Advanced Higher Physics

Quanta and Waves

Notes

Name……………………………………………………….

Key Area Notes, Examples and Questions

Other Notes

Key Area: An Introduction to Quantum Theory

Success Criteria

1.1 I understand the challenges to classical physics theory from experimental observations of black Body radiation, the photoelectric effect, models of the atom and the De Broglie wavelength.

1.2 I can use $\lambda = \frac{h}{p}$ to solve problems involving the De Broglie wavelength of a particle and its momentum.

1.3 I can describe the Bohr Model of the atom.

1.4 I can solve problems involving the angular momentum of an electron and its principal quantum number using the relationship $mvr = \frac{nh}{2\pi}$

1.5 I can describe experimental evidence for wave particle duality.

1.6 I know that quantum mechanics only calculates the probabilities of properties such as position, momentum and energy.

1.7 I understand the uncertainty principle in terms of; position and momentum, energy and time.

1.8 I understand how the uncertainty principle leads to quantum tunnelling.

1.9 I can solve problems involving the uncertainty principal relationships.

1.1 I understand the challenges to classical physics theory from experimental observations of black Body radiation, the photoelectric effect, models of the atom and the De Broglie wavelength.

Classical Physics

Classical physics includes

- Newtonian mechanics which is all the physics derived from Newton's three laws, Newton's law of gravitation, the law of conservation of energy, the law of conservation of momentum, velocity, acceleration etc.
- Electromagnetism, electric fields, charges, current, voltage, e-m spectrum, inverse square law etc.

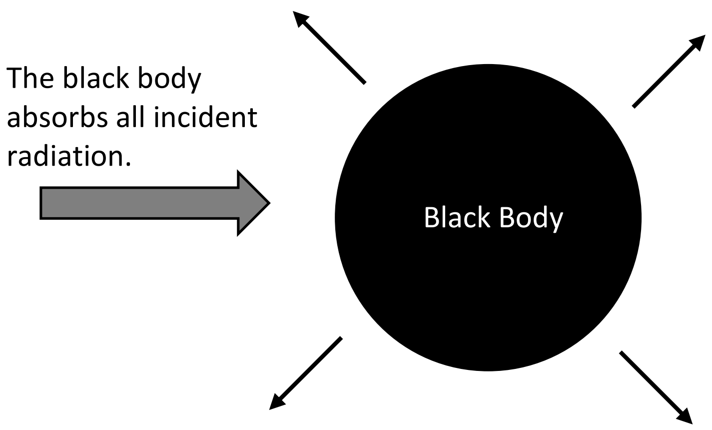
Most of the physics you have learned in National 5 and Higher is classical physics. The exceptions to this are the quantum mechanics of the photoelectric effect, $E = hf$, line spectra, some of the semiconductor physics and relativity.

Before quantum mechanics and relativity theories were developed in the early 20th century all physics was classical physics.

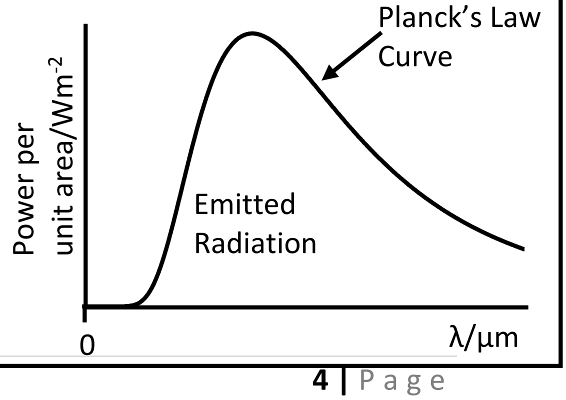
Development in physics has not stopped and other theories and models have been built on quantum mechanics and relativity. e.g. Yang-Mills theory which describes the behaviour of elementary particles and the Big Bang model of cosmology.

