are no forces present, not even gravity, the book stays suspended in space, at rest, exactly where the observer placed it. If the observer then took the book and threw it across the room, he would observe the book moving in a straight line at constant velocity, as shown in figure 7.2(b).

Let us now consider an elevator on earth where the supporting cables have broken and the elevator goes into free-fall. An observer inside the freely falling elevator places a book in front of himself and then releases it. The book appears to that freely falling observer to be at rest exactly where the observer placed it, figure 7.2(c). (Of course, an observer outside the freely falling elevator would observe both the man and the book in free-fall but with no relative motion with respect to each other.) If the freely falling observer now takes the book and throws it across the elevator room he would observe that the book travels in a straight line at constant velocity, figure 7.2(d).

Because an inertial frame is defined by Newton's first law as a frame in which a body at rest, remains at rest, and a body in motion at some constant velocity continues in motion at that same constant velocity, we must conclude from the illustration of figure 7.2 that the freely falling frame of reference acts exactly as an inertial coordinate system to anyone inside of it. *Thus, the acceleration due to gravity has been transformed away by accelerating the coordinate system by the same amount as the acceleration due to gravity.* If the elevator were completely closed, the observer could not tell whether he was in a freely falling elevator or in a space station in interstellar space.

The equivalence principle allows us to treat an accelerated frame of reference as equivalent to an inertial frame of reference with gravity present, figure 7.1, or to consider an inertial frame as equivalent to an accelerated frame in which gravity is absent, figure 7.2. By placing all frames of reference on the same footing, Einstein was then able to *postulate* ***the general theory of relativity, namely, the laws of physics are the same in all frames of reference.***

From his general theory of relativity, Einstein was quick to see its relation to gravitation when he

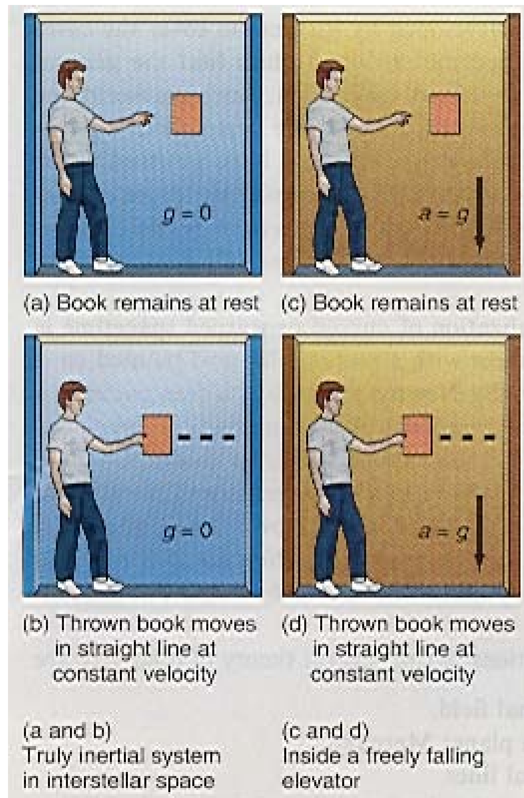

Figure 7.2 A freely falling frame of reference is locally the same as an inertial frame of reference.

said, *"It will be seen from these reflections that in pursuing the General Theory of Relativity we shall be led to a theory of gravitation, since we are able to produce a gravitational field merely by changing the system of coordinates.* ***It will also be obvious that the principle of the constancy of the velocity of light in vacuo must be modified."*** [2]

Although the general theory was developed by Einstein to cover the cases of accelerated reference frames, it soon became obvious to him that the general theory had something quite significant to say about gravitation. *Since the world line of an accelerated particle in spacetime is curved, then by the principle of equivalence, a particle moving under the effect of gravity must also have a curved world line in spacetime. Hence, the mass that is responsible for causing the gravitational field, must warp spacetime to make the world lines of spacetime curved.* This is

[2] "The Foundation of the General Theory of Relativity" from A. Einstein, *Annalen der Physik* 49, 1916 in *The Principle of Relativity,* Dover Publishing Co.

sometimes expressed as, matter warps spacetime and spacetime tells matter how to move. We will go into the details of curved spacetime in much greater detail in the next chapter. For now let us look at the problem from a purely physical point of view.

7.2 The Gravitational Red Shift

Although the General Theory of Relativity was developed by Einstein to cover the cases of accelerated reference frames, it soon became obvious to him that the general theory had something quite significant to say about gravitation. Let us consider the two clocks *A* and *B* located at the top and bottom of the rocket, respectively, in figure 7.3(a). The rocket is in interstellar space where we assume that all gravitational fields, if any, are effectively zero. The rocket is accelerating uniformly, as shown. Located in this interstellar space is a clock *C*, which is at rest. At the instant that the top of the rocket accelerates past clock *C*, clock *A* passes clock *C* at the speed v_A. Clock *A*, the moving clock, when observed from clock *C*, the stationary clock, shows an elapsed time Δt_A, given in chapter 1 by the time dilation equation 1.64 as

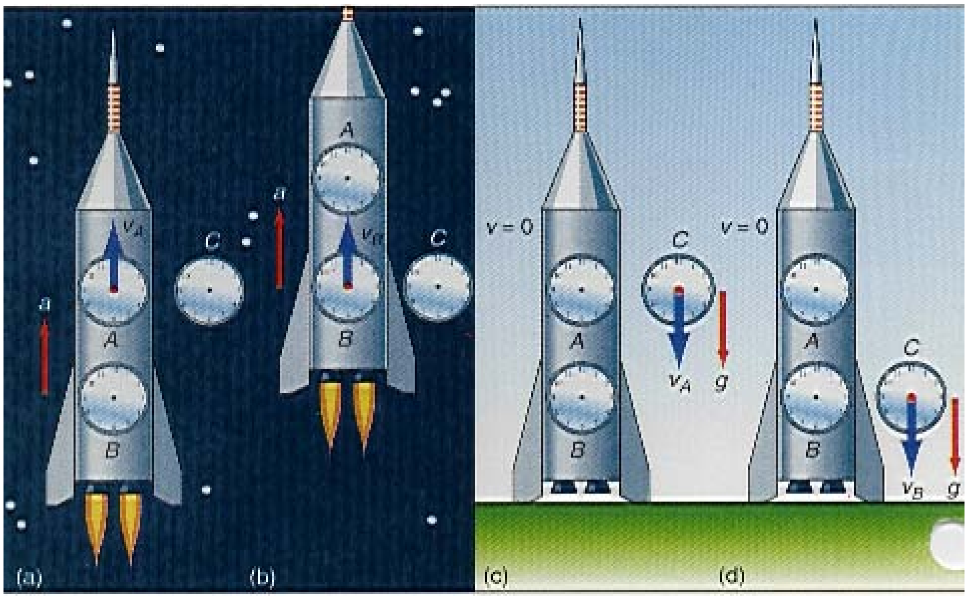

Figure 7.3 A clock in a gravitational field.

$$\Delta t_C = \frac{\Delta t_A}{\sqrt{1 - v_A^2/c^2}} \tag{7.4}$$

And since $\sqrt{1 - v_A^2/c^2}$ is less than 1, then $\Delta t_C > \Delta t_A$, and the moving clock *A* runs slow compared to the stationary clock *C*.

