Advanced Higher Physics

Electromagnetism

Notes

Name…………………………………………………………….

Key Area Notes, Examples and Questions

Other Notes

Key Area: Fields

Success Criteria

1.1 I can define electric field strength.

1.2 I can draw electric field patterns around single charges, a system of two charges and a uniform electric field.

1.3 I can solve problems involving electric fields and the forces produced on charged particles.

1.4 I can define electric potential.

1.5 I can state that the energy required to move a charge between two points in an electric field is independent of the path taken.

1.6 I can solve problems involving electric potential.

1.7 I can solve problems on the motion and energy of charged particles in uniform electric fields.

1.8 I know the definition of the Electron Volt (eV) and can convert between electron volts and joules.

1.9 I explain the magnetic effect called ferromagnetism which occurs in certain metals.

1.10 I can draw magnetic field line patterns.

1.11 I can solve problems involving the magnetic induction formed around a current carrying wire.

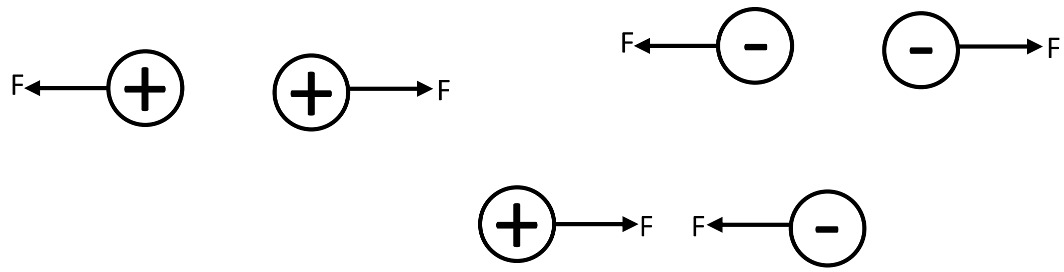
1.12 I can solve problems involving charged particles in magnetic fields in terms of their; mass, velocity, charge, radius of their path and the magnetic induction of the magnetic field.

1.13 I can solve problems involving the forces acting on a current carrying wire in a magnetic field.

1.14 I can state comparisons between nuclear, electromagnetic and gravitational forces in terms of relative magnitude and range.

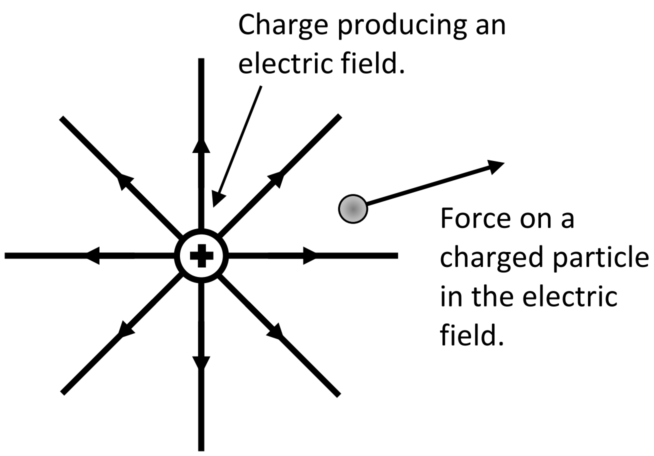
1.1 I can define electric field strength and know its relationship to the force produced on a charged particle.

Charged particles exert forces on other charged particles.



A charged particle produces an electric field which occupies the surrounding space. The electric field exerts a force on other charged particles.

Electric field strength, E, at any point is the force applied per unit positive charge at that point. Electric Field Strength is a vector quantity with units of Newtons per Coulomb (NC^{-1}). or in Volts per Metre (Vm^{-1})



Electric Fields around Point charges
The relationship which gives the electric field produced around a point charge is

$$E = \frac{Q}{4\pi\epsilon_0 r^2}$$

Electric Field Strength (NC^{-1} or Vm^{-1}) → E

Charge of the particle producing the electric field (C). → Q

Permittivity of free space (Fm^{-1}) → ϵ_0

Distance from charge Q (m) → r

Note: Permittivity of Free Space
ϵ_0 is a constant which determines how easily the electric field can permeate a vacuum (free space). The value permittivity for air is very close to ϵ_0 and so can be used for all calculations you will meet. When the electric field is permeating other materials ϵ usually has a much higher value.

Forces Produced by Electric Fields

The relationship between the force produced on a charged particle in an electric field, charge and electric field strength is given by the relationship

$$F = QE$$

Force (N)

Electric Field Strength (NC^{-1} or Vm^{-1})

Charge on the particle in the electric field (C)

Forces - Point Charges

Both the relationships $F = QE$ and $\mathrm{E} = \frac{\mathrm{Q}}{4\pi\epsilon_0 r^2}$ can be combined to give Coulomb's Law

Note that Q in each of these two relationships refer to different charges.

$$F = \frac{Q_1 Q_2}{4\pi\epsilon_0 r^2}$$

Charge on the particle in the electric field (C)

Charge of the particle producing the electric field (C).

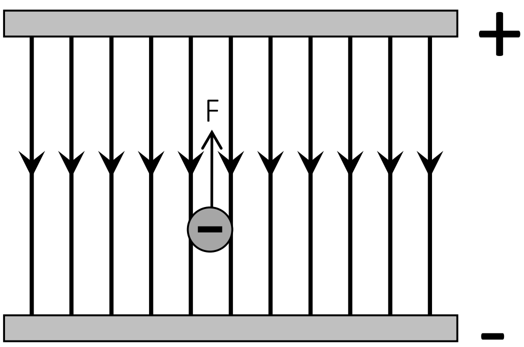
Electric Force (N)

Distance of Q_1 from Q_2 (m)

Permittivity of free space (Fm^{-1})

> **Note**
> Like Newton's Law of Gravitation, Coulomb's Law is an inverse square law.

Forces - Uniform Electric fields



From the Higher Physics course

$E_w = Fd$ and $E_W = QV$

Combining these gives

$$F = \frac{QV}{d}$$

Equating this to the definition of an electric field, $F = QE$ gives

$$\frac{QV}{d} = QE$$

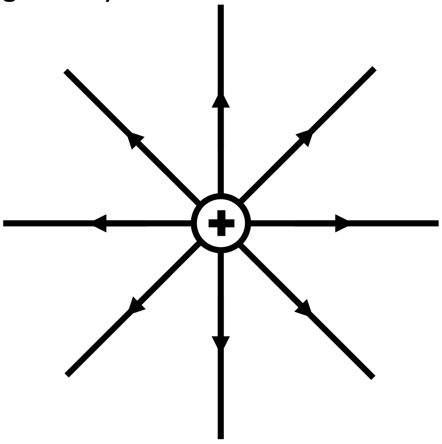
Which simplifies to

$$V = Ed$$

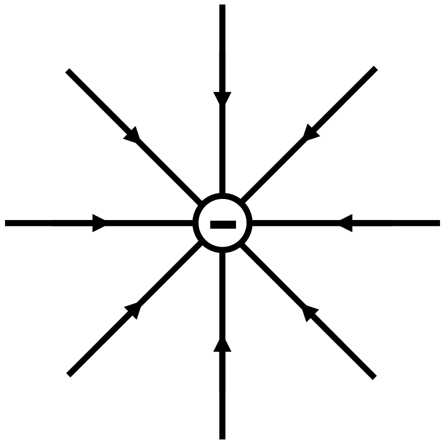
When performing calculations with uniform fields between charged plates $V = Ed$ is usually used as the voltage across the plates is usually known.

1.2 I can draw electric field patterns around single charges, a system of two charges and a uniform electric field.

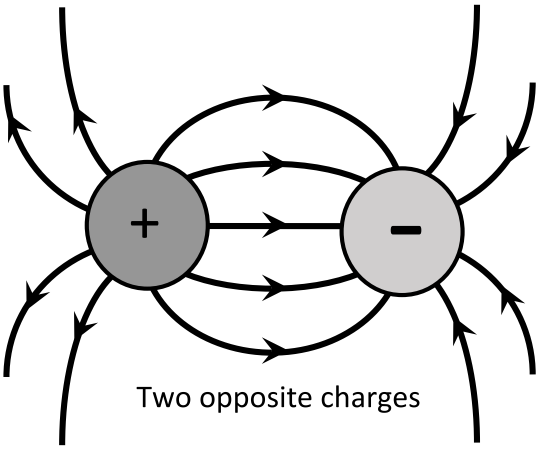
Also see Higher Physics Particle and Waves Notes section 2.2.



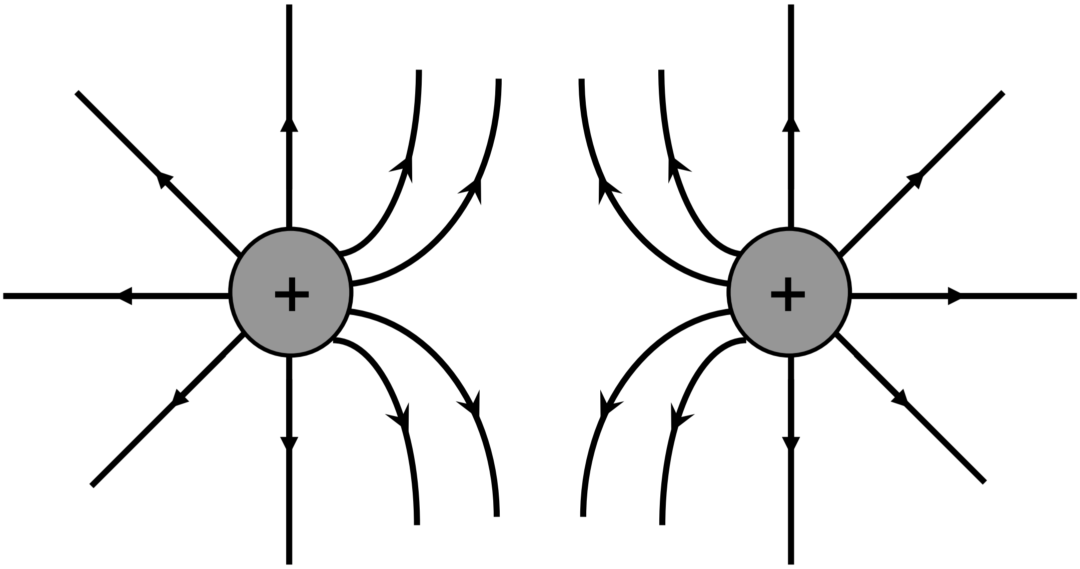
Single positive charge



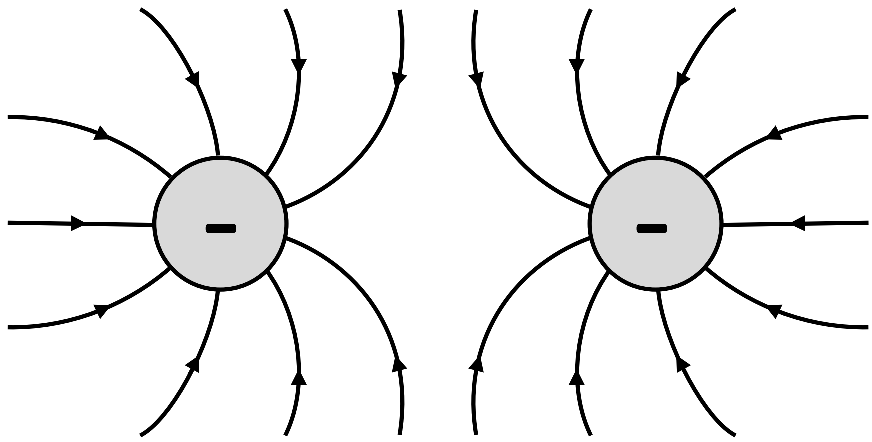
Single negative charge



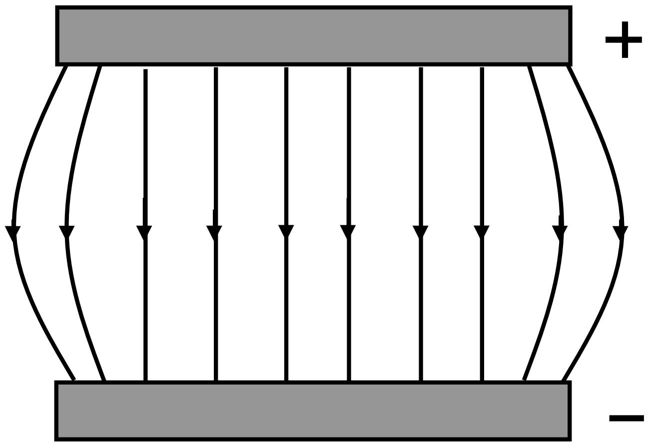
Two opposite charges



Two positive charges



Two negative charges



Parallel plates produce a uniform field between the plates.
The curvature of the field lines at the edges of the plates is small and frequently not shown in diagrams.

Parallel Charged Plates

1.3 I can solve problems involving electric fields and the forces produced on charged particles.

Example 1

Find the magnitude of electric field strength at the Bohr radius $(5.29 \times 10^{-11}\text{m})$ of a hydrogen atom.

Solution 1

$r = 5.29 \times 10^{-11}\text{m}$

$\epsilon_0 = 8.85 \times 10^{-12}\text{Fm}^{-1}$

$Q = 1.6 \times 10^{-19}\text{C}$

$$E = \frac{Q}{4\pi\epsilon_0 r^2}$$

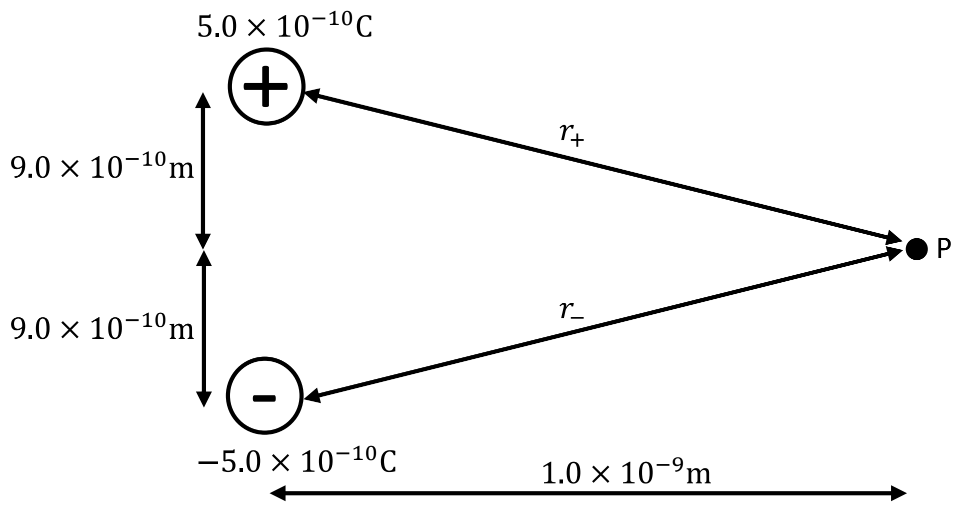
$$E = \frac{1.6 \times 10^{-19}}{4\pi \times 8.85 \times 10^{-12} \times 5.29 \times 10^{-11}}$$

$$E = 27\text{NC}^{-1}$$

Electromagnetism problem book pages 6 and 7, questions 1 to 7.