Challenges to Classical Physics – Blackbody Radiation

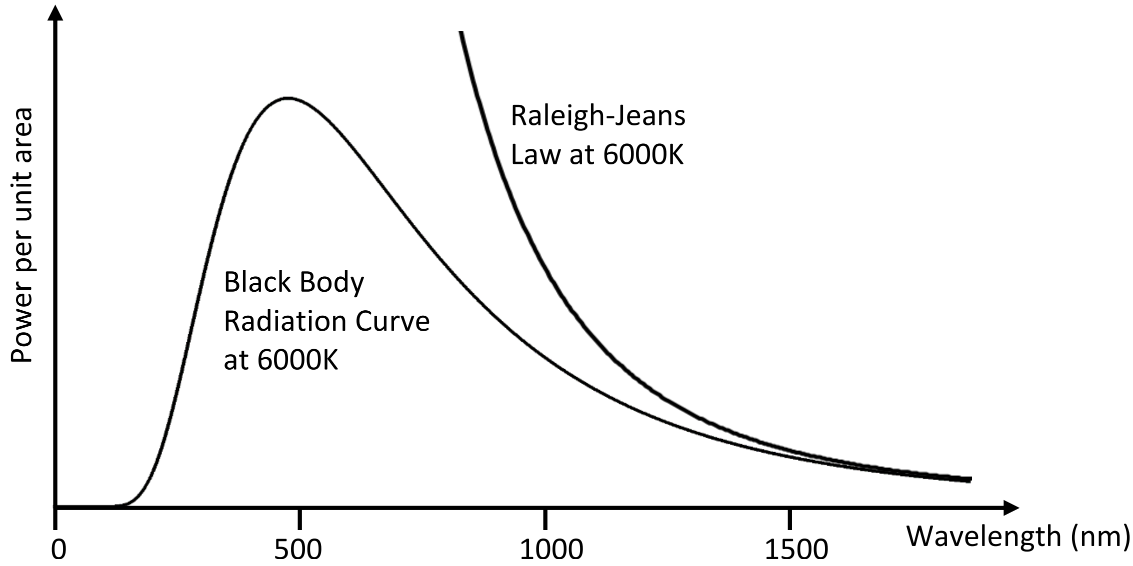
A black body is an idealised object which absorbs all radiation. Black bodies also emit radiation as a continuous spectrum which follows the shape shown given by Planck's Law. Hot, opaque surfaces such as the surface of a star or a heated cavity emit radiation which closely approximates a black body radiation curve.



To remain at the same temperature a black body must emit the same energy as it absorbs or is internally generated.
The black body emits radiation over a range of wavelengths which depends only on the temperature. The distribution of these wavelengths and power emitted follows Planck's law which is shown on the graph below.



The graph below shows the prediction from classical physics of the emission spectrum from a heated object. This is called the Raleigh-Jeans Law. This classical physics law predicts that an infinite amount of radiation is emitted at short wavelengths. If this were the case, all objects above absolute zero would instantly emit radiation to bring its temperature to 0K. This scenario is called the ultraviolet catastrophe and clearly does not occur.



This failure of classical physics was resolved by Max Planck who introduced the idea that energy is discontinuous (quantised) i.e. occurs in "jumps". This is an important idea in quantum mechanics which allowed Max Planck to derive the physics that correctly predicted the black body radiation curve.

Challenges to Classical Physics – Photoelectric Effect

In the Higher Physics course you examined the photoelectric effect (See Section 5 in the Higher Physics Particle and Waves Notes). This occurs when light of a high enough frequency incident on a metal causes the emission of electrons.

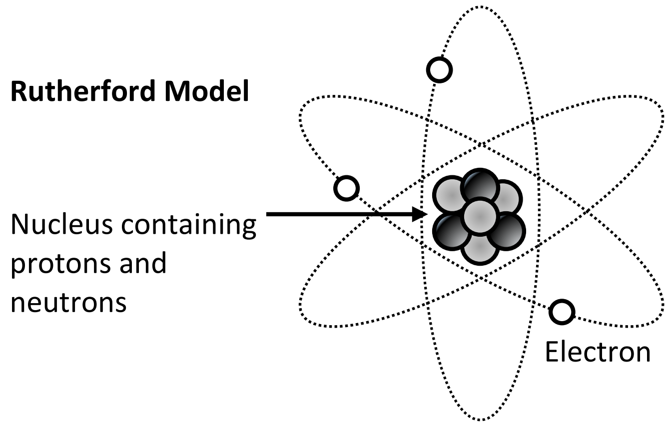
The classical physics could not account for

- the threshold frequency
- electrons being emitted immediately
- the kinetic energy of the electrons not depending on irradiance
- the kinetic energy of the electrons depending on frequency

These problems were resolved by Einstein's model of the photoelectric effect. This is a quantum mechanical model which takes the energy of the incident light as being quantised i.e. photons (see 5.1 in the Higher Physics Particles and Waves notes). The energy of the photons is given by the relationship $E = hf$.

Challenges to Classical Physics – Models of the Atom

The Rutherford model of the atom consists of a small dense nucleus containing neutral neutrons and positively charge protons. Around the outside of the atom electrons "orbit" the nucleus.



The Rutherford model of the atom however cannot be correct. The electrons going around the nucleus are constantly being accelerated by the centripetal force caused by the attraction between the protons and electrons. (see section Rotational Dynamics and Astrophysics notes section 2). As accelerating charges emit radiation all electrons should, by this model, radiate all their energy and collapse into the nucleus.

This problem was partially resolved by the Niels Bohr who developed the model of the atom described in section 1.3.

Challenges to Classical Physics – De Broglie Wavelength

Classical physics describes phenomena using particles and waves which were regarded as separate entities.

Experiments with the scattering of x-ray by Arthur Compton showed that, although normally regarded as wave, x-rays could also have particle like properties when interacting with matter.

Experiments with electrons by G P Thomson showed that although normally regarded as a particle electrons could form a diffraction pattern showing that they could have wave like properties.