A few moments later, clock *B* passes clock *C* at the speed v_B, as in figure 7.3(b). The speed v_B is greater than v_A because of the acceleration of the rocket. Let us read the same time interval Δt_C on clock *C* when clock *B* passes as we did for clock *A* so the two clocks can be compared. The difference in the time interval between the two clocks, *B* and *C*, is again given by the time dilation equation 1.64 as

$$\Delta t_C = \frac{\Delta t_B}{\sqrt{1 - v_B^2/c^2}} \tag{7.5}$$

Because the time interval Δt_C was set up to be the same in both equations 7.4 and 7.5, the two equations can be equated to give a relation between clocks *A* and *B*. Thus,

$$\frac{\Delta t_A}{\sqrt{1-v_A^2/c^2}} = \frac{\Delta t_B}{\sqrt{1-v_B^2/c^2}}$$

Rearranging terms, we get

$$\frac{\Delta t_A}{\Delta t_B} = \frac{\left(1-v_A^2/c^2\right)^{1/2}}{\left(1-v_B^2/c^2\right)^{1/2}}$$

$$\frac{\Delta t_A}{\Delta t_B} = \left(1-v_A^2/c^2\right)^{1/2}\left(1-v_B^2/c^2\right)^{-1/2} \tag{7.6}$$

But the two terms on the right-hand side of equation 7.6 can be expanded by the binomial theorem, equation 1.33, as

$$(1-x)^n = 1 - nx + \frac{n(n-1)x^2}{2!} - \frac{n(n-1)(n-2)x^3}{3!} + \ldots \tag{1.33}$$

This is a valid series expansion for $(1-x)^n$ as long as x is less than 1. In this particular case,

$$x = v^2/c^2$$

which is much less than 1. In fact, since x is very small, it is possible to simplify the binomial theorem to

$$(1-x)^n = 1 - nx \tag{1.34}$$

Hence,

$$\left(1-v_A^2/c^2\right)^{1/2} = 1 - \left(\frac{1}{2}\right)\frac{v_A^2}{c^2} = 1 - \frac{v_A^2}{2c^2}$$

and

$$\left(1-v_B^2/c^2\right)^{-1/2} = 1 - \left(\frac{-1}{2}\right)\frac{v_B^2}{c^2} = 1 + \frac{v_B^2}{2c^2}$$

where again the assumption is made that v is small enough compared to c, to allow us to neglect the terms x^2 and higher in the expansion. Thus, equation 7.6 becomes

$$\frac{\Delta t_A}{\Delta t_B} = \left(1 - \frac{v_A^2}{2c^2}\right)\left(1 + \frac{v_B^2}{2c^2}\right)$$
$$= 1 + \frac{v_B^2}{2c^2} - \frac{v_A^2}{2c^2} - \frac{1}{4}\frac{v_B^2 v_A^2}{c^4}$$

The last term is set equal to zero on the same assumption that the speeds v are much less than c. Finally, rearranging terms,

$$\frac{\Delta t_A}{\Delta t_B} = 1 + \left(\frac{v_B^2}{2} - \frac{v_A^2}{2}\right)\frac{1}{c^2} \tag{7.7}$$

But by Einstein's principle of equivalence, we can equally well say that the rocket is at rest in the gravitational field of the earth, whereas the clock C is accelerating toward the earth in free-fall. When the clock C passes clock A it has the instantaneous velocity v_A, figure 7.3(c), and when it passes clock B it has the instantaneous velocity v_B, figure 7.3(b). We can obtain the velocities v_A and v_B by the law of conservation of energy, that is,

$$\frac{1}{2}mv^2 + \text{PE} = E_0 = \text{Constant} = \text{Total energy} \tag{7.8}$$

The total energy per unit mass, found by dividing equation 7.8 by m, is

$$\frac{v^2}{2} + \frac{PE}{m} = \frac{E_0}{m}$$

The conservation of energy per unit mass when clock C is next to clock A, obtained with the aid of figure 7.4, is

$$\frac{v_A^2}{2} + \frac{mgh_A}{m} = \frac{E_0}{m}$$

or

$$\frac{v_A^2}{2} + gh_A = \frac{E_0}{m} \tag{7.9}$$

Similarly, when the clock C is next to clock B, the conservation of energy per unit mass becomes

$$\frac{v_B^2}{2} + gh_B = \frac{E_0}{m} \tag{7.10}$$

Subtracting equation 7.9 from equation 7.10, gives

$$\frac{v_B^2}{2} + gh_B - \frac{v_A^2}{2} - gh_A = \frac{E_0}{m} - \frac{E_0}{m} = 0$$

Hence,

$$\frac{v_B^2}{2} - \frac{v_A^2}{2} = gh_A - gh_B = gh \tag{7.11}$$

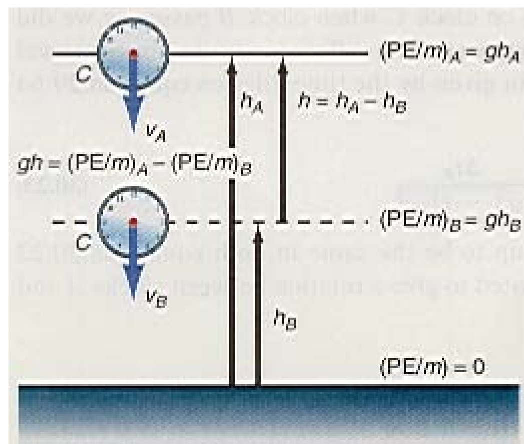

Figure 7.4 Freely falling clock C.