Example 2 - Electric Dipole

The diagram below shows two charged particles in an arrangement called an electric dipole. Find the electric field at point P.



Solution 2 – Electric Dipole

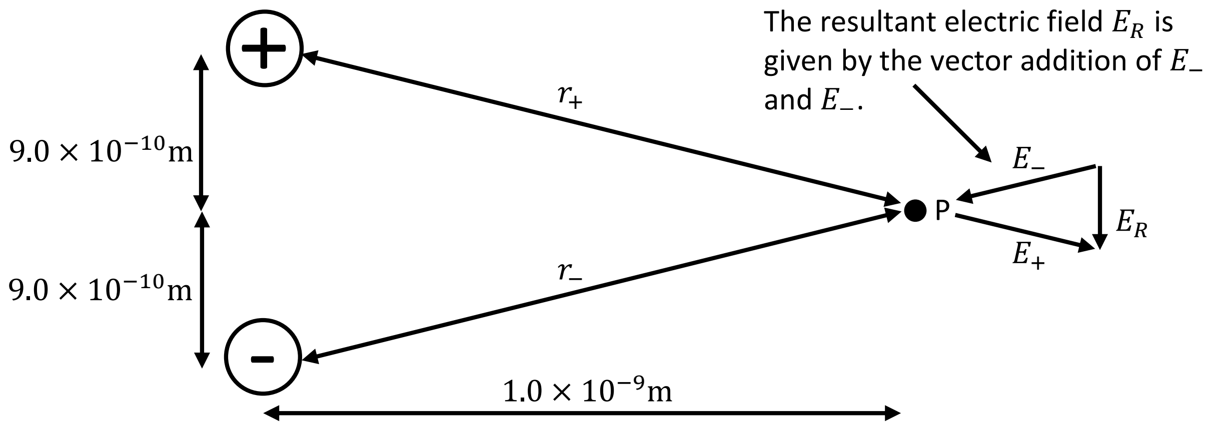
$$r_+ = r_- = \sqrt{(9.0 \times 10^{-10})^2 + (1.0 \times 10^{-9})^2} = 1.345 \times 10^{-9}\text{m}$$

Electric field due to the positive charge

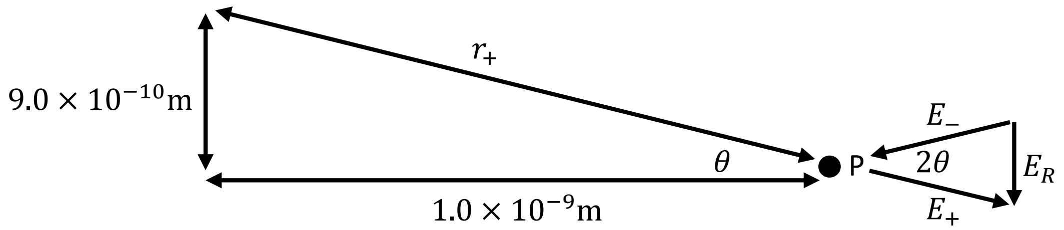
$$E_+ = \frac{Q_+}{4\pi\epsilon_0 r_+^2} = \frac{5.0 \times 10^{-10}}{4 \times \pi \times 8.85 \times 10^{-12} \times (1.345 \times 10^{-9})^2} = 2.485 \times 10^{18}\text{NC}^{-1}$$

Electric field due to the negative charge

$$E_{+} = \frac{Q_{+}}{4\pi\epsilon_0 r_{-}^2} = \frac{-5.0 \times 10^{-10}}{4 \times \pi \times 8.85 \times 10^{-12} \times (1.345 \times 10^{-9})^2} = -2.485 \times 10^{18}\mathrm{NC^{-1}}$$



To find the resultant electric field first find the angle θ.



$$\tan\theta = \frac{9.0 \times 10^{-10}}{1.0 \times 10^{-9}}$$

$$\theta = \tan^{-1}\left(\frac{9.0 \times 10^{-10}}{1.0 \times 10^{-9}}\right) = 41.99°$$

Using the cosine rule

$$E_R = \sqrt{|E_-|^2 + |E_+|^2 - 2|E_-||E_+|\cos 2\theta}$$

As $\quad |E_-|^2 = |E_+|^2 = |E|^2$ and $|E_-||E_+| = |E|^2$

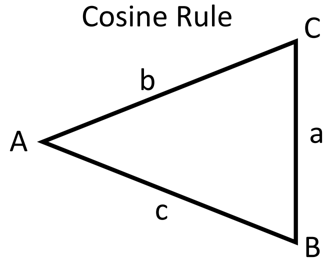
$$E_R = \sqrt{2|E|^2 - 2|E|^2\cos 2\theta}$$

$$E_R = |E|\sqrt{2(1 - \cos 2\theta)}$$

$$E_R = 2.485 \times 10^{18} \times \sqrt{2(1 - \cos(2 \times 41.99))}$$

$E_R = 3.3 \times 10^{18}\mathrm{NC^{-1}}$ vertically downward

Cosine Rule



$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$a = \sqrt{b^2 + c^2 - 2bc\cos A}$$

Electromagnetism problem book page 9, question 13.

Example 3 – Coulomb's Law

Helium $^4_2\mathrm{He}$ consists of two protons and two neutrons. Calculate the ratio, $\frac{\text{Electrostaic Force}}{\text{Gravitational Force}}$, for the two protons when they are $10^{-15}\mathrm{m}$ apart.

Solution 3 – Coulomb's Law

Electric Force

$$F_E = \frac{Q_1 Q_2}{4\pi\epsilon_0 r^2} = \frac{1.6 \times 10^{-19} \times 1.6 \times 10^{-19}}{4\pi \times 8.85 \times 10^{-12} \times (10^{-15})^2} = 230\mathrm{N}$$

Gravitational Force

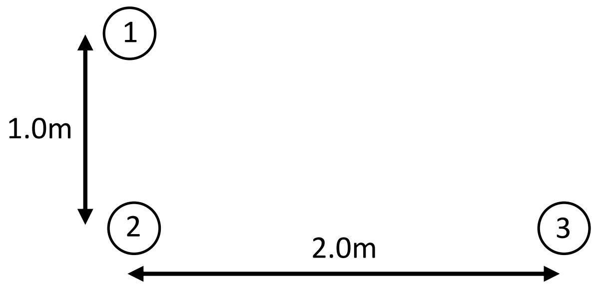
$$F_G = \frac{G m_1 m_2}{r^2} = \frac{6.67 \times 10^{-11} \times 1.673 \times 10^{-27} \times 1.673 \times 10^{-27}}{(10^{-15})^2} = 1.87 \times 10^{-34}\mathrm{N}$$

$$\frac{F_E}{F_G} = \frac{230}{1.87 \times 10^{-34}} = 1.2 \times 10^{36}$$

Note how the electric force is much larger than the gravitational force.

Example 4 – Coulomb's Law

Three charged objects are fixed in position. Each has a charge of $+120\mu\mathrm{C}$. Calculate the magnitude of the force on charge 2.



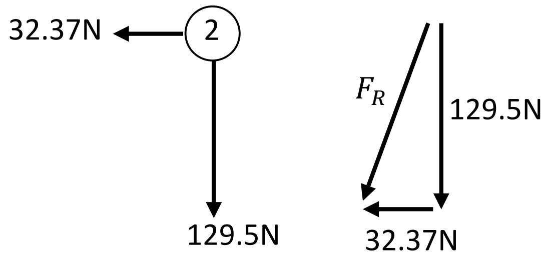
Solution 4 Coulomb's Law

Force on charge 2 due to charge 3

$$F_{23} = \frac{Q_2 Q_3}{4\pi\epsilon_0 r^2} = \frac{120 \times 10^{-6} \times 120 \times 10^{-6}}{4\pi \times 8.85 \times 10^{-12} \times 2.0^2} = 32.37\mathrm{N}$$

Force on charge 2 due to charge 1

$$F_{21} = \frac{Q_2 Q_3}{4\pi\epsilon_0 r^2} = \frac{120 \times 10^{-6} \times 120 \times 10^{-6}}{4\pi \times 8.85 \times 10^{-12} \times 1.0^2} = 129.5\mathrm{N}$$



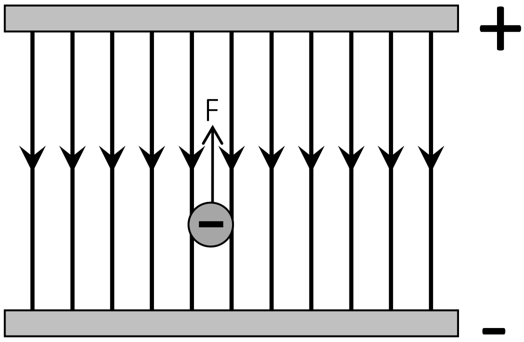
Force F_{23} and F_{21} are both vectors

$$F_R = \sqrt{129.5^2 + 32.37^2} = 133\mathrm{N}$$

Electromagnetism problem book pages 4 to 6, questions 1 to 11; page 8 question 10

Example 5 – Uniform Electric Fields

Two charged plates 1.0cm apart have a voltage of 4000V placed across them.



a. Find the electric field strength between the two plates.

b. If an electron is placed between the plates, calculate the electric force on the electron.

Solution Example 5 – Uniform Electric Fields.

a. $d = 1.0\text{cm} = 0.01\text{m}$

$$V = Ed \quad \Rightarrow \quad E = \frac{V}{d}$$

$$E = \frac{4000}{0.01} = 4.0 \times 10^5 \text{Vm}^{-1}$$

b. $F = QE$

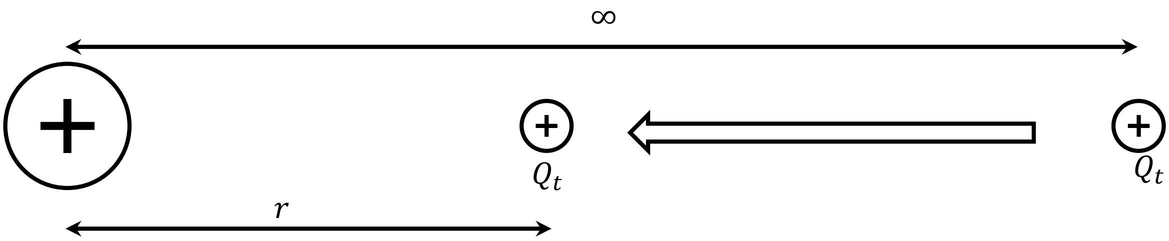
$$F = 1.6 \times 10^{-19} \times 4.0 \times 10^5$$

$$F = 6.4 \times 10^{-14}\text{N}$$

Electromagnetism problem book pages 7 to 9, questions 8, 9, 11 to 13.

1.4 I can define electric potential.

Electric potential at a point is the work done in moving a unit positive charge Q_t from infinity to that point. Note the similarity between this definition and the definition of gravitational potential.



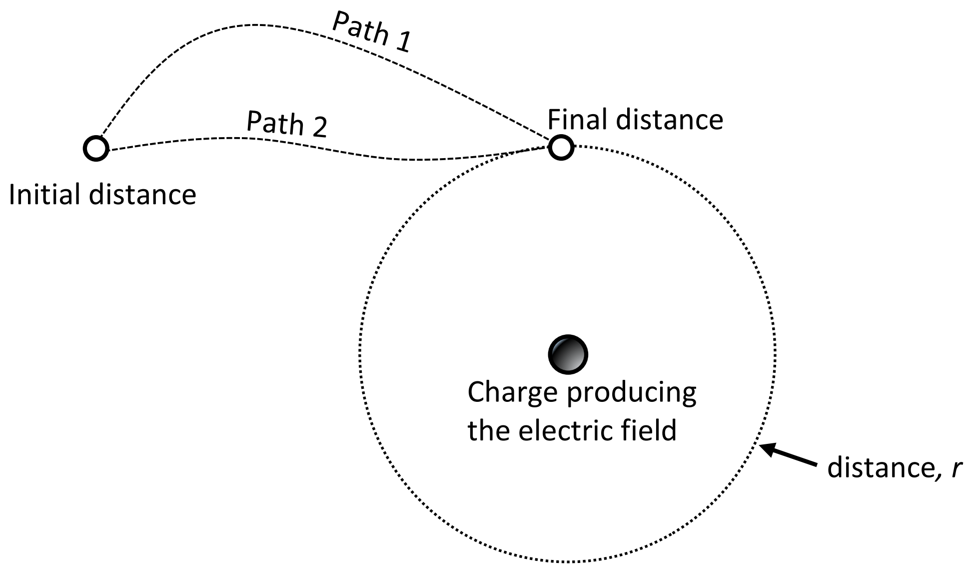
$$V = \frac{Q}{4\pi\epsilon_0 r}$$

- Electric potential (V)
- Charge (C)
- Permittivity of free space (Fm^{-1})
- Distance (m)

Note
Electric potential is a scalar quantity.

1.5 I can state that the energy required to move a charge between two points in an electric field is independent of the path taken.

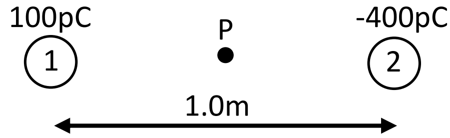
From the electric potential relationship, the electric potential energy of a unit charge depends on the distance from the charge producing the field. The distances between the initial and final positions determine the energy required to move the charge. Whether the charge follows path 1 or path 2 the energy change will be the same.



1.6 I can solve problems involving electric potential.

Example 1

Calculate the electric potential at point P midway between the two charges.



Solution 1

$$r = \frac{1.0}{2} = 0.50\text{m}$$

$$V_1 = \frac{Q}{4\pi\epsilon_0 r} = \frac{100 \times 10^{-12}}{4\pi \times 8.85 \times 10^{-12} \times 0.50} = 1.80\text{V}$$

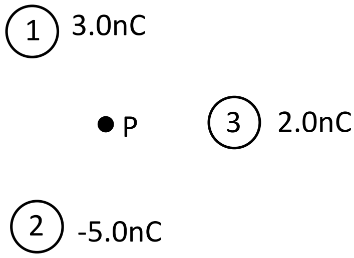
$$V_2 = \frac{Q}{4\pi\epsilon_0 r} = \frac{-400 \times 10^{-12}}{4\pi \times 8.85 \times 10^{-12} \times 0.50} = -7.19\text{V}$$

Potential at P

$$V_P = 1.80 - 7.19 = -5.4\text{V}$$

Example 2

Three small charges are each 4.0cm from the point P. Calculate the electric potential at point P.