This was resolved by De Broglie (pronounced De Broy) when he proposed that particle and wave like properties are linked by $\lambda = \frac{h}{p}$. See section 1.2

1.2 I can use $\lambda = \dfrac{h}{p}$ to solve problems involving the De Broglie wavelength of a particle and its momentum.

Wavelength (m)

Planck's Constant (Js)

$$\lambda = \frac{h}{p}$$

Linear Momentum ($kgms^{-1}$)

<u>Example 1</u>

An electron is accelerated from rest through a voltage of 2.0kV. Find the De Broglie wavelength of the electron.

<u>Solution 1</u>

This calculation involves several steps

Use $E = QV$ to find E.

Use $E = \frac{1}{2}mv^2$ to find v.

Use $p = mv$ to find p.

Then use

$\lambda = \dfrac{h}{p}$ to find λ.

From the data sheet

$Q = 1.6 \times 10^{-19}\mathrm{C}$

$m = 9.11 \times 10^{-31}\mathrm{kg}$

$h = 6.63 \times 10^{-34}\mathrm{Js}$

$$E = QV$$
$$E = 1.6 \times 10^{-19} \times 2.0 \times 10^{3}$$
$$E = 3.2 \times 10^{-16}\mathrm{J}$$

$$E = \frac{1}{2}mv^2 \quad \Rightarrow \quad v = \sqrt{\frac{2E}{m}}$$
$$v = \sqrt{\frac{2 \times 3.2 \times 10^{-16}}{9.11 \times 10^{-31}}}$$
$$v = 2.65 \times 10^{7}\mathrm{ms^{-1}}$$

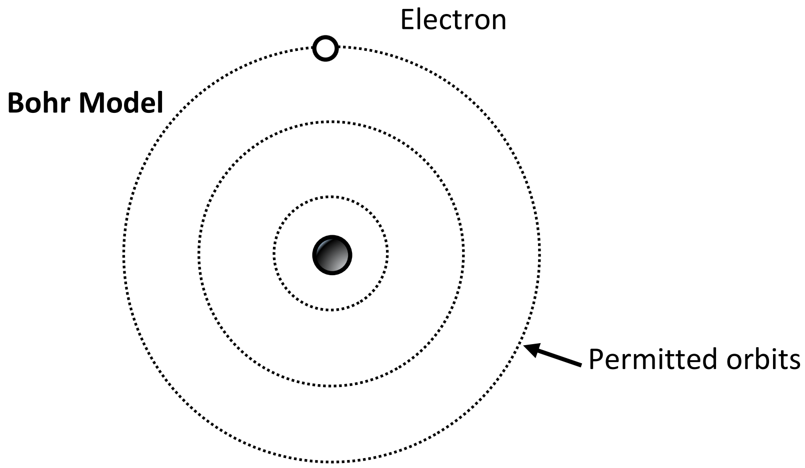
$$p = mv$$
$$p = 9.11 \times 10^{-31} \times 2.65 \times 10^{7}$$
$$p = 2.41 \times 10^{-23}\mathrm{kgms^{-1}}$$

$$\lambda = \frac{h}{p}$$
$$\lambda = \frac{6.63 \times 10^{-34}}{2.41 \times 10^{-23}}$$
$$\lambda = 2.7 \times 10^{-11}\mathrm{m}$$

Quanta and Waves problem book pages 4 and 5, questions 5 to 11.

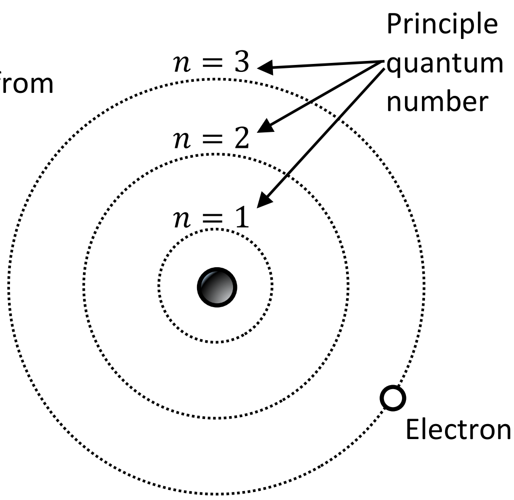
1.3 I can describe the Bohr Model of the atom.

Also, see Section 4.8 to 4.10 in the Higher Physics Particles and Waves Notes.



The Bohr model is a development of the Rutherford model of the atom. Bohr postulated several features of his model

- Electrons have circular orbits around the nucleus.
- The electrons can only orbit without radiating energy in certain orbits. Each orbit has a certain energy.
- The angular momentum is quantised.
- Electrons are only allowed to jump from one allowed orbit to another by emitting or absorbing a photon. The frequency of the photon is given by $E = hf$, where the energy is the difference in energy levels between the orbits.



Each of the permitted orbits is labelled with a number from closest to the nucleus outwards. This is the principle quantum number.