where h is the distance between A and B, and gh is the gravitational potential energy per unit mass, which is sometimes called the *gravitational potential.* Substituting equation 7.11 back into equation 7.7, gives

$$\frac{\Delta t_A}{\Delta t_B} = 1 + \frac{gh}{c^2} \tag{7.12}$$

For a clearer interpretation of equation 7.12, let us change the notation slightly. Because clock B is closer to the surface of the earth where there is a stronger gravitational field than there is at a height h above the surface where the gravitational field is weaker, we will let

$$\Delta t_B = \Delta t_g$$

and

$$\Delta t_A = \Delta t_f$$

where Δt_g is the elapsed time on a clock in a strong **g**ravitational field and Δt_f is the elapsed time on a clock in a weaker gravitational field. If we are far enough away from the gravitational mass, we can say that Δt_f is the elapsed time in a gravitational-field-**f**ree space. With this new notation equation 7.12 becomes

$$\frac{\Delta t_f}{\Delta t_g} = 1 + \frac{gh}{c^2} \tag{7.13}$$

or

$$\Delta t_f = \Delta t_g \left(1 + \frac{gh}{c^2}\right) \tag{7.14}$$

Since $(1 + gh/c^2) > 0$, *the elapsed time on the clock in the gravitational-field-free space* Δt_f *is greater than the elapsed time on a clock in a gravitational field* Δt_g. ***Thus, the time elapsed on a clock in a gravitational field is less than the time elapsed on a clock in a gravity-free space. Hence, a clock in a gravitational field runs slower than a clock in a field-free space.***

Thus, equation 7.14 gives the slowing down of a clock in a gravitational field. Compare this to equation 2.24, the time dilation formula, which shows the slowing down of a moving clock.

$$\Delta t = \frac{\Delta t_0}{\sqrt{1 - v^2/c^2}} \tag{2.24}$$

Equation 2.24 says that a clock on earth reads a longer time interval Δt than the clock at rest in the moving rocket ship Δt_0. Or as is sometimes said, moving clocks slow down. Thus, if the moving clock slows down, a smaller time duration is indicated on the moving clock than on a stationary clock.

Example 7.1

A clock in a gravitational field. A clock in a gravitational field on the earth ticks off a time interval of 10.0 hr. What time would elapse at a height of 1,000,000 km above the surface of earth.

Solution

The time elapsed in the gravitational free area is found from equation 7.14 as

$$\Delta t_f = \Delta t_g\left(1 + \frac{gh}{c^2}\right)$$

$$\Delta t_f = 10.0 \text{ hr}\left(1 + \frac{(9.80 \text{ m/s}^2)(1.00 \times 10^9 \text{ m}}{\left(3.00 \times 10^8 \text{ m/s}\right)^2}\right)$$

$$\Delta t_f = 10.000000109 \text{ hr}$$

The time in the gravity free space is greater than the time elapsed in the gravitational field area. But as you can see, the difference is still very small.

To go to this Interactive Example click on this sentence.

We can find a further effect of the slowing down of a clock in a gravitational field by placing an excited atom in a gravitational field, and then observing a spectral line from that atom far away from the gravitational field. The speed of the light from that spectral line is, of course, given by

$$c = \lambda\nu = \frac{\lambda}{T} \tag{7.15}$$

where λ is the wavelength of the spectral line, ν is its frequency, and T is the period or time interval associated with that frequency. Hence, if the time interval $\Delta t = T$ changes, then the wavelength of that light must also change. Solving for the period or time interval from equation 7.15, we get

$$T = \frac{\lambda}{c} \tag{7.16}$$

Substituting T from 7.16 for Δt in equation 7.13, we get

$$T_f = T_g\left(1+\frac{gh}{c^2}\right) \tag{7.17}$$

$$\frac{\lambda_f}{c} = \frac{\lambda_g}{c}\left(1+\frac{gh}{c^2}\right)$$

$$\lambda_f = \lambda_g\left(1+\frac{gh}{c^2}\right) \tag{7.18}$$

where λ_g is the wavelength of the emitted spectral line in the gravitational field and λ_f is the wavelength of the observed spectral line in gravity-free space, or at least farther from where the atom is located in the gravitational field. Because the term $(1 + gh/c^2)$ is a positive number, it follows that

$$\lambda_f > \lambda_g \tag{7.19}$$

That is, *the wavelength observed in the gravity-free space is greater than the wavelength emitted from the atom in the gravitational field.* Recall from College Physics that the visible portion of the electromagnetic spectrum runs from violet light at around 380.0 nm to red light at 720.0 nm. Thus, red light is associated with longer wavelengths. *Hence, since $\lambda_f > \lambda_g$, the wavelength of the spectral line increases toward the red end of the spectrum, and the entire process of the slowing down of clocks in a gravitational field is referred to as the* ***gravitational red shift.***

A similar analysis in terms of frequency can be obtained from equation 7.14 as,

$$T_f = T_g\left(1+\frac{gh}{c^2}\right) \tag{7.14}$$

and since

$$T = \frac{1}{\nu} \tag{7.20}$$

equation 7.17 becomes

$$\frac{1}{\nu_f} = \frac{1}{\nu_g}\left(1+\frac{gh}{c^2}\right) \tag{7.21}$$

Solving for ν_f gives

$$\nu_f = \frac{\nu_g}{\left(1+\frac{gh}{c^2}\right)} = \nu_g\left(1+\frac{gh}{c^2}\right)^{-1} \tag{7.22}$$

Using the binomial theorem

$$\left(1+\frac{gh}{c^2}\right)^{-1} = \left(1-\frac{gh}{c^2}\right) \tag{7.23}$$

Substituting equation 7.23 into equation 7.23 gives

$$\nu_f = \nu_g\left(1-\frac{gh}{c^2}\right) \tag{7.24}$$

Where now the frequency observed in the gravitational-free space is less than the frequency emitted in the gravitational field because the term $\left(1-\frac{gh}{c^2}\right)$ is less than one. The change in frequency per unit frequency emitted, found from equation 7.24, is

$$\nu_f - \nu_g = -\frac{gh}{c^2}\nu_g$$

$$\frac{\nu_g - \nu_f}{\nu_g} = \frac{gh}{c^2}$$

$$\frac{\Delta\nu}{\nu_g} = \frac{gh}{c^2} \tag{7.25}$$

The gravitational red shift was confirmed on the earth by an experiment by R. V. Pound and G. A. Rebka at Harvard University in 1959 using a technique called the *Mossbauer effect*. Gamma rays were emitted from radioactive cobalt in the basement of the Jefferson Physical Laboratory at Harvard University. These gamma rays traveled 22.5 m, through holes in the floors, up to the top floor. The difference between the emitted and absorbed frequency of the gamma ray was found to agree with equation 7.25.

Example 7.2

Gravitational frequency shift. Find the change in frequency per unit frequency for a γ-ray traveling from the basement, where there is a large gravitational field, to the roof of the building, which is 22.5 m higher, where the gravitational field is weaker.

Solution

The change in frequency per unit frequency, found from equation 7.25, is

$$\frac{\Delta\nu}{\nu_g} = \frac{gh}{c^2}$$

$$= \frac{\left(9.80 \text{ m/s}^2\right)\left(22.5 \text{ m}\right)}{\left(3.00\times 10^8 \text{ m/s}\right)^2}$$

$$= 2.45 \times 10^{-15}$$

As you can see, the change in frequency per unit frequency is very small.