Solution 2

$$r = 4.0\text{cm} = 0.04\text{m}$$

$$V_1 = \frac{Q}{4\pi\epsilon_0 r} = \frac{3.0 \times 10^{-9}}{4\pi \times 8.85 \times 10^{-12} \times 0.04} = 674.4\text{V}$$

$$V_2 = \frac{Q}{4\pi\epsilon_0 r} = \frac{-5.0 \times 10^{-9}}{4\pi \times 8.85 \times 10^{-12} \times 0.04} = -1124\text{V}$$

$$V_3 = \frac{Q}{4\pi\epsilon_0 r} = \frac{2.0 \times 10^{-9}}{4\pi \times 8.85 \times 10^{-12} \times 0.04} = 449.6\text{V}$$

Potential at P

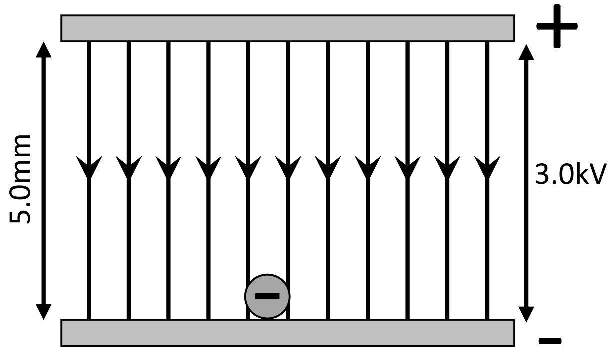
$$V_P = 674.4 - 1124 + 449.6 = 0\text{V}$$

Electromagnetism problem book pages 9 to 11, questions 1 to 14.

1.7 I can solve problems on the motion and energy of charged particles in uniform electric fields.

<u>Example 1 Motion parallel to the electric field</u>

A particle of mass $2.5 \times 10^{-8}\text{kg}$ with a charge of $-4.0 \times 10^{-10}\text{C}$ starts on the negative plate in the diagram shown. The voltage across the plates is 3.0kV.



a. Find the electric field strength between the plates.

b. Find the electric force acting on the charged particle.

c. find the speed of the mass as it strikes the positive plate.

<u>Solution 1 Motion parallel to the electric field</u>

a. $d = 5.0\text{mm} = 5.0 \times 10^{-3}\text{m}$
$V = 3.0\text{kV} = 3.0 \times 10^{3}\text{V}$

$$V = Ed \quad \Rightarrow \quad E = \frac{V}{d}$$

$$E = \frac{3.0 \times 10^{3}}{5.0 \times 10^{-3}}$$

$$E = 6.0 \times 10^{5}\text{Vm}^{-1}$$

b. $Q = -4.0 \times 10^{-10}\text{C}$

$$F = QE$$

$$F = -4.0 \times 10^{-10} \times 6.0 \times 10^{5}$$

$F = -2.4 \times 10^{-4}\text{N}$ The negative sign indicates that the force is in the opposite direction to the electric field lines.

c. Work done on the charged particle $E_W = QV$
Kinetic energy of the particle $E_k = \frac{1}{2}mv^2$
As all work done will be converted to kinetic energy
$\frac{1}{2}mv^2 = QV$

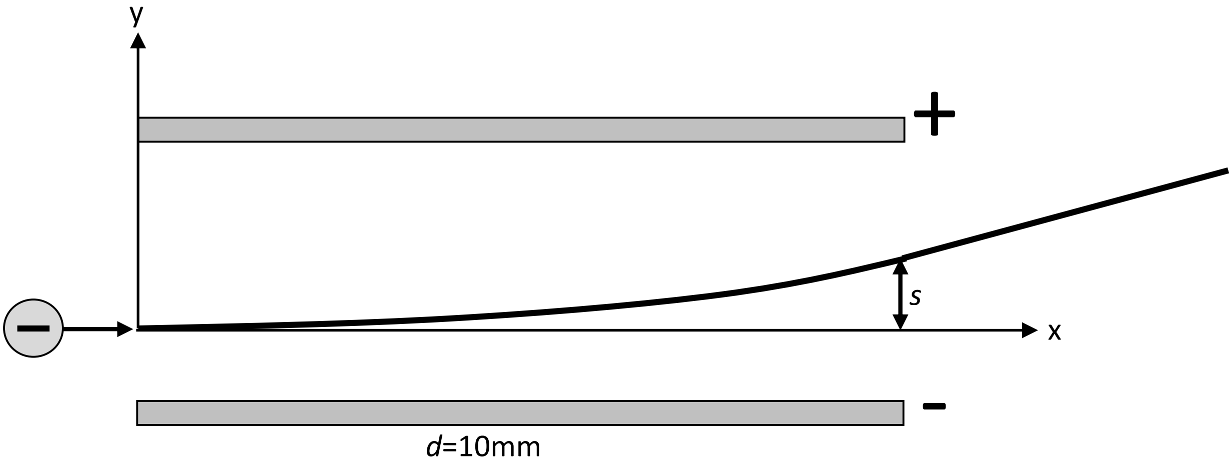
$$v = \sqrt{\frac{2QV}{m}}$$

$$v = \sqrt{\frac{2 \times 4.0 \times 10^{-10} \times 3.0 \times 10^{3}}{2.5 \times 10^{-8}}}$$

$$v = 9.8\text{ms}^{-1}$$

Example 2 Motion perpendicular to the electric field

An electron in an oscilloscope is fired at a speed of $1.0 \times 10^6 \mathrm{ms}^{-1}$ through charged deflection plates of length 10mm. If the strength of the electric field between the plates is $1.0 \times 10^4 \mathrm{NC}^{-1}$, calculate the deflection, *s*, of the electron when leaving the plates.



Solution 2 Motion perpendicular to the electric field

x-direction

Calculate the time taken for the electron to pass the length of the plates. The component of the electron's velocity in the x-direction is constant as the electric field will accelerate the electron vertically.

$d = 10\mathrm{mm} = 10 \times 10^{-3}\mathrm{m}$

$$s = vt \quad \Rightarrow \quad t = \frac{s}{v}$$

$$t = \frac{10 \times 10^{-3}}{1.0 \times 10^6}$$

$$t = 1.0 \times 10^{-8}\mathrm{s}$$

y-direction

Find the y-direction acceleration then use $s = ut + \frac{1}{2}at^2$ to find the displacement.

$$a = \frac{F}{m} \text{ and } F = QE$$

$$a = \frac{QE}{m}$$

$$a = \frac{1.6 \times 10^{-19} \times 1.0 \times 10^4}{9.11 \times 10^{-31}}$$

$$a = 1.76 \times 10^{15}\mathrm{ms}^{-1}$$

Substituting a into $s = ut + \frac{1}{2}at^2$

$$s = 0 \times 1.0 \times 10^{-8} + \frac{1}{2} \times 1.76 \times 10^{15} \times (1.0 \times 10^{-8})^2$$

$$s = 0.088\mathrm{m}$$

Electromagnetism problem book pages 13 to 17, questions 1 to 12.

1.8 I know the definition of the Electron Volt (eV) and can convert between electron volts and joules.

The electron volt (eV) is a unit of energy not voltage. It is defined as the work done on an electron as it is moved between two points with potential difference of 1 volt.

$$E_w = QV$$
$$E_w = 1.6 \times 10^{-19} \times 1 = 1.6 \times 10^{-19}\text{J}$$

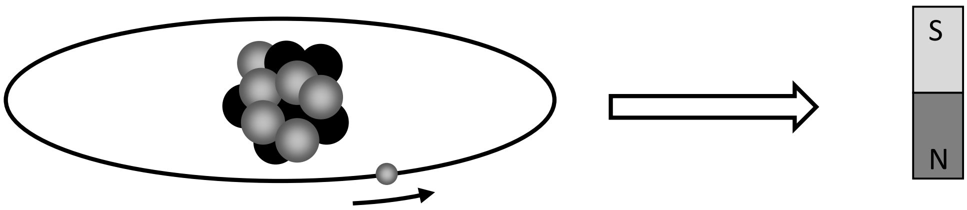
The electron volt is frequently used when dealing with the energy of subatomic particles and atomic processes. It is also used with $E = mc^2$ to express the mass of subatomic particles in terms of energy.

To convert joules to electron volts divide by 1.6×10^{-19}.
To convert from electron volts to joules multiply by 1.6×10^{-19}.

Electromagnetism problem book page 18, questions 16 to 19.

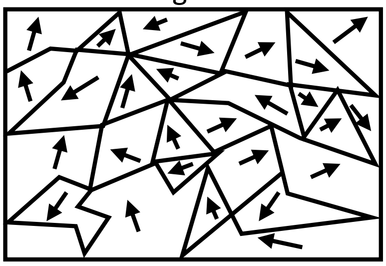
1.9 I explain the magnetic effect called ferromagnetism which occurs in certain metals.

The motion of electrons in around the nucleus produce a magnetic dipole similar to a bar magnet.



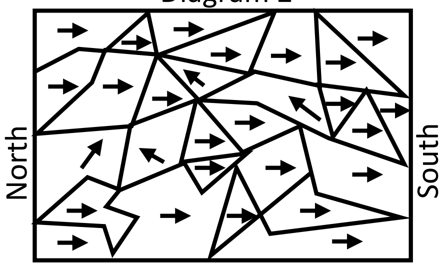
In most materials, the magnetic fields produced by the electrons cancel to produce no effect. In some materials, the magnetic fields of each atom combine to produce an overall ferromagnetic effect.
The magnetic fields produced by groups of atoms form regions called domains. In each domain, the magnetic fields of the atoms all line up in the same direction. This effectively makes each domain a dipole magnet. Material in which magnetic domains form are called ferromagnetic. There are few ferromagnetic materials. Examples are iron, nickel and cobalt.

Diagram 1



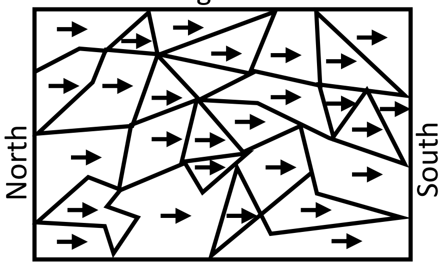
Randomly orientated domains.
The material is not magnetised.

Diagram 2



Most domains aligned.
Material is partially
magnetised but unsaturated.

Diagram 3



All domains aligned.
Material is magnetised
and saturated.

Diagram 1 shows a ferromagnetic material where the magnetic domains are aligned randomly. The magnetic fields cancel leaving the material unmagnetized.
Diagram 2 shows a ferromagnetic material after being affected by an external magnetic field. The atoms within the magnetic domains are rotated to align with the externally applied magnetic field. This alignment remains after the external magnetic field is removed leaving the material magnetised. It is now a permanent magnet. in this case some domains remain randomly orientated and some are aligned. This means that the material is only partially magnetised.
Diagram 3 shows a material where all the magnetic domains have been aligned using a strong external magnetic field. This leaves a strong saturated magnet. The magnetisation of the saturated magnet cannot be further increased.

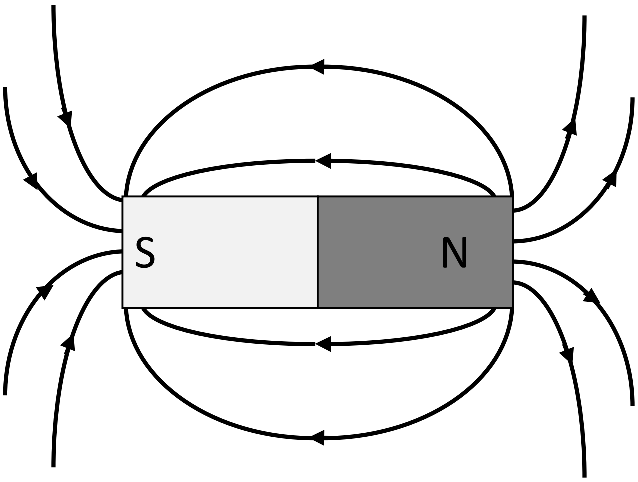
Demagnetisation
Anything that disrupts the alignment of the domains will demagnetise a permanent magnet. This can be done by

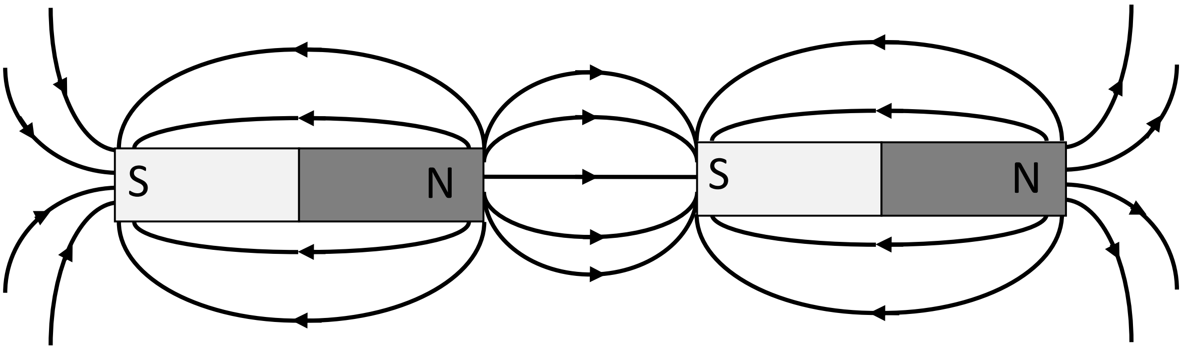
- placing the magnet in an external alternating magnetic field. This is a common way to demagnetise materials.
- repeatedly striking the magnet.
- heating the magnet above its curie temperature. Above this temperature the atoms in the ferromagnetic material have sufficient kinetic energy to rotate to random directions.

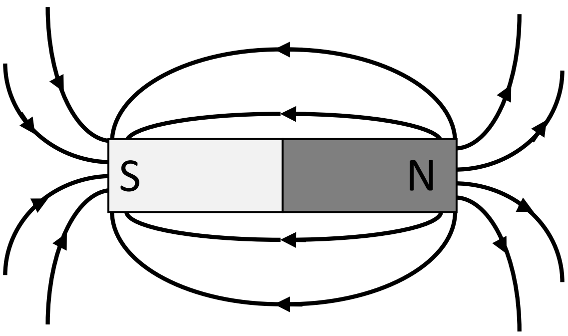
Electromagnetism problem book page 19, questions 1 to 4.

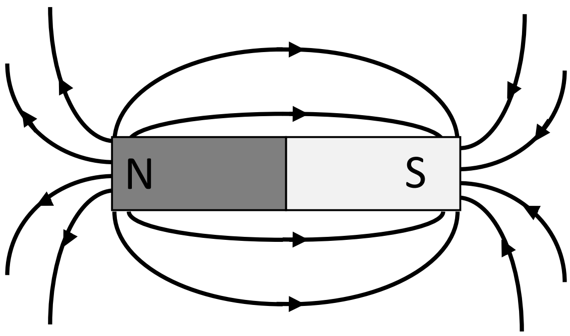
1.10 I can draw magnetic field line patterns.

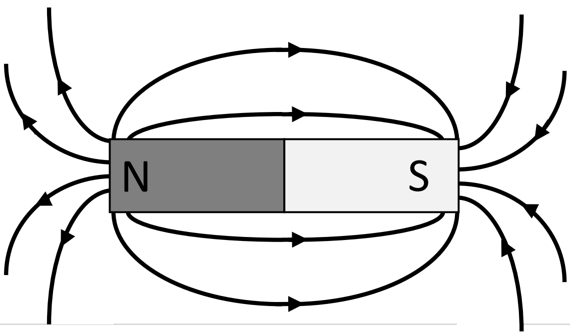
Magnetic field lines point from north to south. The spacing between the field line indicates the strength on the magnetic field. The closer the lines the stronger the field.