Successes of the Bohr Model
The Bohr model of the atom correctly predicts the frequencies of electromagnetic radiation emitted by the hydrogen atom and ionised helium.

Problems with the Bohr Model
The Bohr model works well for hydrogen and ionised helium, allowing the energy levels to be calculated. The interactions between multiple electrons in heavier atoms means that the energy levels predicted by the Bohr model are not correct.
The Bohr model also predicts a non-zero value for angular momentum of the electron in the lowest energy level ($n = 1$). This is not the case for hydrogen where the lowest angular momentum is zero.

1.4 I can solve problems involving the angular momentum of an electron and its principal quantum number using the relationship $mvr = \frac{nh}{2\pi}$.

In the Bohr Model of the atom the relationship between the angular momentum of an electron and its principal quantum number is given by.

Planck's Constant (Js)

Principle Quantum Number

$$mvr = \frac{nh}{2\pi}$$

Mass of the Electron (kg)

Speed of the Electron (ms^{-1})

Radius of the orbit (m)

$mvr =$ Angular momentum of the electron

Example

The radius of the $n = 1$ orbit of an electron in a hydrogen atom is $5.29 \times 10^{-11}\text{m}$. Calculate the speed of the electron in this orbit.

Solution

$$\left.\begin{aligned} m &= 9.11 \times 10^{-31}\text{kg} \\ h &= 6.63 \times 10^{-34}\text{Js} \end{aligned}\right\} \text{From the data sheet}$$

$$r = 5.29 \times 10^{-11}\text{m}$$

$$mvr = \frac{nh}{2\pi}$$

$$9.11 \times 10^{-31} \times v \times 5.29 \times 10^{-11} = \frac{1 \times 6.63 \times 10^{-34}}{2\pi}$$

$$v = \frac{1 \times 6.63 \times 10^{-34}}{2\pi \times 9.11 \times 10^{-31} \times 5.29 \times 10^{-11}}$$

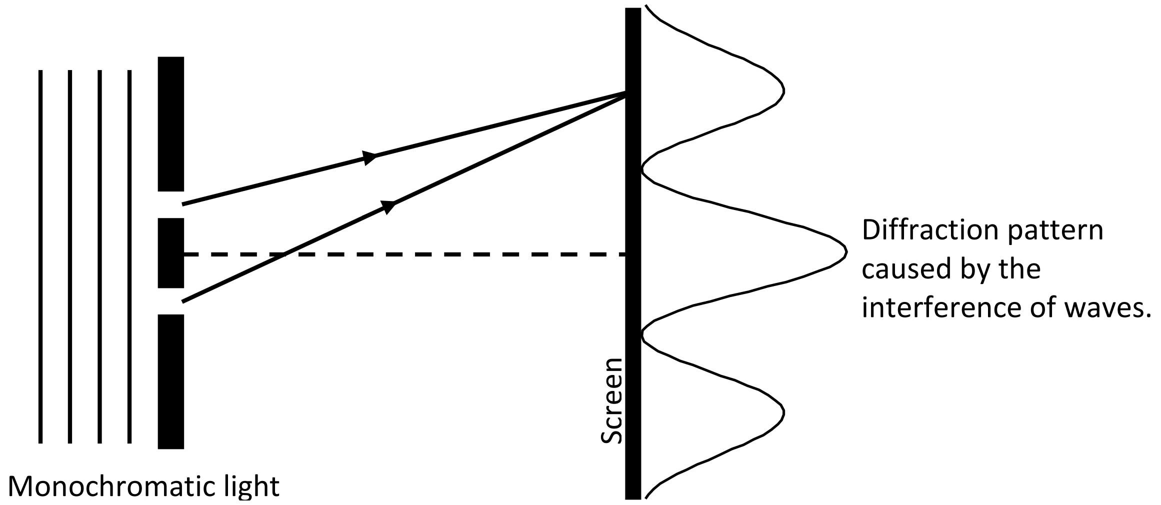
$$v = 2.19 \times 10^{6}\text{ms}^{-1}$$

Quanta and Waves problem book page 5, question 12.