To go to this Interactive Example click on this sentence.

The experiment was repeated by Pound and J. L. Snider in 1965, with another confirmation. Since then the experiment has been repeated many times, giving an accuracy to the gravitational red shift to within 1%.

Further confirmation of the gravitational red shift came from an experiment by Joseph Hafele and Richard Keating. Carrying four atomic clocks, previously synchronized with a reference clock in Washington, D.C., Hafele and Keating flew around the world in 1971. *On their return they compared their airborne clocks to the clock on the ground and found the time differences associated with the time dilation effect and the gravitational effect exactly as predicted. Further tests with atomic clocks in airplanes and rockets have added to the confirmation of the gravitational red shift.*

7.3 The Gravitational Red Shift by the Theory of Quanta

The effect of acceleration and gravitation on the time recorded on a clock was derived in the last section by observing how a clock slows down in a gravitational field. The effect is of course known as the gravitational red shift. A remarkably simple derivation of this red shift can also be obtained by treating light as a particle, a photon, in a gravitational field.

Let an atom at the surface of the earth emit a photon of light of frequency ν_g. As you recall from your course in Modern Physics, a photon of light has the energy $E = h\nu$, where h is Planck's constant = $(6.625 \times 10^{-34}$ J s) and ν is the frequency of the light associated with the photon. Let this particular photon have the energy

$$E_g = h\nu_g \tag{7.26}$$

The subscript g is to remind us that this is a photon in the gravitational field. Let us assume that the light source was pointing upward so that the photon travels upward against the gravitational field of the earth until it arrives at a height y above the surface, as shown in figure 7.5. (We have used y for the height instead of h, as used previously, so as not to confuse the height with Planck's constant h.) As the photon rises it must do work against the gravitational field. When the photon arrives at the height y, its energy E_f must be diminished by the work it had to do to get there. Thus

$$E_f = E_g - W \tag{7.27}$$

Because the gravitational field is weaker at the height y than at the surface, the subscript f has been used on E to indicate that this is the energy in the weaker field or even in a field-free space. The work done by the photon in climbing to the height y is the same as the potential energy of the photon at the height y. Therefore,

$$W = \text{PE} = mgy \tag{7.28}$$

Substituting equation 7.28 and the values of the energies back into equation 7.27, gives

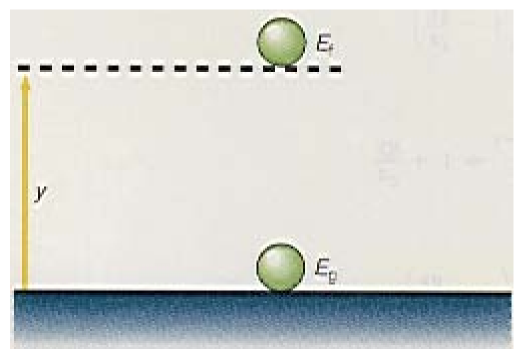

Figure 7.5 A photon in a gravitational field.

$$h\nu_f = h\nu_g - mgy \tag{7.29}$$

But the mass of the emitted photon is

$$m = \frac{E_g}{c^2} = \frac{h\nu_g}{c^2}$$

Placing this value of the mass back into equation 7.29, gives

$$h\nu_f = h\nu_g - \frac{h\nu_g}{c^2}gy$$

or

$$\nu_f = \nu_g\left(1 - \frac{gy}{c^2}\right) \tag{7.30}$$

Equation 7.30 says that the frequency of a photon associated with a spectral line that is observed away from the gravitational field is less than the frequency of the spectral line emitted by the atom *in the gravitational field itself.* Since the frequency ν is related to the wavelength λ by $c = \lambda\nu$, the observed wavelength in the field-free space λ_f is longer than the wavelength emitted by the atom in the gravitational field λ_g. Therefore, the observed wavelength is shifted toward the red end of the spectrum. Note the equation 7.30 is the same as equation 7.24 that we derived in the last section from a different point of view. *The slowing down of a clock in a gravitational field follows directly from equation* (7.30) by noting that the frequency ν is related to the period of time T by $\nu = 1/T$. Hence

$$\frac{1}{T_f} = \frac{1}{T_g}\left(1 - \frac{gy}{c^2}\right)$$

$$T_f = \frac{T_g}{1 - gy/c^2}$$

$$T_f = T_g\left(1 - \frac{gy}{c^2}\right)^{-1}$$

But by the binomial theorem,

$$\left(1 - \frac{gy}{c^2}\right)^{-1} = 1 + \frac{gy}{c^2}$$

Thus,

$$T_f = T_g\left(1 + \frac{gy}{c^2}\right) \tag{7.31}$$

Equation 7.31 is identical to equation 7.17. Finally calling the period of time T an elapsed time, Δt, we have

$$\Delta t_f = \Delta t_g\left(1 + \frac{gy}{c^2}\right) \tag{7.32}$$

which is identical to equation 7.14, which shows the slowing down of a clock in a gravitational field.

We can also show that the slowing of a clock in a gravitational field is identical to the slowing down of a clock by the Lorentz transformation. Consider the term gy/c^2 in equations 7.31 or 7.32. From the law of conservation of energy we have

$$\text{PE} = \text{KE}$$
$$mgy = \tfrac{1}{2}mv^2$$

and hence

$$gy = \frac{v^2}{2}$$

Replacing this in equation 7.32 gives

$$\Delta t_f = \Delta t_g\left(1 + \tfrac{1}{2}\frac{v^2}{c^2}\right) \tag{7.33}$$

Using the binomial theorem in reverse

$$1 - nx = (1 - x)^n$$

with $x = v^2/c^2$ and $n = -1/2$, we get

$$\left(1 + \frac{1}{2}\frac{v^2}{c^2}\right) = \left[1 - \left(-\frac{1}{2}\right)\frac{v^2}{c^2}\right] = \left(1 - \frac{v^2}{c^2}\right)^{-1/2} = \frac{1}{\sqrt{1 - v^2/c^2}} \tag{7.34}$$

Replacing this value in equation 7.32 we get

$$\Delta t_f = \frac{\Delta t_g}{\sqrt{1 - v^2 / c^2}} \tag{7.35}$$

But the time elapsed on the clock at rest in the gravitational field, Δt_g, is the same as the time elapsed Δt_o on a clock at rest in the moving coordinate system and Δt_f the time elapsed on the clock in the gravity free space is the same as the time elapsed Δt on the moving coordinate system. Hence equation 7.35 becomes

$$\Delta t = \frac{\Delta t_0}{\sqrt{1 - v^2 / c^2}} \tag{7.36}$$

But this is exactly the Lorentz transformation equation for time dilation. This is very significant here, because it is derived on the basis of the gravitational red shift by the theory of quanta and not on inertial motion, yet the results are identically the same.