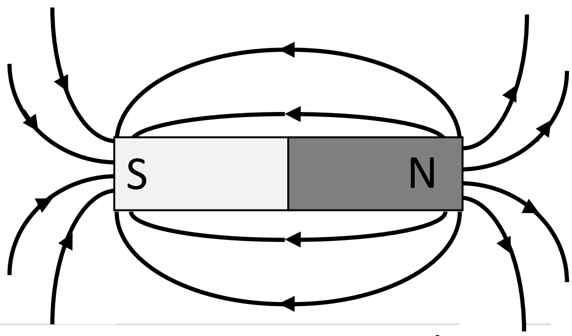


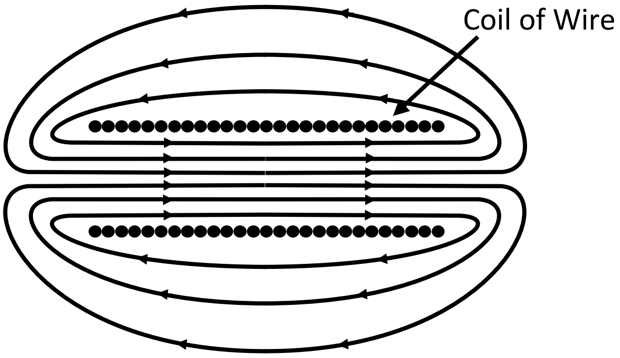




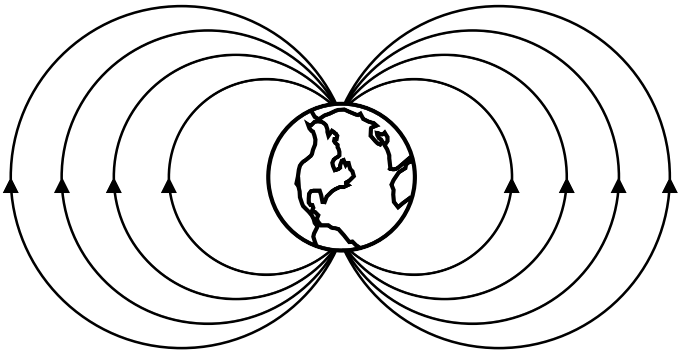




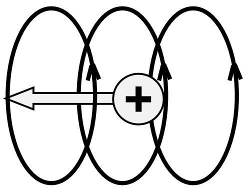




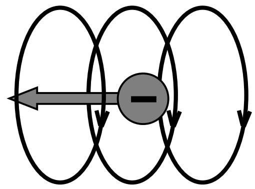
Solenoid
A solenoid (inductor) consists of a coil of wire. Passing a current through the coil produces a magnetic field.



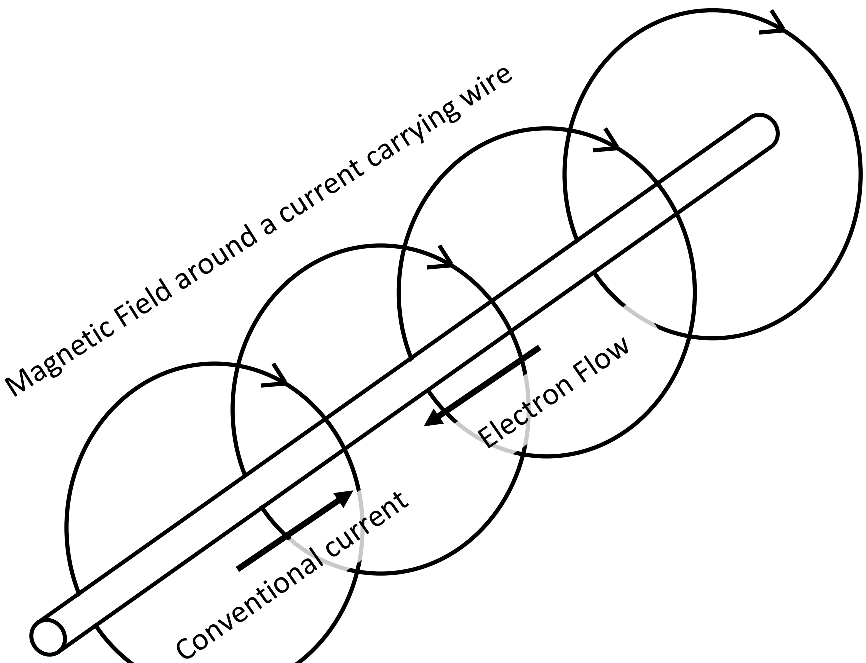
Earth's Magnetic Field
The Earth's liquid iron rich core produces currents which create a magnetic field. This field is similar in shape to a dipole magnet in the core of the Earth.



Magnetic field around a moving positive charge.

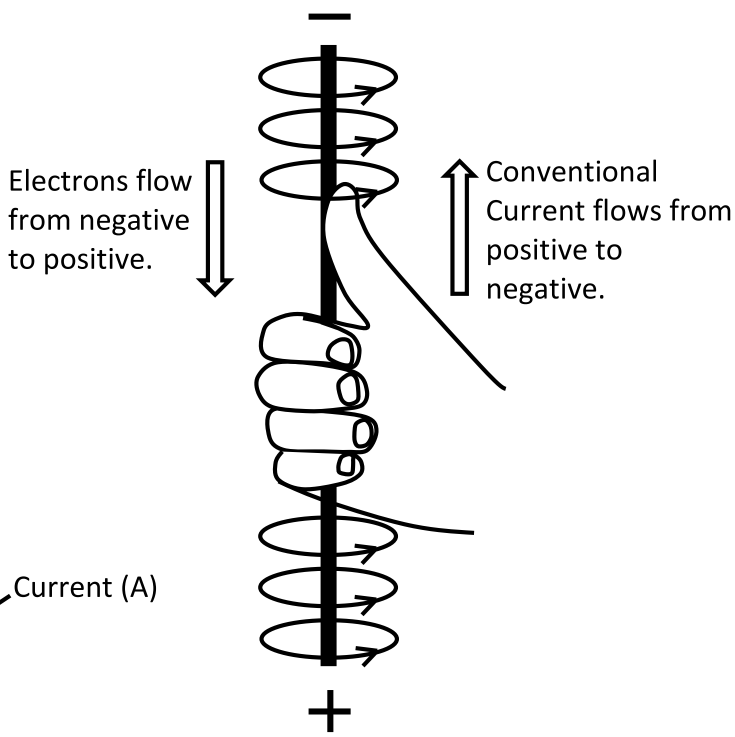


Magnetic field around a moving negative charge.



1.11 I can solve problems involving the magnetic induction formed around a current carrying wire.

Current in a wire produces a magnetic field which forms closed loops around the wire. The direction of the magnetic field is given by the right-hand rule. The thumb follows the direction of the conventional current. The curl of the fingers gives the direction of the magnetic field. The strength of the magnetic field is called the magnetic induction and is given by



Permeability of free space (Hm^{-1})

Magnetic Induction (T)

Current (A)

$$B = \frac{\mu_0 I}{2\pi r}$$

Distance from the wire (m)

Magnetic induction is measured in Tesla (T).

Example

When a voltage of 6.0V is placed across the ends of a straight wire a magnetic induction of $1.5 \times 10^{-5}\text{T}$ is formed at 10mm from a wire. Find the resistance of the wire.

Solution

$\mu_0 = 4\pi \times 10^{-7}\text{Hm}^{-1}$
$r = 10\text{mm} = 10 \times 10^{-3}\text{m}$
$B = 1.5 \times 10^{-5}\text{T}$

$$B = \frac{\mu_0 I}{2\pi r} \Rightarrow I = \frac{2\pi r B}{\mu_0}$$

$$I = \frac{2\pi \times 10 \times 10^{-3} \times 1.5 \times 10^{-5}}{4\pi \times 10^{-7}}$$

$$I = 0.75\text{A}$$

$$V = IR \Rightarrow R = \frac{V}{I}$$

$$R = \frac{6.0}{0.75}$$

$$R = 8.0\Omega$$

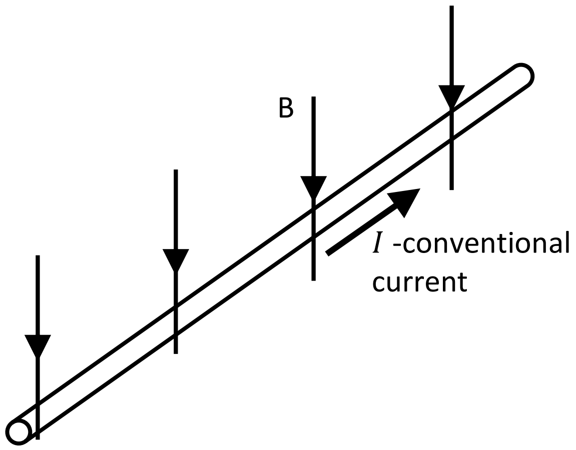
Electromagnetism problem book page 21, questions 1 to 4.

1.12 I can solve problems involving charged particles in magnetic fields in terms of their; mass, velocity, charge, radius of their path and the magnetic induction of the magnetic field.

This is covered in section 2.4 of the Quanta and Waves Notes.

1.13 I can solve problems involving the forces acting on a current carrying wire in a magnetic field.

When a current carrying wire is placed in magnetic field there will be a force on the wire. In section 2.4 in quanta and waves the force on moving charged particles in a magnetic field was found using the relationship $F = qvB$. This can be extended to the charges moving in a wire giving the relationship below. The direction of the force on the wire can be found using the right-hand rule given in section 2.4 of quanta and waves. This must be done with the velocity direction given by the direction of the conventional current.



Current (A)

Length of the wire (m)

Force on the wire (N)

$$F = IlB\sin\theta$$

Angle between the wire and the magnetic induction (°)

Magnetic Induction (T)

Example 1

A wire carrying a current of 6.0 A has 0.50 m of its length placed in a magnetic field of magnetic induction 0.20 T. Calculate the size of the force on the wire if it is placed:

a. at right angles to the direction of the field
b. at 45° to the direction of the field
c. along the direction of the field (i.e. lying parallel to the field lines).

Solution 2

a. When the field is at a right angle to the wire $\theta = 90°$.
$F = IlB \sin\theta$
$F = 6.0 \times 0.50 \times 0.20 \times \sin 90° = 0.60\text{N}$

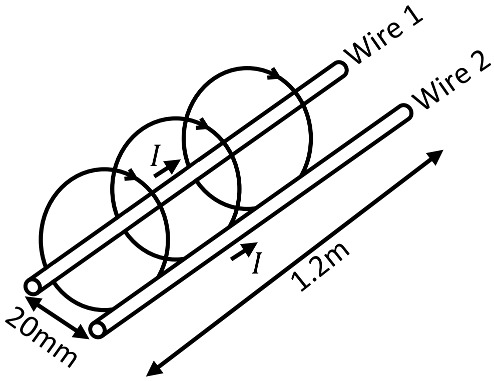
b. $F = 6.0 \times 0.50 \times 0.20 \times \sin 45° = 0.42\text{N}$

c. $\theta = 0°, \sin 0°\, 0, \text{so } F = 0N$

Example 2

Two wires each carrying 2.0A are placed 20mm apart.

a. Calculate the force produced on wire 2.
b. Do the wires attract or repel each other?



Solution 2

a. Use $B = \frac{\mu_0 I}{2\pi r}$ to find magnetic induction at wire 2.
Then use $F = IlB \sin\theta$ to find the force on the wire.

$I = 2.0\text{A}$
$r = 20\text{mm} = 20 \times 10^{-3}\text{m}$
$\theta = 90°$

$$B = \frac{\mu_0 I}{2\pi r}$$

$$B = \frac{4\pi \times 10^{-7} \times 2.0}{2\pi \times 20 \times 10^{-3}}$$

$$B = 2.0 \times 10^{-5}\text{T}$$

$$F = IlB \sin\theta$$

$$F = 2.0 \times 1.2 \times 2.0 \times 10^{-5} \times \sin 90°$$

b. Wires attract. Use the right hand rule from section 2.4 in the Quanta and Waves Notes.

Electromagnetism problem book pages 21 to 26, questions 1 to 13.

1.14 I can state comparisons between nuclear, electromagnetic and gravitational forces in terms of relative magnitude and range.

The table below compares the relative strength of the nuclear, electromagnetic and gravitational forces taking the nuclear force to have a value of 1.

Force	Relative Magnitude	Range (metres)
Strong	1	10^{-15}
Electromagnetic	10^{-3}	Infinite
Gravity	10^{-41}	Infinite

Key Area: Circuits

Success Criteria

2.1 I can describe the variation of current and potential difference with time in a CR circuit during charging and discharging.

2.2 I can define the time constant for a CR circuit and use this to and solve problems.

2.3 I can define capacitive reactance.

2.4 I can solve problems involving capacitive reactance, voltage, current frequency and capacitance.

2.5 I understand how an inductor is constructed.

2.6 I understand electromagnetic induction and the factors which affect the induction of a current in an inductor.

2.7 I can state what is meant by the self-inductance of a coil.

2.8 I know the effect of placing an iron core inside an inductor.

2.9 I understand Lenz’s law and the effect back E.M.F has on the current in a circuit.

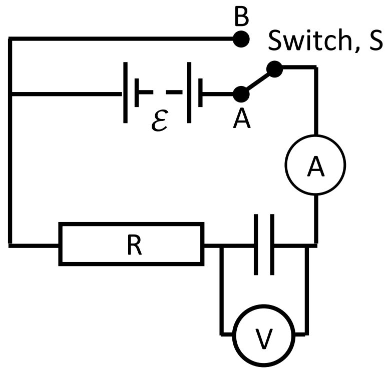
2.10 I can solve problems involving back E.M.F and the energy stored in a capacitor.

2.11 I can define inductive reactance.

2.12 I can solve problems involving inductive reactance, voltage, current frequency and inductance.

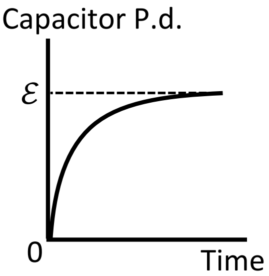
2.1 I can describe the variation of current and potential difference with time in a CR circuit during charging and discharging.

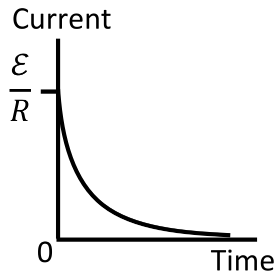
See section 4.8 and 4.9 in the Higher Physics Electricity notes



The circuit shown contains a capacitor and resistor in series. This is a CR circuit. When switch S is moved to position A the capacitor charges. When moved to position B the capacitor discharges.
The graphs of current potential difference and against time across the capacitor against time are shown below.