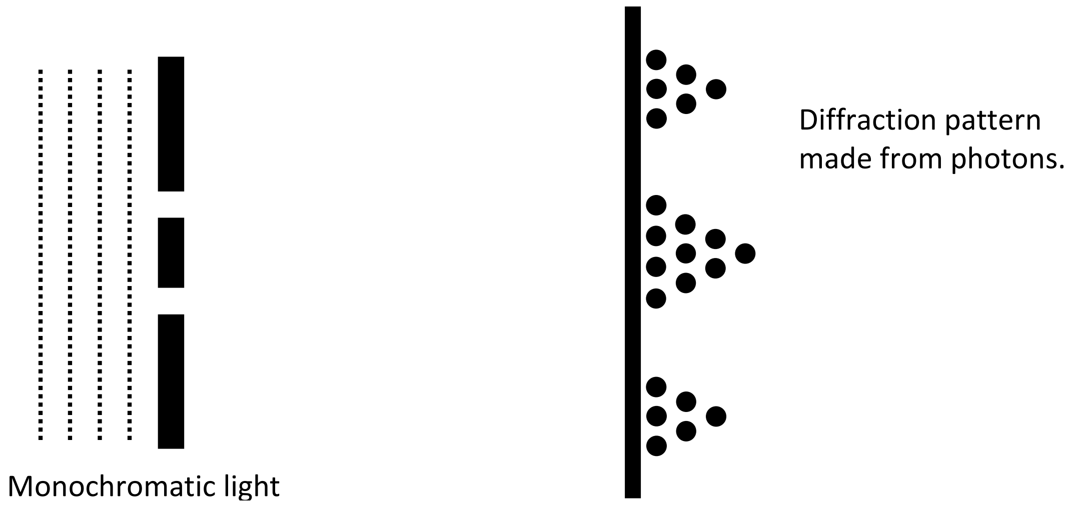
1.5 I can describe experimental evidence for wave particle duality.

There is experimental evidence which shows that light can behave both like a wave and as a particle.

Double Slit Experiment
When monochromatic light is incident on a double slit, the light passing through each slit interferes causing a diffraction pattern the screen. Interference is a test for waves so this shows that light is a wave.



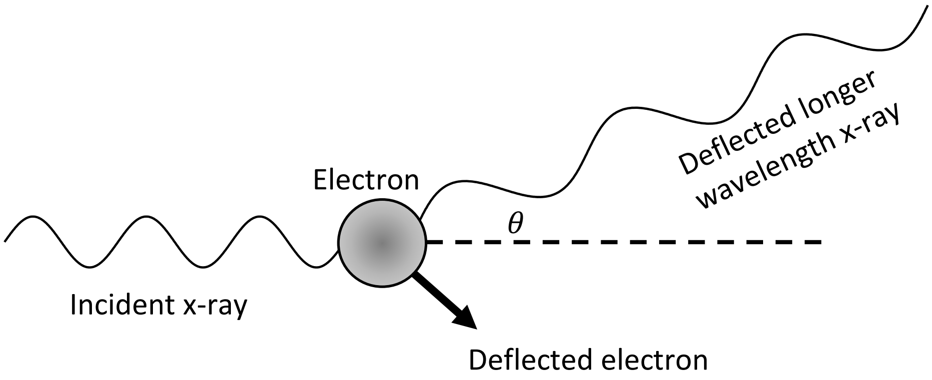
When the brightness of the source is reduced so that only individual photons pass through the slits the interference pattern is still formed. This is however made from single photons. This shows that light is made from particles.



Photoelectric effect
This is described in Section 1.1 in these notes and Section 5 in the Higher Physics Particle and Waves Notes.

Compton Scattering

When x-rays interact with electrons the incident x-ray is deflected through an angle with its wavelength increased. There is a relationship between the change in wavelength and the scattering angle which cannot be explained by treating x-rays as waves. To explain Compton scattering x-rays must be regarded as particles carrying momentum with some of the momentum being transferred to the electron..



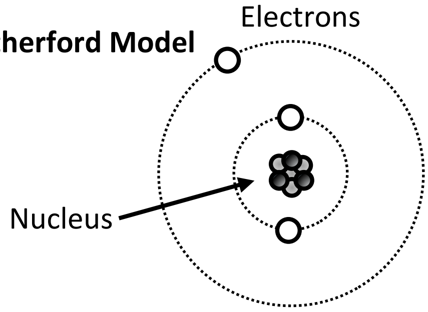
1.6 I know that quantum mechanics only calculates the probabilities of properties such as position, momentum and energy.

Quantum mechanics was developed to explain the experimental observations described in section 1.1. that could not be explained by classical physics. The development of wave particle duality of quantum mechanics allowed the nature of matter to be described more accurately . At the core of quantum mechanics is the realisation that unpredictability is at the heart of the nature of matter.
A classical physics, Newtonian, view allows all future properties of a system to be calculated if the starting properties (position, momentum etc.) are known.
Quantum mechanics showsthat we can only calculate probabilities of positions, momentum and energy of particles.

1.7 I understand the uncertainty principle in terms of; position and momentum, energy and time.

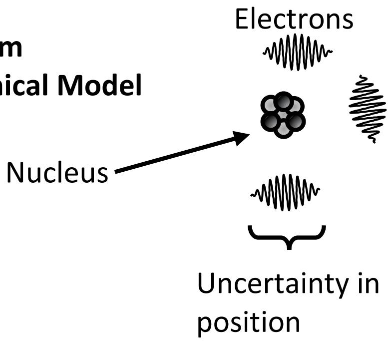
Position and Momentum

Rutherford Model



In the classical, Rutherford, model of the atom the position and momentum of electrons can be measured to any precision required.