Before leaving this section we should point out that for the general case, the value for the acceleration due to gravity g that we have been using in all these equations is not a constant but varies with the mass of the main object and the distance that the second object is located with respect to the main object. Recall that the acceleration due to gravity comes from Newton's second law

$$F = ma = mg$$

and Newton's law of universal gravitation

$$F = \frac{Gm_e m}{r^2}$$

where m_e is the mass of the earth, and r is the distance from the center of the earth to the location of the second mass. Equating the two values of F gives

$$mg = \frac{Gm_e m}{r^2}$$

Therefore the acceleration due to gravity is determined from

$$g = \frac{Gm_e}{r^2} \tag{7.37}$$

So for any particular problem dealing with time dilation and gravitational acceleration, you can determine g by equation 7.37 if necessary.

7.4 An Accelerated Clock and the Lorentz Transformation Equations

An extremely interesting consequence of the gravitational red shift can be formulated by invoking Einstein's principle of equivalence discussed in at the beginning of this chapter. Calling the inertial system containing gravity the K system and the accelerated frame of reference the K' system, Einstein stated, "we assume that we may just as well regard the system K as being in a space free from a gravitational field if we then regard K as uniformly accelerated." *Einstein's principle of equivalence was thus stated as: on a local scale the physical effects of a gravitational field are indistinguishable from the physical effects of an accelerated coordinate system. "Hence the systems K and K' are equivalent with respect to all physical processes, that is, the laws of nature with respect to K are in entire agreement with those with respect to K'."* Einstein then *postulated his theory of general relativity, as: The laws of physics are the same in all frames of reference.*

Since a clock slows down in a gravitational field, equation 7.32, using the equivalence principle, an accelerated clock should also slow down. Replacing the acceleration due to gravity g by the acceleration of the clock a, equation 7.32 becomes

$$\Delta t_f = \Delta t_a \left(1 + \frac{ay}{c^2}\right) \tag{7.38}$$

Note that the subscript g on Δt_g in equation 7.32 has now been replaced by the subscript a, giving Δt_a, to indicate that this is the time elapsed on the accelerated clock. Notice from equation 7.38 that

$$\Delta t_f > \Delta t_a$$

indicating that time slows down on the accelerated clock. *That is, an accelerated clock runs more slowly than a clock at rest.* In chapter 1 we saw, using the Lorentz transformation equations, that a clock at rest in a moving coordinate system slows down, and called the result the Lorentz time dilation. However, nothing was said at that time to show how the coordinate system attained its velocity. *Except for zero velocity, all bodies or reference systems must be accelerated to attain a velocity. Thus, there should be a relation between the Lorentz time dilation and the slowing down of an accelerated clock.* Let us change our notation slightly and call Δt_f the time Δt in a stationary coordinate system and Δt_a the time interval on a clock that is at rest in a coordinate system that is accelerating to the velocity v. Assuming that the acceleration is constant, we can use the kinematic equation

$$v^2 = v_0^2 + 2ay$$

Further assuming that the initial velocity v_0 is equal to zero and solving for the quantity ay we obtain

$$ay = \frac{v^2}{2} \tag{7.39}$$

Substituting equation 7.39 into equation 7.38, yields

$$\Delta t = \Delta t_a \left(1 + \frac{v^2}{2c^2}\right) \tag{7.40}$$

Using the binomial theorem in reverse

$$1 - nx = (1 - x)^n$$

with $x = v^2/c^2$ and $n = -1/2$, we get

$$\left(1 + \frac{v^2}{2c^2}\right) = \left[1 - \left(-\frac{1}{2}\right)\frac{v^2}{c^2}\right] = \left(1 - \frac{v^2}{c^2}\right)^{-1/2} = \frac{1}{\sqrt{1 - v^2/c^2}} \tag{7.41}$$

Equation 7.40 becomes

$$\Delta t = \frac{\Delta t_a}{\sqrt{1 - v^2/c^2}} \tag{7.42}$$

But this is exactly the time dilation formula, equation 2.24, found by the Lorentz transformation. *Thus the Lorentz time dilation is a special case of the slowing down of an accelerated clock.* This is a very important result. *Therefore, it is more reasonable to take the slowing down of a clock in a gravitational field, and thus by the principle of equivalence, the slowing down of an accelerated clock as the more basic physical principle. The Lorentz transformation for time dilation can then be derived as a special case of a clock that is accelerated from rest to the velocity v.*

Example 7.3

Comparison of time dilation by Lorentz's time dilation equation and the slowing down of an accelerated clock. (a) A rocket ship is moving at a speed of 1610 km/s = 1.61×10^6 m/s. A clock on the rocket ship ticks off a time interval of 1.00 hr. Using the Lorentz time dilation equation, find the time elapsed on the clock on earth. (b) To arrive at the speed of 1610 km/s, the rocket ship accelerates at 9.80 m/s^2. How far must the rocket travel to arrive at this velocity. A clock on the rocket ticks off a time interval of 1.00 hr. Find the time recorded on the earth clock using the equation for time dilation of an accelerated clock. Compare the results of the Lorentz time dilation equation and the results of time dilation for an accelerated clock.

Solution

a. The time elapsed on the clock on earth is found by the Lorentz time dilation formula equation 2.24 as

$$\Delta t = \frac{\Delta t_0}{\sqrt{1 - v^2/c^2}}$$
$$= \frac{1.00 \text{ hr}}{\sqrt{1 - (1.61 \times 10^6 \text{ m/s})^2/(3.00 \times 10^8 \text{ m/s})^2}}$$
$$= 1.0000144 \text{ hr}$$

As we would expect with time dilation, this relatively large speed of 1610 km/s = 3,600,000 mph, the difference in the clocks is very small. That is, a difference of 0.0000144 hr = 0.051843 seconds between the moving clock and the stationary clock in a period of time of 1.00 hours.

b. To find the difference in time using the concept of an accelerated clock we start with equation 7.38

$$\Delta t = \Delta t_a \left(1 + \frac{ay}{c^2}\right)$$

Now in order to compare the accelerated clock with the Lorentz clock we have to know the acceleration of the rocket that gave it its speed, and the distance the rocket moved during this acceleration so it could attain its speed. We can obtain this simply from the kinematic equation

$$v^2 = v_0^2 + 2ay \tag{7.43}$$

We assume the rocket starts from rest so $v_0 = 0$, and equation 7.43 becomes

$$v^2 = 2ay \tag{7.44}$$

Since the velocity v of the Lorentz rocket must be the same as the velocity v of the accelerated rocket we solve equation 7.44 for ay as

$$ay = v^2/2 \tag{7.45}$$

Now we have to pick a reasonable value for the acceleration a, and then we can solve for the distance the rocket has to move to acquire the velocity v. That is,

$$y = \frac{v^2}{2a} \tag{7.46}$$

We will assume a value of a of 9.80 m/s^2. This acceleration is the same as the acceleration due to gravity and an astronaut could easily take it for the long time necessary for the rocket to acquire the velocity desired. We can change the acceleration value later to some other value if we wish. The distance y that the rocket must move to attain this velocity is found from equation 7.46 as

$$y = \frac{v^2}{2a} = \frac{\left(1.61\times10^6 \text{ m/s}\right)^2}{2\times9.8 \text{ m/s}^2}$$
$$y = 1.32 \times 10^{11} \text{ m}$$