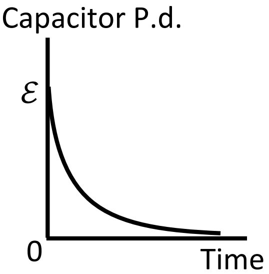
Charging

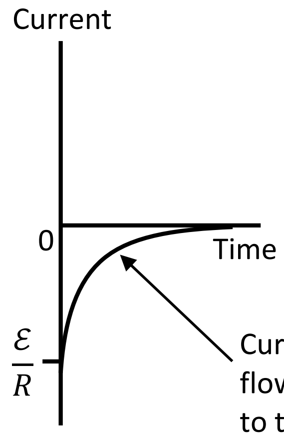




Switch in position A. The capacitor is charging.
The voltage rises from zero until it reaches the e.m.f. of the battery, $\mathcal{E}$.
The current has an initial value of $\frac{\mathcal{E}}{R}$ which falls towards zero.

Discharging



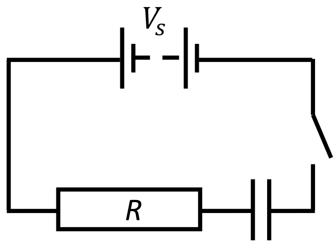


Current is negative as it is flowing in the opposite direction to the charging current.

Switch in position B. The capacitor is discharging.
The voltage falls towards zero from an initial value of $\mathcal{E}$.
The current has an initial value of $-\frac{\mathcal{E}}{R}$ which falls towards zero.

Revision of higher physics capacitors
Electromagnetism problem book pages 27 to 30, questions 1 to 7.

2.2 I can define the time constant for a CR circuit and use this to and solve problems.



In a circuit containing a capacitor and resistor, the relationships which define the charging current and potential difference across a capacitor are

$$I = \frac{V_C}{R} e^{-\frac{t}{RC}} \quad \text{and} \quad V_C = V_S\left[1 - e^{-\frac{t}{RC}}\right]$$

You do not need to know or be able to use these relationships.

Where
V_s - EMF of the supply
V_C - potential difference across the capacitor
R - resistance in the circuit
I - Charging current
C – Capacitance in the circuit

The term RC in these relationships is called the time constant.

Resistance (Ω)

Time Constant (s) → $t = RC$ ← Capacitance (F)

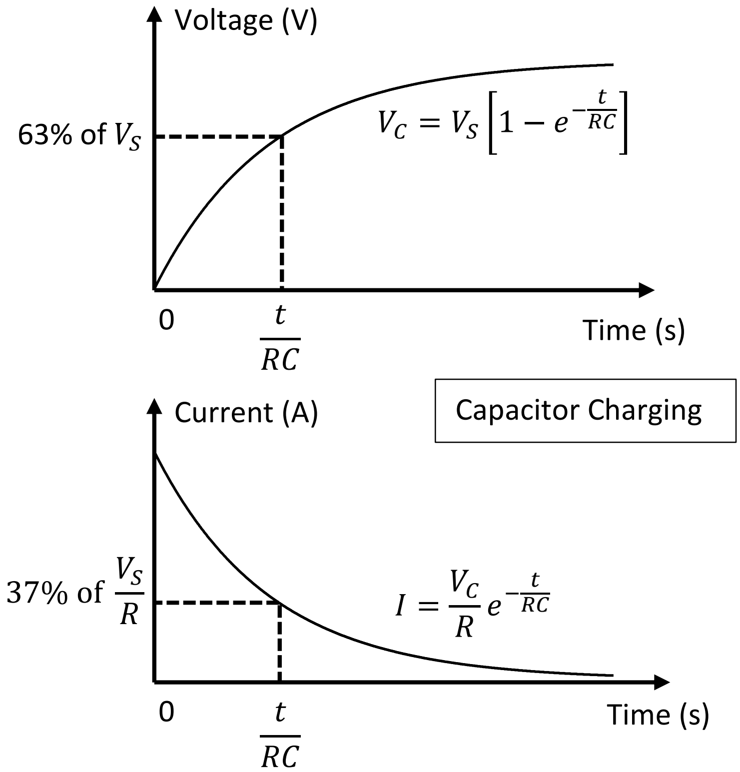
Note
The t in this relationship is a **constant**. It is a different quantity to the **variable** t in the above relationships for I and V_C.

- A large value of time constant gives a long charging and discharging time.
- A small value of time constant give a short charging and discharging time.

The exponential relationship of the charging curves means that the time taken for the voltage to reach the EMF of the supply and the charging current to decrease to zero is not easily determined. The time constant is used to make the charging and discharging times of CR circuits easy to compare.

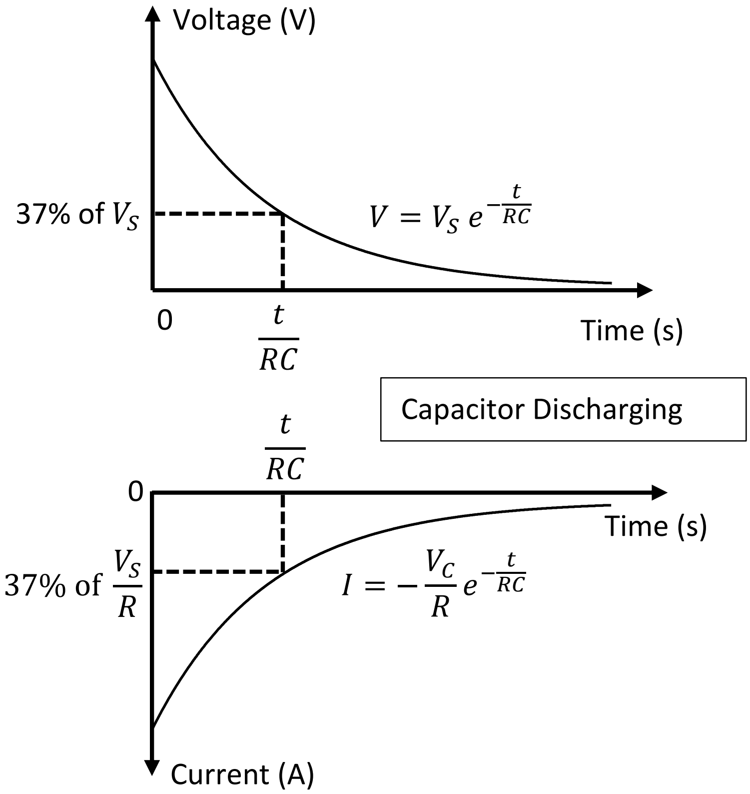
When the capacitor is charging the time constant represents the time taken for

- the voltage across the capacitor to increase to 63% of the supply EMF.
- The current in the circuit to decrease by 63% to 37% of the initial charging current.



When the capacitor is discharging the time constant represents the time taken for

- the voltage across the capacitor to decrease by 63% to 37% of the supply EMF.
- The current in the circuit to decrease by 63% to 37% of the initial discharge current.



Example 1

You are given the following components.

10MΩ Resistor	20µF Capacitor
1MΩ Resistor	20pF Capacitor
10kΩ Resistor	20nF Capacitor

a. Which the combination of a single resistor and single capacitor in series give the longest charging time.

b. Calculate the time constant for the combination found in part a.

Solution 1

a. For the longest time the time constant, RC, must be have the largest value. R and C have the largest values so choose 10MΩ resistor and a 20µF capacitor.

b. $t = RC$

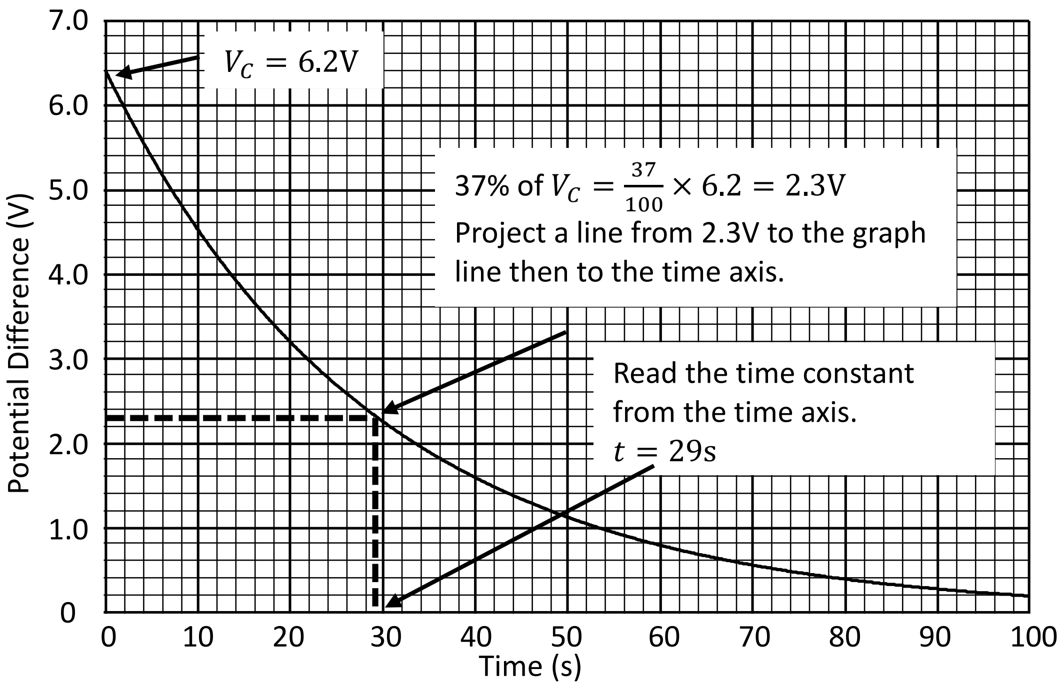
$t = 10 \times 10^{6} \times 10 \times 10^{-6}$

$t = 100\text{s}$

Example Finding the time constant from a graph

The variation of potential difference across a capacitor in a of an RC circuit as it discharges is shown below. The time constant for this circuit can be found by

- Reading the initial voltage V_C.
- Calculating 37% of V_C.
- Tracing a line from 37% of V_C to the graph line then down to the time axis.
- Reading the time constant value from the time axis.

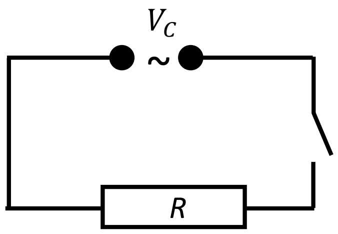


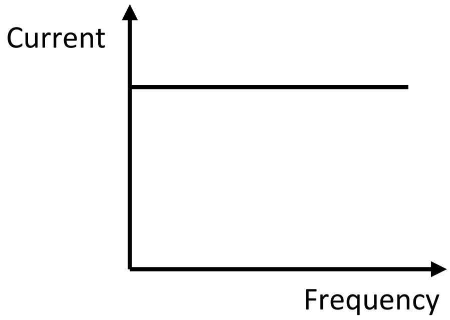
Electromagnetism problem book pages 30 to 31, questions 8 to 12.

2.3 I can define capacitive reactance.

Capacitive reactance is the opposition to a.c. current by the capacitance of a capacitor.

In a circuit containing resistance only, the frequency of the supply has no effect on the current in the circuit

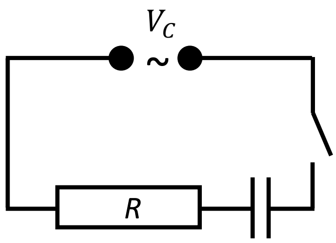


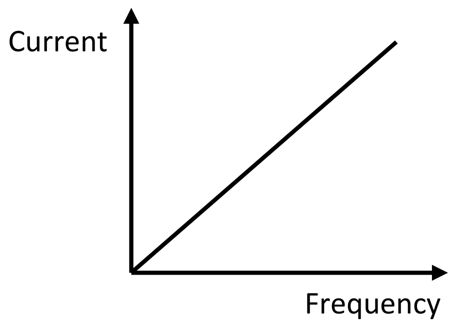


Ohm's Law applies to resistance only circuits

So $R = \frac{V_C}{I}$

In a circuit containing a resistor and capacitor (an RC circuit) the current depends on the frequency of the supply.





The resistance in an RC circuit is fixed. The quantity **capacitive reactance** is defined to take into account the variation in current with frequency.

$$X_C = \frac{V}{I}$$

Capacitive Reactance (Ω) → X_C; Voltage across the capacitor (V) → V; Current (A) → I

Compare with inductive reactance in section 2.11.

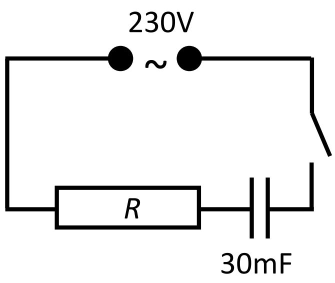
$$X_C = \frac{1}{2\pi f C}$$

Capacitive Reactance (Ω) → X_C; Capacitance (F) → C; Supply frequency (Hz) → f

2.4 I can solve problems involving capacitive reactance, voltage, current frequency and capacitance.

Example 1

The circuit shown runs from the UK mains at 230V, 50Hz. Calculate the capacitive reactance in the circuit.



Solution 1

$f = 50\text{Hz}$
$C = 30\text{mF} = 30 \times 10^{-3}\text{F}$
$V = 230\text{V}$

$$X_C = \frac{1}{2\pi f C}$$

$$X_C = \frac{1}{2\pi \times 50 \times 30 \times 10^{-3}}$$

$$X_C = 0.11\Omega$$

Example

A circuit containing capacitive components is designed in the US to operate at 100V derived from mains 60Hz supply. It is shipped to the UK where it is operated at 100V derived from the main 50Hz supply. It is found that the power output from the circuit is reduced. Explain why.

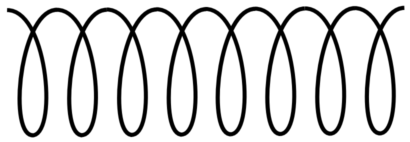
Solution

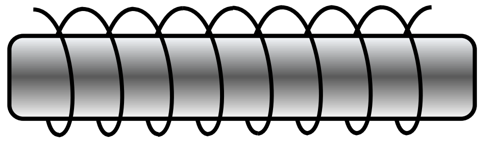
The frequency of the supply is decreased so the capacitive reactance in the circuit will be increased as $X_C = \frac{1}{2\pi f C}$. As the capacitive reactance is increased the current in the circuit will decrease as $X_C = \frac{V}{I}$. The reduced current reduces the power output of the circuit.

Electromagnetism problem book pages 31 to 33, questions 1 to 6.

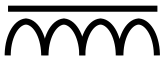
2.5 I understand how an inductor is constructed.

An inductor consists of a coil of wire which can contain and metal core





Symbol for an inductor without a core



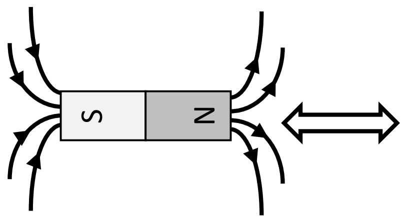
Symbol for an inductor with a core

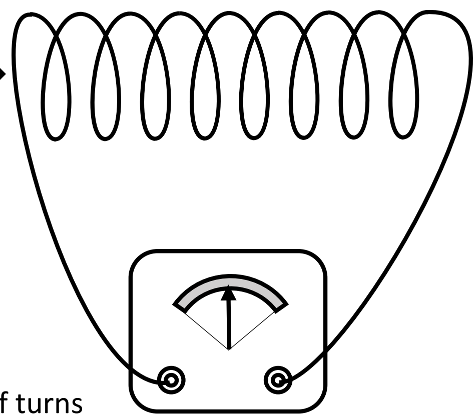
The inductance of an inductor depends on

- The number of turns per metre. The greater the number of turns per meter the larger the inductance.
- Having an iron core. Inductors with an iron core have a higher inductance than an inductor without a core.