Quantum Mechanical Model



In the quantum mechanical model the position and momentum of electrons cannot both be measured to any required precision at the same time. The probabilistic nature of matter limits the precision to which measurements can be taken. This is called the uncertainty principle. The uncertainty principle also applies to all other particles.

The limit that the uncertainty principle places on the measurement of position and momentum is given by the relationship

Planck's Constant (Js)

Uncertainty in position (m)

$$\Delta x \Delta p \geq \frac{h}{4\pi}$$

Uncertainty in Momentum ($kgms^{-1}$)

Energy and Time

The uncertainty principle can also be stated in in terms of energy and time.

Uncertainty in energy (J)

Planck's Constant (Js)

$$\Delta E \Delta t \geq \frac{h}{4\pi}$$

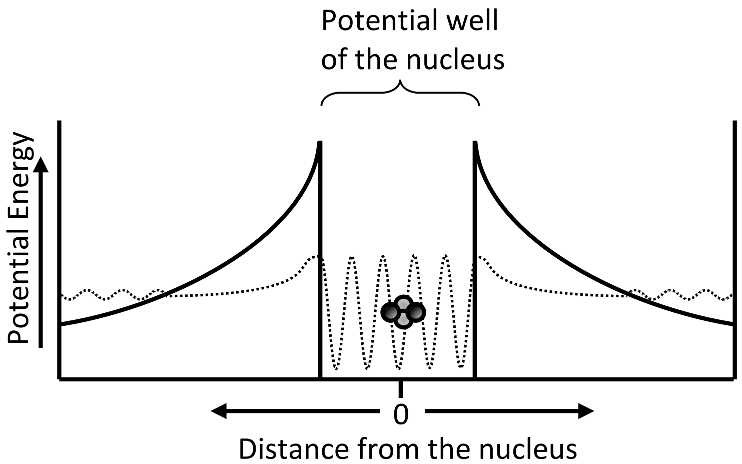
Uncertainty in time (s)

1.8 I understand how the uncertainty principle leads to quantum tunnelling.

Alpha Decay

Consider the protons in the nucleus of an unstable atom which will emit alpha radiation. When the proton is in the nucleus the strong force binds the protons and neutrons together. When outwith the range of the strong force the electrostatic repulsion will force the protons apart. This situation produces a potential well with the protons having insufficient energy to overcome the potential barrier around the nucleus and soremain bound inside.

The uncertainty principle in terms of time $\Delta E \Delta t \geq \frac{h}{4\pi}$ means that for short periods of time the protons can "borrow" sufficient energy to move to the other side of the potential barrier and escape the nucleus.



The dotted line in the diagram shows a wave that represents the probability of a proton being a particular position. The greater the amplitude of the wave the higher the probability that the proton will be in that position. Notice that there is probability that the proton can be in the nucleus, within the potential barrier or outside the potential barrier. Even though the probability of the proton being outside the nucleus is low it still exists so the alpha particle can escape from the nucleus after a period of time. Quantum tunnelling has several useful application such as the scanning tunnelling electron microscope, understanding nuclear fusion and quantum computing.

1.9 I can solve problems involving the uncertainty principal relationships.

Both the relationships $\Delta x \Delta p \geq \frac{h}{4\pi}$ and $\Delta E \Delta t \geq \frac{h}{4\pi}$ can be used to solve problems.

<u>Example</u>

A alpha particle is moving at $5\% \pm 0.05\%$ of the speed of light. Calculate the minimum uncertainty in its position.

<u>Solution</u>

For the minimum uncertainty $\Delta x \Delta p = \frac{h}{4\pi}$

From the data sheet

$m = 6.645 \times 10^{-27}\text{kg}$

$h = 6.63 \times 10^{-34}\text{Js}$

$v = 3.0 \times 10^{8}\text{ms}^{-1}$

$$p = mv = 6.645 \times 10^{-27} \times \frac{5}{100} \times 3.0 \times 10^{8} = 9.97 \times 10^{-20}\text{kgms}^{-1}$$

$\Delta p = 0.1\%$ of $9.97 \times 10^{-20}\text{kgms}^{-1}$ 0.1% as $\pm 0.05\%$ has a total uncertainty of 0.1%

$$\Delta p = \frac{0.1}{100} \times 9.97 \times 10^{-20} = 9.97 \times 10^{-23}\text{kgms}^{-1}$$

Substituting this value and h in $\Delta x \Delta p = \frac{h}{4\pi}$ gives

$$\Delta x \times 9.97 \times 10^{-23} = \frac{6.63 \times 10^{-34}}{4\pi}$$

$$\Delta x = \frac{6.63 \times 10^{-34}}{4\pi \times 9.97 \times 10^{-23}}$$

$$\Delta x = 5.3 \times 10^{-13}\text{m}$$

Quanta and Waves problem book page 4, question 1 to 4.