The value of ay that we need for equation 7.46 is found as

$$ay = (9.80 \text{ m/s}^2)(\ 1.32 \times 10^{11} \text{ m}) = 1.30 \times 10^{12} \text{ m}^2\text{s}^2$$

We can now determine the time Δt_f from equation 7.38 as

$$\Delta t_f = \Delta t_a\left(1+\frac{ay}{c^2}\right)$$

$$\Delta t_f = 1.00 \text{ hr}\left(1+\frac{(1.30\times10^{12} \text{ m}^2\text{s}^2}{\left(3.00\times10^8 \text{ m/s}\right)^2}\right)$$

$$\Delta t_f = 1.0000144 \text{ hr}$$

The difference in the value of Δt found with the Lorentz time dilation equation and the time Δt found with the accelerated clock time dilation equation is
Lorentz time dilation = 1.0000144E+00 hr
Accelerated time dilation = 1.0000144E+00 hr
And is essentially zero. Hence the accelerated time dilation is the same as the Lorentz time dilation. Therefore we can assume that a Lorentz clock is equivalent to a gravitational clock and an accelerated clock.

To go to this Interactive Example click on this sentence.

Just as the slowing down of a clock in a gravitational field can be attributed to the warping of spacetime by the mass, it is reasonable to assume that the slowing down of the accelerated clock can also be thought of as the warping of spacetime by the increased mass, due to the increase in the velocity of the accelerating mass.

The Lorentz length contraction can also be derived from this model by the following considerations. Consider the emission of a light wave in a gravitational field. We will designate the wavelength of the emitted light by λ_g, and the period of the light by T_g. The velocity of the light emitted in the gravitational field is given by

$$c_g = \frac{\lambda_g}{T_g} \tag{7.47}$$

We will designate the velocity of light in a region far removed from the gravitational field as c_f for the velocity in a field-**f**ree region. The velocity of light in the field-free region is given by

$$c_f = \frac{\lambda_f}{T_f} \tag{7.48}$$

where λ_f is the wavelength of light, and T_f is the period of the light as observed in the field-free region. If the gravitating mass is not too large, then we can make the reasonable assumption that the velocity of light is the same in the gravitational field region and the field-free region, that is, $c_g = c_f$. We can then equate equation 7.47 to equation 7.48 to obtain

$$\frac{\lambda_g}{T_g} = \frac{\lambda_f}{T_f}$$

Solving for the wavelength of light in the field-free region, we get

$$\lambda_f = \frac{T_f}{T_g}\lambda_g$$

Substituting the value of T_f from equation 7.31 into this we get

$$\lambda_f = \frac{T_g}{T_g}\left(1 + \frac{gy}{c^2}\right)\lambda_g$$

$$\lambda_f = \left(1 + \frac{gy}{c^2}\right)\lambda_g \tag{7.49}$$

Equation 7.49 gives the wavelength of light λ_f in the gravitational-field-free region. By the principle of equivalence, the wavelength of light emitted from an accelerated observer, accelerating with the constant acceleration a through a distance y is obtained from equation 7.49 as

$$\lambda_0 = \left(1 + \frac{ay}{c^2}\right)\lambda_a \tag{7.50}$$

where λ_0 is the wavelength of light that is observed in the region that is not accelerating, that is, the wavelength observed by an observer who is at rest. This result can be related to the velocity v that the accelerated observer attained during the constant acceleration by the kinematic equation

$$v^2 = v_0^2 + 2ay$$

Further assuming that the initial velocity v_0 is equal to zero and solving for the quantity ay we obtain

$$ay = \frac{v^2}{2}$$

Substituting this result into equation 7.50 we obtain

$$\lambda_0 = \left(1 + \frac{v^2}{2c^2}\right)\lambda_a \tag{7.51}$$

Using the binomial theorem in reverse as in equation 7.41,

$$\left(1 + \frac{v^2}{2c^2}\right) = \frac{1}{\sqrt{1 - v^2/c^2}}$$

equation 7.51 becomes

$$\lambda_0 = \frac{\lambda_a}{\sqrt{1 - v^2/c^2}}$$

Solving for λ_a we get

$$\lambda_a = \lambda_0 \sqrt{1 - v^2/c^2} \tag{7.52}$$

But λ is a length, in particular λ_a is a length that is observed by the observer who has accelerated from 0 up to the velocity v and is usually referred to as L, whereas λ_0 is a length that is observed by an observer who is at rest relative to the measurement and is usually referred to as L_0. Hence, we can write equation 7.52 as

$$L = L_0 \sqrt{1 - v^2/c^2} \tag{7.53}$$

But equation 7.53 is the Lorentz contraction of special relativity. *Hence, the Lorentz contraction is a special case of contraction of a length in a gravitational field, and by the principle of equivalence, a rod L_0 that is accelerated to the velocity v is contracted to the length L.* (That is, if a rod of length L_0 is at rest in a stationary spaceship, and the spaceship accelerates up to the velocity v, then the observer on the earth would observe the contracted length L.) *Hence, the acceleration of the rod is the basic physical principle underlying the length contraction. It is physically the acceleration that gives the object its velocity that is used in the original Lorentz equations.*

Example 7.4

Comparison of length contraction by Lorentz-Fitzgerald Contraction and the length contraction equation caused by an acceleration.

a. *Using the Lorentz-Fitzgerald contraction equation*. What is the length of a meter stick at rest on earth, when it is observed by an astronaut moving at a speed of $v = 1.61 \times 10^6$ m/s = 3,600,000 miles/hr.

b. *Length contraction for an accelerated rod.* To arrive at the speed of 1.61×10^6 m/s, the rocket ship accelerates at 9.80 m/s^2. How far must the rocket travel to arrive at this velocity. Find the length contraction of the meter stick caused by this acceleration.

c. Compare the results of the Lorentz-Fitzgerald length contraction and the accelerated length contraction.