2.6 I understand electromagnetic induction and the factors which affect the induction of a current in an inductor.

Magnetic induction occurs when the movable charges in a conductor are subject to a changing magnetic field. This causes them to move producing an electrical current. The diagram below shows a magnet being moved in and out of a coil of wire. This will produce a voltage reading on the voltmeter.



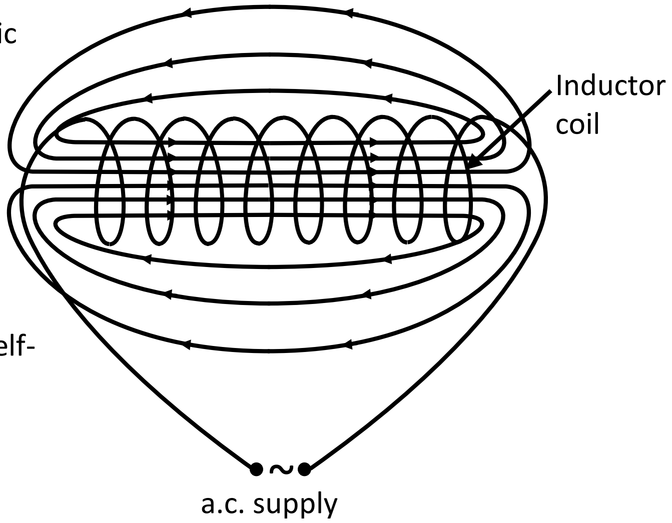


The factors which affect the voltage produced are:

- **The speed of the magnet.** The faster the magnet the greater the rate of change of the magnetic field, the greater the induced voltage.
- **The strength of the magnetic field.** The greater the magnetic field the greater the voltage induced.
- **Number of turns on the coil.** The greater the number of turns the larger the voltage produced.
- **Direction of the magnet field.** Reversing the magnet reverses the polarity of the voltage.
- **Direction of motion.** Reversing the direction of motion of the magnet reverses the polarity of the voltage.

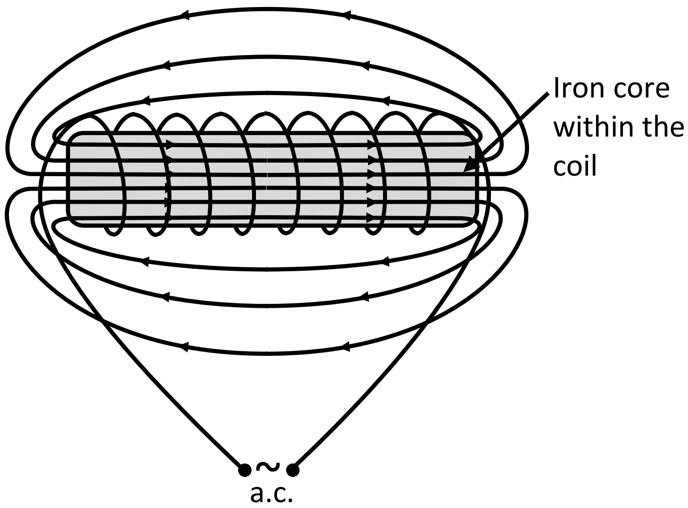
2.7 I can state what is meant by the self-inductance of a coil.

When current passes through a wire, a magnetic field is produced (see section 1.10). When formed into an inductor coil the magnetic field shown is produced. When connected to an a.c. supply, the magnetic field produced by the coil will alternate along with the flow of current.
The alternating magnetic field produced by the inductor coil induces E.M.F in the coil. This is self-inductance.



2.8 I know the effect of placing an iron core inside an inductor.

Placing an iron core within the inductor increases the magnetic field produced. The iron core is within the magnetic field produced by the inductor coil. The makes the core a magnet (See section 1.9) which increases the magnetic induction. The increased magnetic induction produces a greater back E.M.F.

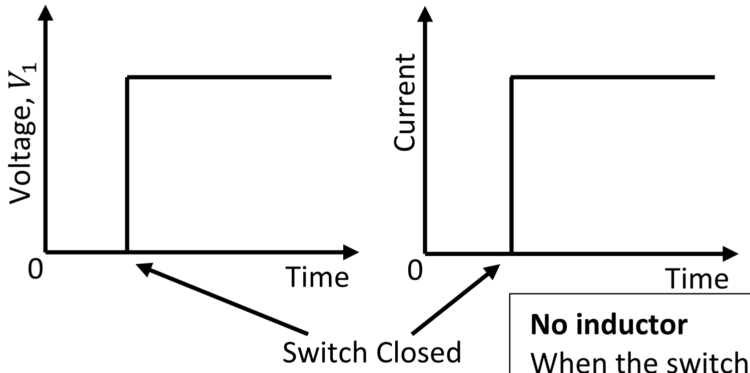


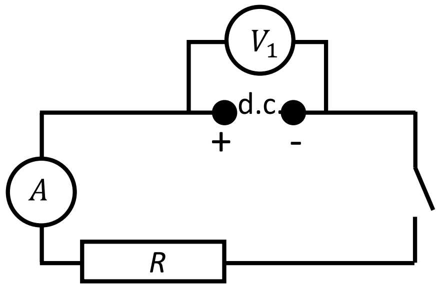
Electromagnetism problem book page 34, questions 1 and 2.

2.9 I understand Lenz's law and the effect back E.M.F has on the current in a circuit.

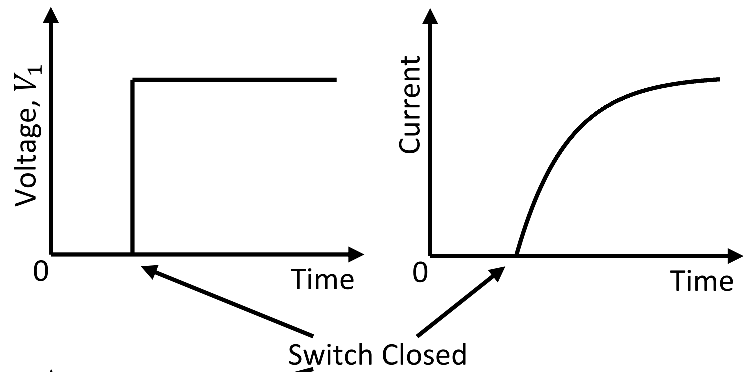
Lenz's law states that the E.M.F produced by self inductance will oppose the current which produced it. The E.M.F produced by self inductance is called back E.M.F.

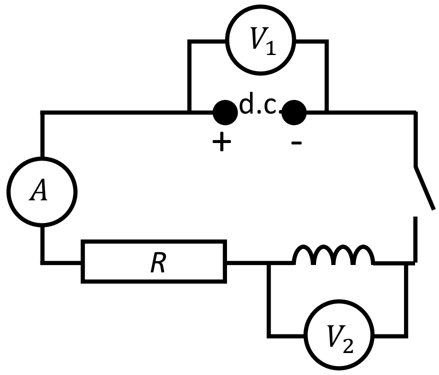
To see the effect of self inductance and back E.M.F. compare the circuits below with and without an inductor.

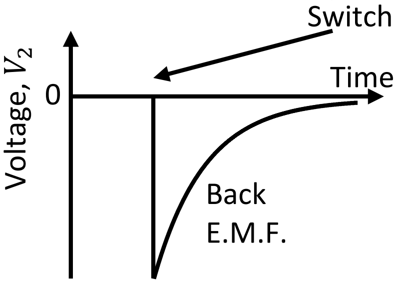




No inductor
When the switch is closed the supply voltage, V_1, and circuit current immediately rise.



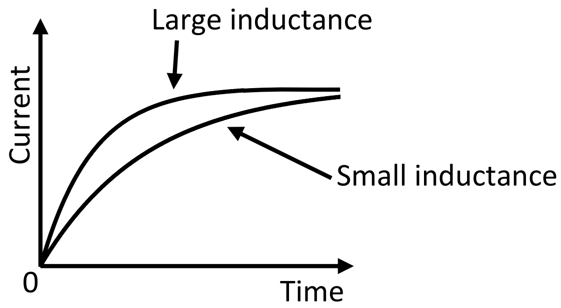




With an inductor
When the switch is closed, the supply voltage rises immediately. As the current rises in the inductor the self-inductance produces a back E.M.F., V_2. This opposes the E.M.F. from the supply reducing the overall E.M.F. in the circuit. This limits the rate of increase in current in the circuit. As the current rises from zero to the value given by $\frac{V_1}{R}$ its rate of change decreases which decreases the back E.M.F.

Comparing a large inductance to a small inductance

The larger the inductance of an inductor the greater its effect on a changing current in a circuit.



2.10 I can solve problems involving back E.M.F and the energy stored in a capacitor.

The back E.M.F. produced by an inductor is given by

$$\epsilon = -L\frac{dI}{dt}$$

Back E.M.F

Inductance (H)

Rate of change of current (As^{-1})

The unit of inductance is the Henry (H).

The negative sign shows that the back E.M.F is in the opposite direction to the (conventional) current.

Note that the back E.M.F depends on the rate of change of current rather than current. This means that a rapidly changing current, e.g. suddenly switching a circuit off, can produce a much larger back E.M.F. than the supply E.M.F.

The energy stored in an inductor is given by

$$E = \frac{1}{2}LI^2$$

Energy Stored (J)

Inductance (H)

Current (A)

Example 1

An inductor is connected to a 6.0 V d.c. supply which has a negligible internal resistance. The inductor has a resistance of 0.80 Ω. When the circuit is switched on it is observed that the current increases gradually. The rate of growth of the current is 200 As^{-1} when the current in the circuit is 4.0 A.

a. Calculate the induced e.m.f. across the coil when the current is 4.0 A.

b. Hence calculate the inductance of the coil.

c. Calculate the energy stored in the inductor when the current is 4.0 A.

d.i. When is the energy stored by the inductor a maximum?

ii. What value does the current have at this time?

Solution 1

a. Potential difference across the resistive element of the circuit

$$V = IR$$

$$V = 4.0 \times 0.80 = 3.2\text{V}$$

Thus p.d. across the inductor $= 6.0 - 3.2 = 2.8\text{V}$

b. Using

$$\epsilon = -L\frac{dI}{dt}$$

$$L = \frac{\epsilon}{\left(\frac{dI}{dt}\right)}$$

$$L = \frac{2.8}{200} = 0.014\text{H}$$

c. $E = \frac{1}{2}LI^2$

$$E = \frac{1}{2} \times 0.014 \times 4.0^2 = 0.11\text{J}$$

d.i. The energy will be a maximum when the current reaches a maximum steady value.

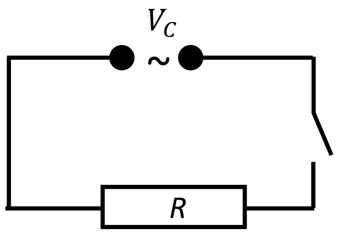
ii. $I_{max} = \frac{V}{R} = \frac{6.0}{0.8} = 7.5\text{A}$

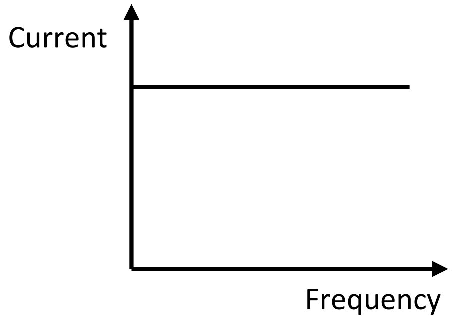
Electromagnetism problem book pages 34 to 38, questions 3 to 8.

2.11 I can define inductive reactance.

Inductive reactance is the opposition to current by the inductance of an inductor.

In a circuit containing resistance only the frequency of the supply has no effect on the current in the circuit

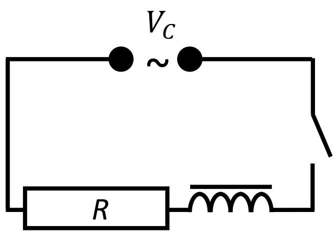


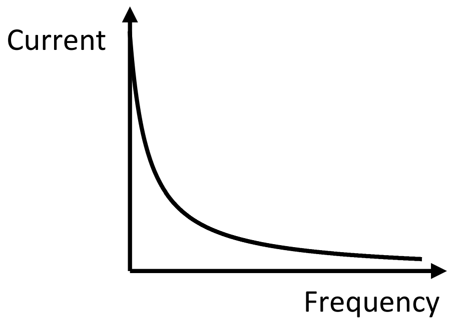


Ohm's Law applies to resistance only circuits

So $R = \frac{V_C}{I}$

With an inductor in a circuit the current depends on the frequency of the supply.





The resistance in an inductive circuit is fixed. The quantity **inductive reactance** is defined to take into account the variation in current with frequency.

$$X_L = \frac{V}{I}$$

- X_L: Inductive Reactance (Ω)
- V: Voltage across the inductor (V)
- I: Current (A)

Compare with capacitive reactance in section 2.3

$$X_L = 2\pi f L$$

- X_L: Inductive Reactance (Ω)
- L: Inductance (H)
- f: Supply frequency (Hz)

2.12 I can solve problems involving inductive reactance, voltage, current frequency and inductance.

Example

An inductor has an inductance of 0.03H. It is connected in a a.c. circuit of 12V 50Hz.

a. Calculate the reactance of the of the inductor.

b. Calculate the R.M.S current in the circuit.

c. The frequency of the circuit is increased to 100Hz. State what happens to the current in the circuit when the frequency is increased.

Solution

a. $X_L = 2\pi f L$

$X_L = 2\pi \times 50 \times 0.03$

$X_L = 9.4\Omega \ \ (9.42\Omega)$

b. $X_L = \frac{V}{I} \ \Rightarrow \ I = \frac{V}{X_L}$

$I = \frac{12}{9.42}$

$I = 1.3\text{A}$

c. Current decreases.

Electromagnetism problem book pages 38 to 41, questions 1 to 7.

Key Area: Electromagnetic Radiation

Success Criteria

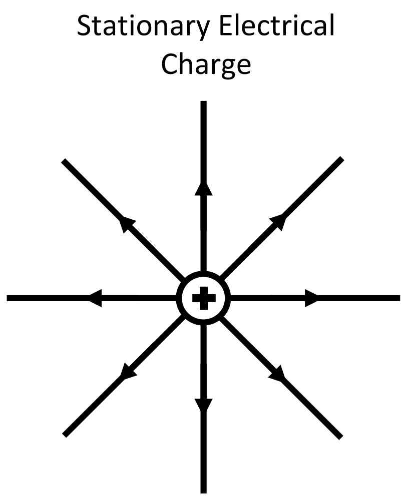
3.1 I know that electricity and magnetism are linked in electromagnetic radiation.