Key Area: Particles from Space

Success Criteria

2.1 I know the origin, composition and energy range of cosmic rays.

2.2 I can describe how cosmic rays interact with the Earth's atmosphere.

2.3 I can describe the solar wind and its interaction with the Earth's magnetic field.

2.4 I can solve problems involving charged particles in magnetic fields in terms of their; mass, velocity, charge, radius of their path and the magnetic induction of the magnetic field.

2.5 I can explain how aurorae are produced in the upper atmosphere.

2.1 I know the origin, composition and energy range of cosmic rays.

Cosmic rays are high energy particles e.g. fast moving protons and electrons or high energy electromagnetic radiation coming from space. Cosmic rays can come from the Sun, other parts of the Milky Way galaxy, supernova explosions, or from other galaxies.

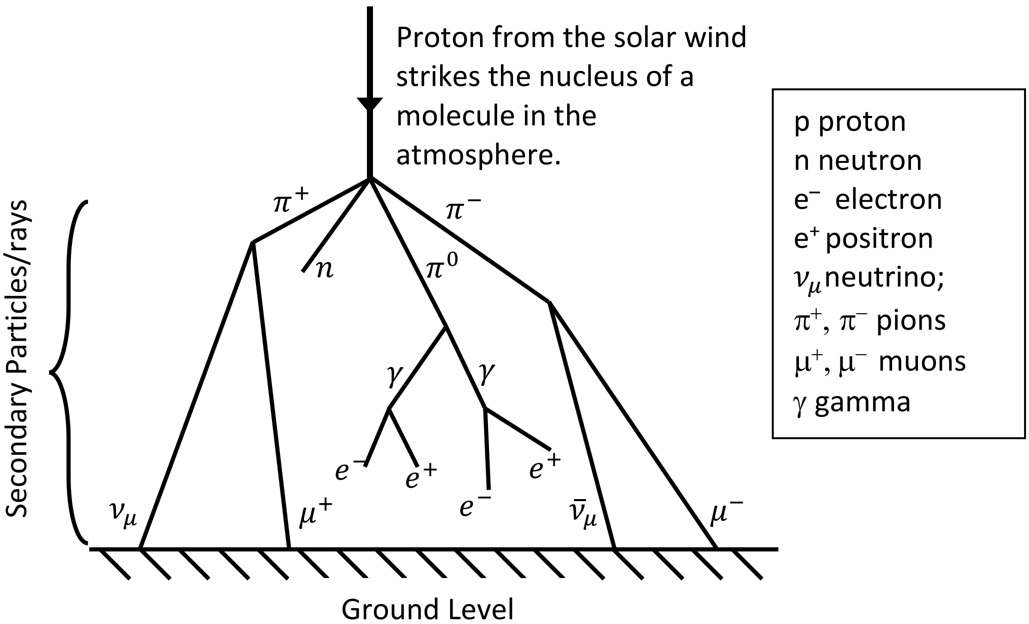
The table below shows the type and percentage composition of cosmic rays.

Type	Approximate Percentage of Cosmic Rays
Protons	89
Alpha particles	9
Carbon, nitrogen and oxygen nuclei	1
Electrons	less than 0.1
Gamma radiation	less than 0.1

The energy range of cosmic rays is $\sim 10^{-10}\mathrm{J}$ $(10^{9}\mathrm{eV})$ to $\sim 10\mathrm{J}$ $(10^{20}\mathrm{eV})$. Contrast this with particle accelerators on Earth. The Large Hadron Collider has currently, for individual protons, reached energies of $1 \times 10^{-6}\mathrm{J}$ $(6.5 \times 10^{12}\mathrm{eV})$.

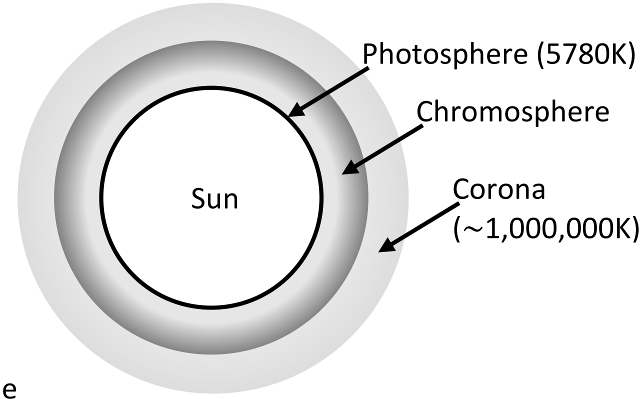
2.2 I can describe how cosmic rays interact with the Earth's atmosphere.

Very few cosmic rays reach the surface of the Earth. They collide with the nuclei of atoms in the Earth's atmosphere to produce many new secondary particles. These particles can go on to collide with other nuclei producing yet more particles. The is called a cosmic air shower.
It is these secondary particles which are detected from the ground.