Solution

a. The length of the meter stick is measured in the frame where it is at rest, and is called its proper length and is denoted by L_0. The length of the meter stick as measured by the astronaut in the moving frame is denoted by L. The Lorentz-Fitzgerald contraction of a meter stick for a speed of 1.61×10^6 m/s is found by the Lorentz equation as

$$\begin{aligned} L &= L_0 \sqrt{1 - \frac{v^2}{c^2}} \\ &= (1.00 \text{ m})\sqrt{1 - \frac{(1.610 \times 10^6 \text{ m/s})^2}{(3.00 \times 10^8 \text{ m/s})^2}} \\ &= (1.00 \text{ m})\sqrt{0.99997} \\ &= 0.9999856 \text{ m} \end{aligned}$$

As we would expect with length contraction, this relatively large speed of 1.61×10^6 m/s, gives a very small difference in the final length.

b. A meter stick is at rest on the earth. A rocket ship now accelerates away from the earth. What is the length contraction caused by the acceleration of the rocket ship away from the earth where the rod is at rest? To find the difference in the length using the concept of an acceleration we start with equation 7.50 as

$$\lambda_0 = \left(1 + \frac{ay}{c^2}\right)\lambda_a \tag{7.50}$$

but change the length designation from λ to *L,* so that equation 7.50 can be rewritten as

$$L_0 = \left(1 + \frac{ay}{c^2}\right)L_a \tag{7.54}$$

Where L_a is the length that is observed by the accelerated observer who has accelerated from 0 up to the velocity v, whereas L_0 is the length that is observed by an observer who is at rest on the earth next to the rod. For this problem the rod is at rest on the earth (L_0), and the rod observed by the astronaut is L_a. Hence we have to rearrange equation 7.54 into the form

$$L_a = \frac{L_0}{\left(1 + \frac{ay}{c^2}\right)} \tag{7.55}$$

Now in order to solve this equation we have to know the acceleration a of the rocket that gave it its speed, and the distance y the rocket moved during this acceleration so it could attain its speed. We can obtain this simply from the kinematic equation as we did in Example 7.3 as

$$v^2 = v_0^2 + 2ay \tag{7.43}$$

We assume the rocket starts from rest so $v_0 = 0$, and equation 7.43 becomes

$$v^2 = 2ay \tag{7.44}$$

Since the velocity v of the Lorentz rocket must be the same as the velocity v of the accelerated rocket we solve equation 7.44 for ay as

$$ay = v^2/2 \tag{7.45}$$

Now we have to pick a reasonable value for the acceleration a, and then we can solve for the distance the rocket has to move to acquire the velocity v. That is,

$$y = \frac{v^2}{2a} \tag{7.46}$$

We will assume a value of a of 9.80 m/s². This acceleration is the same as the acceleration due to gravity and an astronaut could easily take it for the long time necessary for the rocket to acquire the velocity desired. We can change the acceleration value later to some other value if we wish. The distance y that the rocket must move to attain this velocity is found from equation 7.46 as

$$y = \frac{v^2}{2a} = \frac{(1.61 \times 10^6 \text{ m/s})^2}{2 \times 9.8 \text{ m/s}^2}$$

$$y = 1.32 \times 10^{11} \text{ m}$$

The value of ay that we need for equation 7.46 is then found as

$$ay = (9.80 \text{ m/s}^2)(1.32 \times 10^{11} \text{ m}) = 1.30 \times 10^{12} \text{ m}^2\text{s}^2$$

We can now determine the length L_o from equation 7.55 as

$$L_a = \frac{L_0}{\left(1 + \dfrac{ay}{c^2}\right)}$$

$$L_a = \frac{1.00 \text{ m}}{\left(1 + \dfrac{(1.30 \times 10^{12} \text{ m}^2\text{s}^2}{\left(3.00 \times 10^8 \text{ m/s}\right)^2}\right)}$$

$$L_a = 0.9999856 \text{ m}$$

c. Hence the results obtained by the Lorentz-Fitzgerald length contraction and the accelerated length contraction are the same. That is, the length observed by the accelerating astronaut is the same length that we obtained by the Lorentz- Fitzgerald contraction.

The important thing here to observe is that we get the same results by using the Lorentz-Fitzgerald contraction as we do by considering a contraction as the result of gravity or an acceleration.

To go to this Interactive Example click on this sentence.

Thus, both the time dilation and length contraction of special relativity should be attributed to the warping of spacetime by the accelerating mass.

The warping of spacetime by the accelerating mass can be likened to the Doppler effect for sound. Recall that if a source of a sound wave is stationary, the sound wave propagates outward in concentric circles. When the sound source is moving, the waves are no longer circular but tend to bunch up in advance of the moving source. Since light does not require a medium for propagation, the Doppler effect for light is very much different. However, we can speculate that the warping of spacetime by the accelerating mass is comparable to the bunching up of sound waves in air. In fact, if we return to equation 7.30, for the gravitational red shift, and again, using the principle of equivalence, let $g = a$, and dropping the subscript f, this becomes

$$\nu = \nu_a \left(1 - \frac{ay}{c^2}\right) \tag{7.56}$$

Using the kinematic equation for constant acceleration, $ay = v^2/2$. Hence equation 7.56 becomes

$$\nu = \nu_a \left(1 - \frac{v^2}{2c^2}\right) \tag{7.57}$$

Again using the binomial theorem

$$\left(1 - \frac{v^2}{2c^2}\right) = \sqrt{1 - v^2/c^2}$$

Equation 7.56 becomes

$$\nu = \nu_a \sqrt{1 - v^2/c^2} \tag{7.58}$$

Equation 7.58 is called the transverse Doppler effect. It is a strictly relativistic result and has no counterpart in classical physics. The frequency ν_a is the frequency of light emitted by a light source that is at rest in a coordinate system that is accelerating past a stationary observer, whereas ν is the frequency of light observed by the stationary observer. *Notice that the transverse Doppler effect comes directly from the gravitational red shift by using the equivalence principle.* Thus the transverse Doppler effect should be looked on as a frequency shift caused by accelerating a light source to the velocity v.

It is important to notice here that this entire derivation started with the gravitational red shift by the theory of the quanta, then the equivalence principle was used to obtain the results for an accelerating system. The Lorentz time dilation and length contraction came out of this derivation as a special case. Thus, the Lorentz equations should be thought of as kinematic equations, whereas the gravitational and acceleration results should be thought of as a dynamical result.

Time dilation and length contraction have always been thought of as only depending upon the velocity of the moving body and not upon its acceleration. As an example, in Wolfgang Rindler's book *Essential Relativity*,[3] he quotes results of experiments at the CERN laboratory where muons were accelerated. He states "that accelerations up to 10^{19} g (!) do not contribute to the muon time dilation." The only time dilation that could be found came from the Lorentz time dilation formula. They could not find the effect of the acceleration because they had it all the time. The Lorentz time dilation formula itself is a result of the acceleration. Remember, it is impossible to get a nonzero velocity without an acceleration.