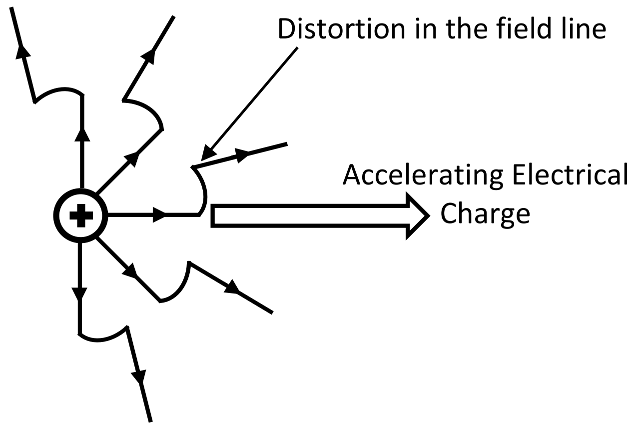
3.2 I understand that electromagnetic radiation is made up of an electric and magnetic field.

3.3 I can solve problems involving the speed of light, the permittivity of free space and the permeability of free space.

3.1 I know that electricity and magnetism are linked in electromagnetic radiation.

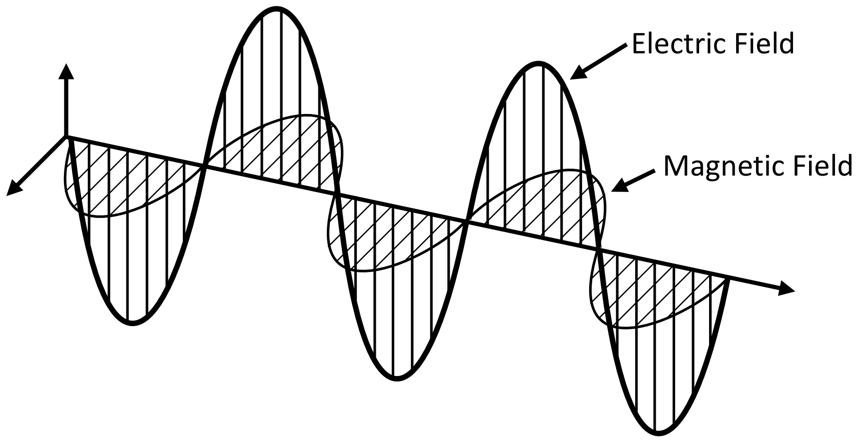
When a charged particle is accelerated the electric field lines surrounding the particle is distorted. This distortion propagates out from the charge at the speed of light. This is the electric field component of electromagnetic radiation. As a changing electric field produces a magnetic field the propagating electric field also produces a propagating magnetic field.





3.2 I understand that electromagnetic radiation is made up of an electric and magnetic field.

Electromagnetic radiation consists of two fields; an electric field and a perpendicular magnetic field. These two fields propagate in phase through space as oscillating waves in a direction perpendicular to both fields. The changing electric field induces a changing magnetic field and the changing magnetic field produces a changing electric field.



3.3 I can solve problems involving the speed of light, the permittivity of free space and the permeability of free space.

The electric and magnetic properties of space are related to the speed of light by the relationship

Speed of light (ms^{-1})

$$c = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$$

Permittivity of free space (Fm^{-1})

Permeability of free space (Hm^{-1})

Example

Calculate the speed of light using the relationship

$$c = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$$

<u>Solution</u>

$\mu_0 = 4\pi \times 10^{-7} \mathrm{Fm}^{-1}$

$\epsilon_0 = 8.85 \times 10^{-12} \mathrm{Hm}^{-1}$

$$c = \frac{1}{\sqrt{8.85 \times 10^{-12} \times 4\pi \times 10^{-7}}}$$

$$c = 3.0 \times 10^{8} \mathrm{ms}^{-1}$$

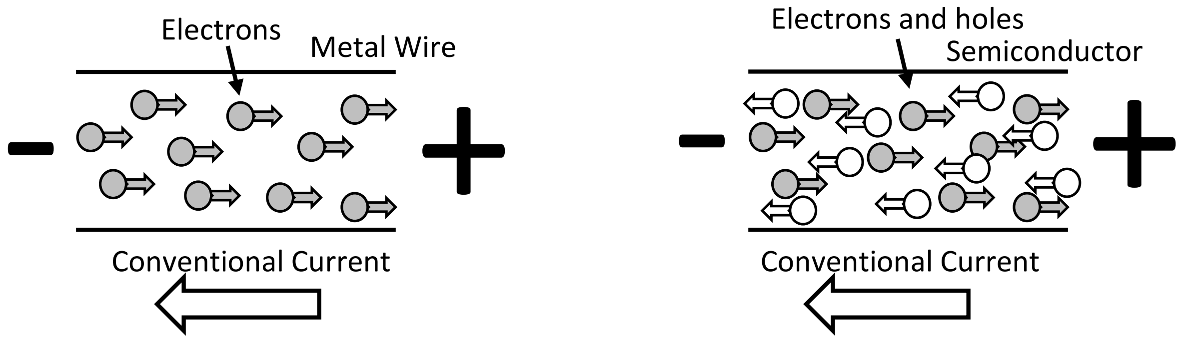
Electromagnetism problem book pages 41 and 42, questions 1 to 4.

Current, Mathematics and Right Hand Rules

This section is background. You will not be examined on this material.

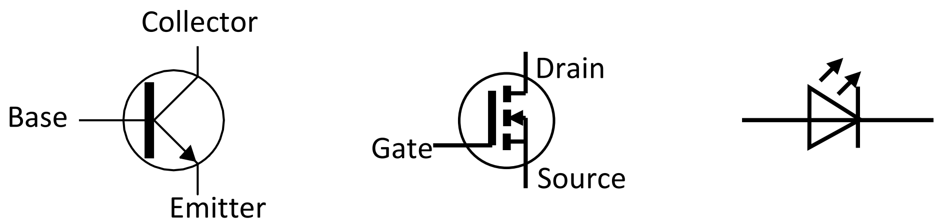
Current

Current is the flow of electrical charges. These charges can be electrons in metals, electrons and holes in semiconductors, ions in solutions or protons in particle accelerators. These can carry a negative charge (electrons, ions) or positive charges (holes, ions, protons).



Defining the direction of current is arbitrary as charges can flow either direction. The direction of **conventional current** is defined as the direction positive charges would move in a circuit.

When dealing with electrical or electronic circuits conventional current rather than electron flow is normally used. This can be see with electronic components that are labelled with arrows. These arrows point in the direction of the conventional current. The triangles in LED and diode symbols also point in the direction of conventional current when forward biased.



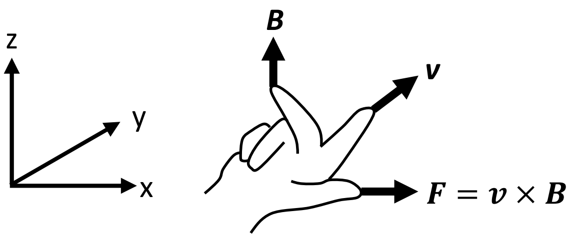
Mathematics and Right Hand Rules
Cartesian axes used in mathematics and the sciences are always right-hand axes. This is an arbitrary choice. It is however the universal choice of axes. For consistency, right hand axes and right hand rules are used in all the notes in the Advanced Higher Physics course.

You will occasionally come across left-hand rules for the Lorentz Force and the directions of magnetic fields. These rules are not wrong and if done correctly will give the same results as the right hand rules. If you use these left hand rules bear in mind that they are not consistent with the vector mathematics used in physics, engineering.

Right Hand Rule for Magnetic Force
Right hand axes are used to define the direction of a vector product of two vectors. This is important when finding the direction of the force on a charged particle caused by a magnetic field.
The magnitude of the magnetic force is given by $F = qvB$. This is a simplified scalar version of the Lorentz Force relationship;

$\boldsymbol{F} = q(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B})$ Where: $\boldsymbol{F}, \boldsymbol{E}$ and $\boldsymbol{B}$ are vectors and the $\times$ symbol is the vector cross product.

The direction of the resultant force vector is defined using right hand axes.



Right Hand Rule for Magnetic Induction Around a Wire
Finding the direction of the magnetic field is around a current carrying wire is also defined by a right hand rule (see section 1.11) which uses conventional current.

Quantities, Units and Multiplication Factors

Quantity	Quantity Symbol	Unit	Unit Abbreviation
capacitance	C	Farad	F
capacitive reactance	X_c	Ohm	Ω
charge	Q	coulomb	C
current	I	Ampere	A
displacement	y, s	metre	m
E.M.F.	ϵ	Volt	V
electric field strength	E	Newton per coulomb	NC^{-1} or Vm^{-1}
energy	E	Joule	J
force	F	newton	N
frequency	f	hertz	Hz
inductance	L	Henry	H
inductive reactance	X_L	Ohm	Ω
magnetic induction	B	Tesla	T
mass	m	kilogram	kg
momentum	p	kilogram metre per second	kgms^{-1}
radius/distance	r	metre	m
resistance	R, r	Ohm	Ω
speed/velocity	v	metre per second	ms^{-1}
time	t	second	s
voltage/Potential difference	V	Volt	V
wavelength	λ	metre	m
work done	E_w	Joule	J

Prefix Name	Prefix Symbol	Multiplication Factor
Pico	p	$\times 10^{-12}$
Nano	n	$\times 10^{-9}$
Micro	µ	$\times 10^{-6}$
Milli	m	$\times 10^{-3}$
Kilo	k	$\times 10^{3}$
Mega	M	$\times 10^{6}$
Giga	G	$\times 10^{9}$
Tera	T	$\times 10^{12}$

You will not be given the tables on this page in any of the tests or the final exam

Relationships required for Physics Advanced Higher

$$v = \frac{ds}{dt}$$

$$a = \frac{dv}{dt} = \frac{d^2s}{dt^2}$$

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$v^2 = u^2 + 2as$$

$$\omega = \frac{d\theta}{dt}$$

$$\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$$

$$\omega = \omega_o + \alpha t$$

$$\theta = \omega_o t + \frac{1}{2}\alpha t^2$$

$$\omega^2 = \omega_o{}^2 + 2\alpha\theta$$

$$s = r\theta$$

$$v = r\omega$$

$$a_t = r\alpha$$

$$a_r = \frac{v^2}{r} = r\omega^2$$

$$F = \frac{mv^2}{r} = mr\omega^2$$

$$T = Fr$$

$$T = I\alpha$$

$$L = mvr = mr^2\omega$$

$$L = I\omega$$

$$E_K = \frac{1}{2}I\omega^2$$

$$F = G\frac{Mm}{r^2}$$

$$V = -\frac{GM}{r}$$

$$v = \sqrt{\frac{2GM}{r}}$$

$$\text{apparent brightness, } b = \frac{L}{4\pi r^2}$$

$$\text{Power per unit area} = \sigma T^4$$

$$L = 4\pi r^2 \sigma T^4$$

$$r_{Schwarzschild} = \frac{2GM}{c^2}$$

$$E = hf$$

$$\lambda = \frac{h}{p}$$

$$mvr = \frac{nh}{2\pi}$$

$$\Delta x\, \Delta p_x \geq \frac{h}{4\pi}$$

$$\Delta E\, \Delta t \geq \frac{h}{4\pi}$$

$$F = qvB$$

$$\omega = 2\pi f$$

$$a = \frac{d^2y}{dt^2} = -\omega^2 y$$

$$y = A\cos\omega t \quad \text{or} \quad y = A\sin\omega t$$

$$v = \pm\omega\sqrt{(A^2 - y^2)}$$

$$E_K = \frac{1}{2}m\omega^2(A^2 - y^2)$$

$$E_P = \frac{1}{2}m\omega^2 y^2$$

$$y = A\sin 2\pi(ft - \frac{x}{\lambda})$$

$$E = kA^2$$

$$\phi = \frac{2\pi x}{\lambda}$$

$$\text{optical path difference} = m\lambda \quad \text{or} \quad \left(m + \frac{1}{2}\right)\lambda$$

where $m = 0, 1, 2....$

$$\Delta x = \frac{\lambda l}{2d}$$

$$d = \frac{\lambda}{4n}$$

$$\Delta x = \frac{\lambda D}{d}$$

$$n = \tan i_P$$

$$F = \frac{Q_1 Q_2}{4\pi\varepsilon_o r^2}$$

$$E = \frac{Q}{4\pi\varepsilon_o r^2}$$

$$V = \frac{Q}{4\pi\varepsilon_o r}$$

$$F = QE$$

$$V = Ed$$

$$F = IlB\sin\theta$$

$$B = \frac{\mu_o I}{2\pi r}$$

$$c = \frac{1}{\sqrt{\varepsilon_o \mu_o}}$$

$$t = RC$$

$$X_C = \frac{V}{I}$$

$$X_C = \frac{1}{2\pi fC}$$

$$\mathcal{E} = -L\frac{dI}{dt}$$

$$E = \frac{1}{2}LI^2$$

$$X_L = \frac{V}{I}$$

$$X_L = 2\pi fL$$

$$\frac{\Delta W}{W} = \sqrt{\left(\frac{\Delta X}{X}\right)^2 + \left(\frac{\Delta Y}{Y}\right)^2 + \left(\frac{\Delta Z}{Z}\right)^2}$$

$$\Delta W = \sqrt{\Delta X^2 + \Delta Y^2 + \Delta Z^2}$$

$$d=\bar{v}t$$
$$s=\bar{v}t$$
$$v=u+at$$
$$s=ut+\frac{1}{2}at^2$$
$$v^2=u^2+2as$$
$$s=\frac{1}{2}(u+v)t$$
$$W=mg$$
$$F=ma$$
$$E_W=Fd$$
$$E_P=mgh$$
$$E_K=\frac{1}{2}mv^2$$
$$P=\frac{E}{t}$$
$$p=mv$$
$$Ft=mv-mu$$
$$F=G\frac{Mm}{r^2}$$
$$t'=\frac{t}{\sqrt{1-\left(\frac{v}{c}\right)^2}}$$
$$l'=l\sqrt{1-\left(\frac{v}{c}\right)^2}$$
$$f_o=f_s\left(\frac{v}{v\pm v_s}\right)$$
$$z=\frac{\lambda_{observed}-\lambda_{rest}}{\lambda_{rest}}$$
$$z=\frac{v}{c}$$
$$v=H_0d$$

$$E_W=QV$$
$$E=mc^2$$
$$E=hf$$
$$E_K=hf-hf_0$$
$$E_2-E_1=hf$$
$$T=\frac{1}{f}$$
$$v=f\lambda$$
$$d\sin\theta=m\lambda$$
$$n=\frac{\sin\theta_1}{\sin\theta_2}$$
$$\frac{\sin\theta_1}{\sin\theta_2}=\frac{\lambda_1}{\lambda_2}=\frac{v_1}{v_2}$$
$$\sin\theta_c=\frac{1}{n}$$
$$I=\frac{k}{d^2}$$
$$I=\frac{P}{A}$$
$$\text{path difference}=m\lambda \text{ or } \left(m+\frac{1}{2}\right)\lambda \text{ where } m=0,\ 1,\ 2...$$
$$\text{random uncertainty}=\frac{\text{max. value - min. value}}{\text{number of values}}$$

$$V_{peak}=\sqrt{2}V_{rms}$$
$$I_{peak}=\sqrt{2}I_{rms}$$
$$Q=It$$
$$V=IR$$
$$P=IV=I^2R=\frac{V^2}{R}$$
$$R_T=R_1+R_2+....$$
$$\frac{1}{R_T}=\frac{1}{R_1}+\frac{1}{R_2}+....$$
$$E=V+Ir$$
$$V_1=\left(\frac{R_1}{R_1+R_2}\right)V_S$$
$$\frac{V_1}{V_2}=\frac{R_1}{R_2}$$
$$C=\frac{Q}{V}$$
$$E=\frac{1}{2}QV=\frac{1}{2}CV^2=\frac{1}{2}\frac{Q^2}{C}$$

Additional Relationships

Circle

$circumference = 2\pi r$

$area = \pi r^2$

Sphere

$area = 4\pi r^2$

$volume = \frac{4}{3}\pi r^3$

Trigonometry

$\sin\theta = \frac{opposite}{hypotenuse}$

$\cos\theta = \frac{adjacent}{hypotenuse}$

$\tan\theta = \frac{opposite}{adjacent}$

$\sin^2\theta + \cos^2\theta = 1$

Moment of inertia

point mass
$I = mr^2$

rod about centre
$I = \frac{1}{12}ml^2$

rod about end
$I = \frac{1}{3}ml^2$

disc about centre
$I = \frac{1}{2}mr^2$

sphere about centre
$I = \frac{2}{5}mr^2$

Table of standard derivatives

$f(x)$	$f'(x)$
$\sin ax$	$a\cos ax$
$\cos ax$	$-a\sin ax$

Table of standard integrals

$f(x)$	$\int f(x)dx$
$\sin ax$	$-\frac{1}{a}\cos ax + C$
$\cos ax$	$\frac{1}{a}\sin ax + C$

Electron Arrangements of Elements

Key

Atomic number
Symbol
Electron arrangement
Name

Transition Elements

Group 1 (1)	Group 2 (2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	Group 3 (13)	Group 4 (14)	Group 5 (15)	Group 6 (16)	Group 7 (17)	Group 0 (18)
1 H 1 Hydrogen																	2 He 2 Helium
3 Li 2, 1 Lithium	4 Be 2, 2 Beryllium											5 B 2, 3 Boron	6 C 2, 4 Carbon	7 N 2, 5 Nitrogen	8 O 2, 6 Oxygen	9 F 2, 7 Fluorine	10 Ne 2, 8 Neon
11 Na 2, 8, 1 Sodium	12 Mg 2, 8, 2 Magnesium											13 Al 2, 8, 3 Aluminium	14 Si 2, 8, 4 Silicon	15 P 2, 8, 5 Phosphorus	16 S 2, 8, 6 Sulphur	17 Cl 2, 8, 7 Chlorine	18 Ar 2, 8, 8 Argon
19 K 2, 8, 8, 1 Potassium	20 Ca 2, 8, 8, 2 Calcium	21 Sc 2, 8, 9, 2 Scandium	22 Ti 2, 8, 10, 2 Titanium	23 V 2, 8, 11, 2 Vanadium	24 Cr 2, 8, 13, 1 Chromium	25 Mn 2, 8, 13, 2 Manganese	26 Fe 2, 8, 14, 2 Iron	27 Co 2, 8, 15, 2 Cobalt	28 Ni 2, 8, 16, 2 Nickel	29 Cu 2, 8, 18, 1 Copper	30 Zn 2, 8, 18, 2 Zinc	31 Ga 2, 8, 18, 3 Gallium	32 Ge 2, 8, 18, 4 Germanium	33 As 2, 8, 18, 5 Arsenic	34 Se 2, 8, 18, 6 Selenium	35 Br 2, 8, 18, 7 Bromine	36 Kr 2, 8, 18, 8 Krypton
37 Rb 2, 8, 18, 8, 1 Rubidium	38 Sr 2, 8, 18, 8, 2 Strontium	39 Y 2, 8, 18, 9, 2 Yttrium	40 Zr 2, 8, 18, 10, 2 Zirconium	41 Nb 2, 8, 18, 12, 1 Niobium	42 Mo 2, 8, 18, 13, 1 Molybdenum	43 Tc 2, 8, 18, 13, 2 Technetium	44 Ru 2, 8, 18, 15, 1 Ruthenium	45 Rh 2, 8, 18, 16, 1 Rhodium	46 Pd 2, 8, 18, 18, 0 Palladium	47 Ag 2, 8, 18, 18, 1 Silver	48 Cd 2, 8, 18, 18, 2 Cadmium	49 In 2, 8, 18, 18, 3 Indium	50 Sn 2, 8, 18, 18, 4 Tin	51 Sb 2, 8, 18, 18, 5 Antimony	52 Te 2, 8, 18, 18, 6 Tellurium	53 I 2, 8, 18, 18, 7 Iodine	54 Xe 2, 8, 18, 18, 8 Xenon
55 Cs 2, 8, 18, 18, 8, 1 Caesium	56 Ba 2, 8, 18, 18, 8, 2 Barium	57 La 2, 8, 18, 18, 9, 2 Lanthanum	72 Hf 2, 8, 18, 32, 10, 2 Hafnium	73 Ta 2, 8, 18, 32, 11, 2 Tantalum	74 W 2, 8, 18, 32, 12, 2 Tungsten	75 Re 2, 8, 18, 32, 13, 2 Rhenium	76 Os 2, 8, 18, 32, 14, 2 Osmium	77 Ir 2, 8, 18, 32, 15, 2 Iridium	78 Pt 2, 8, 18, 32, 17, 1 Platinum	79 Au 2, 8, 18, 32, 18, 1 Gold	80 Hg 2, 8, 18, 32, 18, 2 Mercury	81 Tl 2, 8, 18, 32, 18, 3 Thallium	82 Pb 2, 8, 18, 32, 18, 4 Lead	83 Bi 2, 8, 18, 32, 18, 5 Bismuth	84 Po 2, 8, 18, 32, 18, 6 Polonium	85 At 2, 8, 18, 32, 18, 7 Astatine	86 Rn 2, 8, 18, 32, 18, 8 Radon
87 Fr 2, 8, 18, 32, 18, 8, 1 Francium	88 Ra 2, 8, 18, 32, 18, 8, 2 Radium	89 Ac 2, 8, 18, 32, 18, 9, 2 Actinium	104 Rf 2, 8, 18, 32, 32, 10, 2 Rutherfordium	105 Db 2, 8, 18, 32, 32, 11, 2 Dubnium	106 Sg 2, 8, 18, 32, 32, 12, 2 Seaborgium	107 Bh 2, 8, 18, 32, 32, 13, 2 Bohrium	108 Hs 2, 8, 18, 32, 32, 14, 2 Hassium	109 Mt 2, 8, 18, 32, 32, 15, 2 Meitnerium									

Lanthanides	57 La 2, 8, 18, 18, 9, 2 Lanthanum	58 Ce 2, 8, 18, 20, 8, 2 Cerium	59 Pr 2, 8, 18, 21, 8, 2 Praseodymium	60 Nd 2, 8, 18, 22, 8, 2 Neodymium	61 Pm 2, 8, 18, 23, 8, 2 Promethium	62 Sm 2, 8, 18, 24, 8, 2 Samarium	63 Eu 2, 8, 18, 25, 8, 2 Europium	64 Gd 2, 8, 18, 25, 9, 2 Gadolinium	65 Tb 2, 8, 18, 27, 8, 2 Terbium	66 Dy 2, 8, 18, 28, 8, 2 Dysprosium	67 Ho 2, 8, 18, 29, 8, 2 Holmium	68 Er 2, 8, 18, 30, 8, 2 Erbium	69 Tm 2, 8, 18, 31, 8, 2 Thulium	70 Yb 2, 8, 18, 32, 8, 2 Ytterbium	71 Lu 2, 8, 18, 32, 9, 2 Lutetium
Actinides	89 Ac 2, 8, 18, 32, 18, 9, 2 Actinium	90 Th 2, 8, 18, 32, 18, 10, 2 Thorium	91 Pa 2, 8, 18, 32, 20, 9, 2 Protactinium	92 U 2, 8, 18, 32, 21, 9, 2 Uranium	93 Np 2, 8, 18, 32, 22, 9, 2 Neptunium	94 Pu 2, 8, 18, 32, 24, 8, 2 Plutonium	95 Am 2, 8, 18, 32, 25, 8, 2 Americium	96 Cm 2, 8, 18, 32, 25, 9, 2 Curium	97 Bk 2, 8, 18, 32, 27, 8, 2 Berkelium	98 Cf 2, 8, 18, 32, 28, 8, 2 Californium	99 Es 2, 8, 18, 32, 29, 8, 2 Einsteinium	100 Fm 2, 8, 18, 32, 30, 8, 2 Fermium	101 Md 2, 8, 18, 32, 31, 8, 2 Mendelevium	102 No 2, 8, 18, 32, 32, 8, 2 Nobelium	103 Lr 2, 8, 18, 32, 32, 9, 2 Lawrencium

DATA SHEET

COMMON PHYSICAL QUANTITIES

Quantity	*Symbol*	*Value*	*Quantity*	*Symbol*	*Value*
Gravitational acceleration on Earth	g	$9{\cdot}8\ \mathrm{m\,s^{-2}}$	Mass of electron	m_e	$9{\cdot}11 \times 10^{-31}\ \mathrm{kg}$
Radius of Earth	R_E	$6{\cdot}4 \times 10^{6}\ \mathrm{m}$	Charge on electron	e	$-1{\cdot}60 \times 10^{-19}\ \mathrm{C}$
Mass of Earth	M_E	$6{\cdot}0 \times 10^{24}\ \mathrm{kg}$	Mass of neutron	m_n	$1{\cdot}675 \times 10^{-27}\ \mathrm{kg}$
Mass of Moon	M_M	$7{\cdot}3 \times 10^{22}\ \mathrm{kg}$	Mass of proton	m_p	$1{\cdot}673 \times 10^{-27}\ \mathrm{kg}$
Radius of Moon	R_M	$1{\cdot}7 \times 10^{6}\ \mathrm{m}$	Mass of alpha particle	m_α	$6{\cdot}645 \times 10^{-27}\ \mathrm{kg}$
Mean Radius of Moon Orbit		$3{\cdot}84 \times 10^{8}\ \mathrm{m}$	Charge on alpha particle		$3{\cdot}20 \times 10^{-19}\ \mathrm{C}$
Solar radius		$6{\cdot}955 \times 10^{8}\ \mathrm{m}$	Planck's constant	h	$6{\cdot}63 \times 10^{-34}\ \mathrm{J\,s}$
Mass of Sun		$2{\cdot}0 \times 10^{30}\ \mathrm{kg}$	Permittivity of free space	ε_0	$8{\cdot}85 \times 10^{-12}\ \mathrm{F\,m^{-1}}$
1 AU		$1{\cdot}5 \times 10^{11}\ \mathrm{m}$	Permeability of free space	μ_0	$4\pi \times 10^{-7}\ \mathrm{H\,m^{-1}}$
Stefan-Boltzmann constant	σ	$5{\cdot}67 \times 10^{-8}\ \mathrm{W\,m^{-2}\,K^{-4}}$	Speed of light in vacuum	c	$3{\cdot}0 \times 10^{8}\ \mathrm{m\,s^{-1}}$
Universal constant of gravitation	G	$6{\cdot}67 \times 10^{-11}\ \mathrm{m^3\,kg^{-1}\,s^{-2}}$	Speed of sound in air	v	$3{\cdot}4 \times 10^{2}\ \mathrm{m\,s^{-1}}$

REFRACTIVE INDICES

The refractive indices refer to sodium light of wavelength 589 nm and to substances at a temperature of 273 K.

Substance	*Refractive index*	*Substance*	*Refractive index*
Diamond	2·42	Glycerol	1·47
Glass	1·51	Water	1·33
Ice	1·31	Air	1·00
Perspex	1·49	Magnesium Fluoride	1·38

SPECTRAL LINES

Element	*Wavelength*/nm	*Colour*	*Element*	*Wavelength*/nm	*Colour*
Hydrogen	656	Red	Cadmium	644	Red
	486	Blue-green		509	Green
	434	Blue-violet		480	Blue
	410	Violet	*Lasers*		
	397	Ultraviolet			
	389	Ultraviolet	*Element*	*Wavelength*/nm	*Colour*
Sodium	589	Yellow	Carbon dioxide	9550, 10590	Infrared
			Helium-neon	633	Red

PROPERTIES OF SELECTED MATERIALS

Substance	*Density*/ $\mathrm{kg\,m^{-3}}$	*Melting Point*/ K	*Boiling Point*/K	*Specific Heat Capacity*/ $\mathrm{J\,kg^{-1}\,K^{-1}}$	*Specific Latent Heat of Fusion*/ $\mathrm{J\,kg^{-1}}$	*Specific Latent Heat of Vaporisation*/ $\mathrm{J\,kg^{-1}}$
Aluminium	$2{\cdot}70 \times 10^3$	933	2623	$9{\cdot}02 \times 10^2$	$3{\cdot}95 \times 10^5$	....
Copper	$8{\cdot}96 \times 10^3$	1357	2853	$3{\cdot}86 \times 10^2$	$2{\cdot}05 \times 10^5$	....
Glass	$2{\cdot}60 \times 10^3$	1400	....	$6{\cdot}70 \times 10^2$	....	....
Ice	$9{\cdot}20 \times 10^2$	273	....	$2{\cdot}10 \times 10^3$	$3{\cdot}34 \times 10^5$	....
Glycerol	$1{\cdot}26 \times 10^3$	291	563	$2{\cdot}43 \times 10^3$	$1{\cdot}81 \times 10^5$	$8{\cdot}30 \times 10^5$
Methanol	$7{\cdot}91 \times 10^2$	175	338	$2{\cdot}52 \times 10^3$	$9{\cdot}9 \times 10^4$	$1{\cdot}12 \times 10^6$
Sea Water	$1{\cdot}02 \times 10^3$	264	377	$3{\cdot}93 \times 10^3$	....	....
Water	$1{\cdot}00 \times 10^3$	273	373	$4{\cdot}19 \times 10^3$	$3{\cdot}34 \times 10^5$	$2{\cdot}26 \times 10^6$
Air	1·29	....	....	....	....	....
Hydrogen	$9{\cdot}0 \times 10^{-2}$	14	20	$1{\cdot}43 \times 10^4$	....	$4{\cdot}50 \times 10^5$
Nitrogen	1·25	63	77	$1{\cdot}04 \times 10^3$	....	$2{\cdot}00 \times 10^5$
Oxygen	1·43	55	90	$9{\cdot}18 \times 10^2$	....	$2{\cdot}40 \times 10^4$

The gas densities refer to a temperature of 273 K and a pressure of $1{\cdot}01 \times 10^5$ Pa.