Summary of Basic Concepts

Equivalence principle
On a local scale, the physical effects of a gravitational field are indistinguishable from the physical effects of an accelerated coordinate system. Hence, an accelerated frame of reference is equivalent to an inertial frame of reference in which gravity is present, and an inertial frame is equivalent to an accelerated frame in which gravity is absent.

The general theory of relativity
The laws of physics are the same in all frames of reference (note that there is no statement about the constancy of the velocity of light as in the special theory of relativity).

Gravitational red shift
Time elapsed on a clock in a gravitational field is less than the time elapsed on a clock in a gravity-free space. This effect of the slowing down of a clock in a gravitational field is manifested by observing a spectral line from an excited atom in a gravitational field. The wavelength of the spectral line of that atom is shifted toward the red end of the electromagnetic spectrum.

Photon
A small bundle of electromagnetic energy that acts as a particle of light. The photon has zero rest mass and its energy and momentum are determined in terms of the wavelength and frequency of the light wave.

An Accelerated Clock and the Lorentz Transformation Equations
The Lorentz time dilation is a special case of the slowing down of an accelerated clock. Therefore, it is more reasonable to take the slowing down of a clock in a gravitational field, and thus by the principle of equivalence, the slowing down of an accelerated clock as the more basic physical principle. The Lorentz transformation for time dilation can then be derived as a special case of a clock that is accelerated from rest to the velocity v.

The Lorentz contraction is a special case of contraction of a length in a gravitational field, and by the principle of equivalence, a rod L_0 that is accelerated to the velocity v is contracted to the length L. Hence, the acceleration of the rod is the basic physical principle underlying the length contraction. It is physically the acceleration that gives the object its velocity that is used in the original Lorentz equations.

[3] Springer-Verlag, New York, 1979, Revised 2nd edition, p. 44.

Summary of Important Equations

Slowing down of a clock in a gravitational field
$$\Delta t_f = \Delta t_g \left(1 + \frac{gh}{c^2}\right) \quad (7.14)$$

Gravitational red shift of wavelength
$$\lambda_f = \lambda_g \left(1 + \frac{gh}{c^2}\right) \quad (7.18)$$

Gravitational red shift of frequency
$$\nu_f = \nu_g \left(1 - \frac{gh}{c^2}\right) \quad (7.24)$$

Change in frequency per unit frequency
$$\frac{\Delta \nu}{\nu_g} = \frac{gh}{c^2} \quad (7.25)$$

Slowing down of a clock in a gravitationfield
$$T_f = T_g \left(1 + \frac{gy}{c^2}\right) \quad (7.31)$$

Slowing down of a clock in a gravitational field
$$\Delta t_f = \Delta t_g \left(1 + \frac{gy}{c^2}\right) \quad (7.32)$$

Slowing down of an accelerated clock
$$\Delta t_f = \Delta t_a \left(1 + \frac{ay}{c^2}\right) \quad (7.38)$$

$$\Delta t = \frac{\Delta t_a}{\sqrt{1 - v^2 / c^2}} \quad (7.42)$$

Length contraction in a gravitational field
$$\lambda_f = \left(1 + \frac{gy}{c^2}\right)\lambda_g \quad (7.49)$$

Length contraction in an acceleration
$$\lambda_0 = \left(1 + \frac{ay}{c^2}\right)\lambda_a \quad (7.50)$$

$$L = L_0\sqrt{1 - v^2 / c^2} \quad (7.53)$$

$$L_a = \frac{L_0}{\left(1 + \frac{ay}{c^2}\right)} \quad (7.55)$$

Questions for Chapter 7

1. When light shines on a surface, is momentum transferred to the surface?
2. Could photons be used to power a spaceship through interplanetary space?
3. Which photon, red, green, or blue, carries the most (a) energy and (b) momentum?
4. Ultraviolet light has a higher frequency than infrared light. What does this say about the energy of each type of light?

*5. Why could red light be used in a photographic dark room when developing pictures, but a blue or white light could not?

Problems for Chapter 7

1. A photon has an energy of 5.00 eV. What is its frequency and wavelength?
2. Find the mass of a photon of light of 500.0-nm wavelength.
3. Find the momentum of a photon of light of 500.0-nm wavelength.
4. Find the wavelength of a photon whose energy is 500 MeV.
5. What is the energy of a 650 nm photon?
6. Find the energy of a photon of light of 400.0-nm wavelength.
7. A radio station broadcasts at 92.4 MHz. What is the energy of a photon of this electromagnetic wave?

7.3 The Gravitational Red Shift

8. One twin lives on the ground floor of a very tall apartment building, whereas the second twin lives 200 ft above the ground floor. What is the difference in their age after 50 years?

9. The lifetime of a subatomic particle is 6.25×10^{-7} s on the earth's surface. Find its lifetime at a height of 500 km above the earth's surface.

10. An atom on the surface of Jupiter (g = 23.1 m/s^2) emits a ray of light of wavelength 528.0 nm. What wavelength would be observed at a height of 10,000 m above the surface of Jupiter?

Additional Problems

*11. Using the principle of equivalence, show that the difference in time between a clock at rest and an accelerated clock should be given by

$$\Delta t_R = \Delta t_A \left(1 + \frac{ax}{c^2}\right)$$

where Δt_R is the time elapsed on a clock at rest, Δt_A is the time elapsed on the accelerated clock, a is the acceleration of the clock, and x is the distance that the clock moves during the acceleration.

*12. A particle is moving in a circle of 1.00-m radius and undergoes a centripetal acceleration of 9.80 m/s^2. Using the results of problem 11, determine how many revolutions the particle must go through in order to show a 10% variation in time.

13. The pendulum of a grandfather clock has a period of 0.500 s on the surface of the earth. Find its period at an altitude of 200 km. *Hint:* Note that the change in the period is due to two effects. The acceleration due to gravity is smaller at this height even in classical physics, since

$$g = \frac{GM}{(R+h)^2}$$

To solve this problem, use the fact that the average acceleration is

$$g = \frac{GM}{R(R+h)}$$

and assume that

$$\Delta t_f = \Delta t_g \left(1 + \frac{gh}{c^2}\right)$$

14. Compute the fractional change in frequency of a spectral line that occurs between atomic emission on the earth's surface and that at a height of 325 km.

Interactive Tutorials

15. *Properties of a photon.* A photon of light has a wavelength λ = 420.0 nm, find (a) the frequency ν of the photon, (b) the energy E of the photon, (c) the mass m of the photon, and (d) the momentum p of the photon.

16. *Gravitational red shift.* An atom on the surface of the earth emits a ray of light of wavelength λ_g = 528.0 nm, straight upward. (a) What wavelength λ_f would be observed at a height y = 10,000 m? (b) What frequency ν_f would be observed at this height? (c) What change in time would this correspond to?

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