2.3 I can describe the solar wind and its interaction with the Earth's magnetic field.

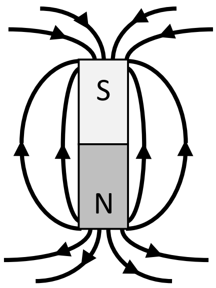
Solar Wind



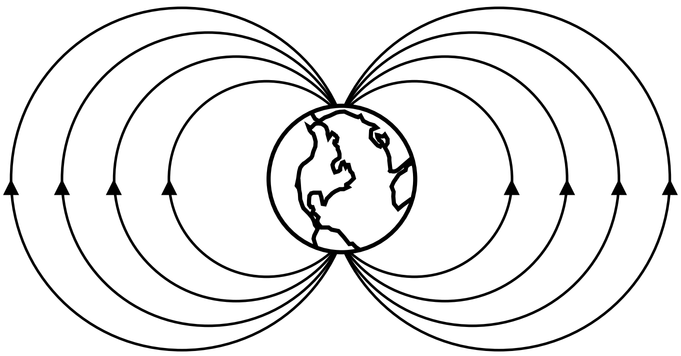
The solar wind is a plasma flowing through the solar system which originates in the atmosphere of the sun.
When the sun is observed (DON'T LOOK AT THE SUN!) what is seen is called the photosphere. The is at the radius from the centre of the Sun where radiation can escape. Beyond the photosphere radius the Sun's atmosphere consists of a further two layers, the chromosphere and the corona.
The solar photosphere emits radiation which approximates a black body temperature of 5780K. The temperature of the Sun's atmosphere varies through the chromosphere to the corona where it can reach 1,000,000K. The high temperatures in the corona mean that some nuclei have sufficient kinetic energy to reach escape speed. The solar wind consists of these higher energy particles released from the corona moving outward from the Sun. The speed of the solar wind varies from 300kms^{-1} to 1000kms^{-1}.

Earth's Magnetic Field

The Earth's liquid iron rich core produces currents which create a magnetic field. This field is similar in shape a dipole magnet in the core of the Earth.

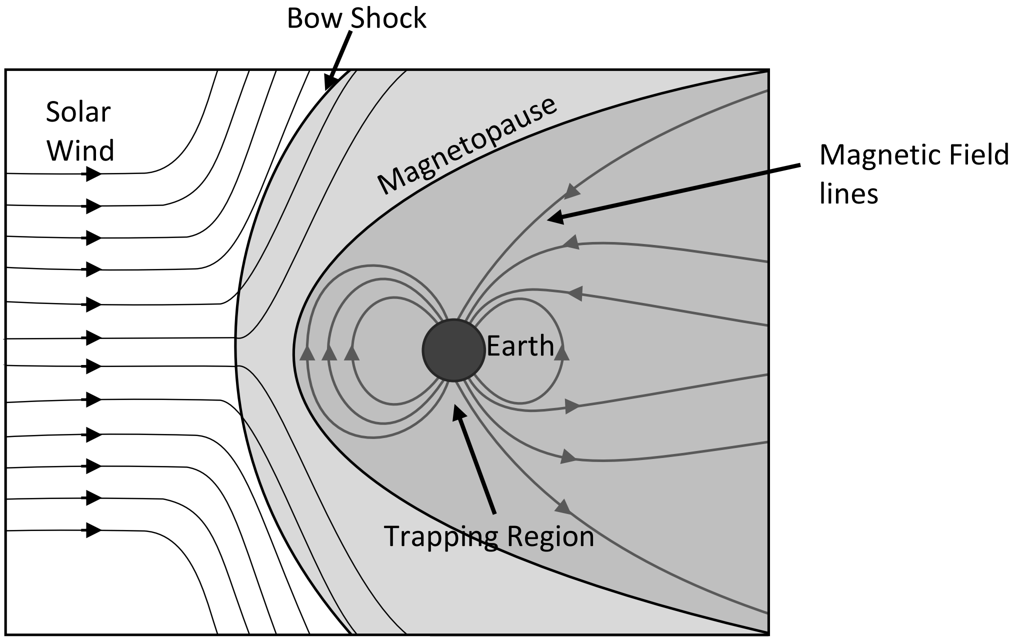


Dipole (bar) Magnet



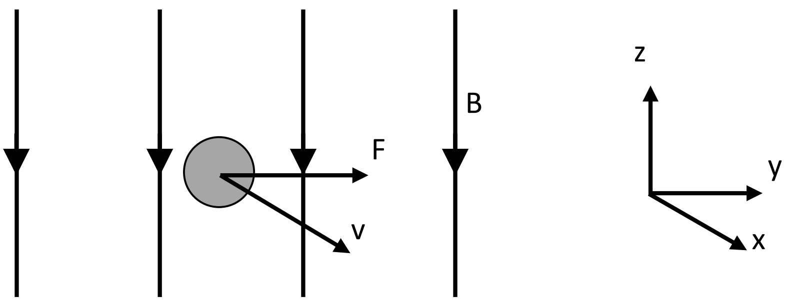
Earth's Magnetic Field

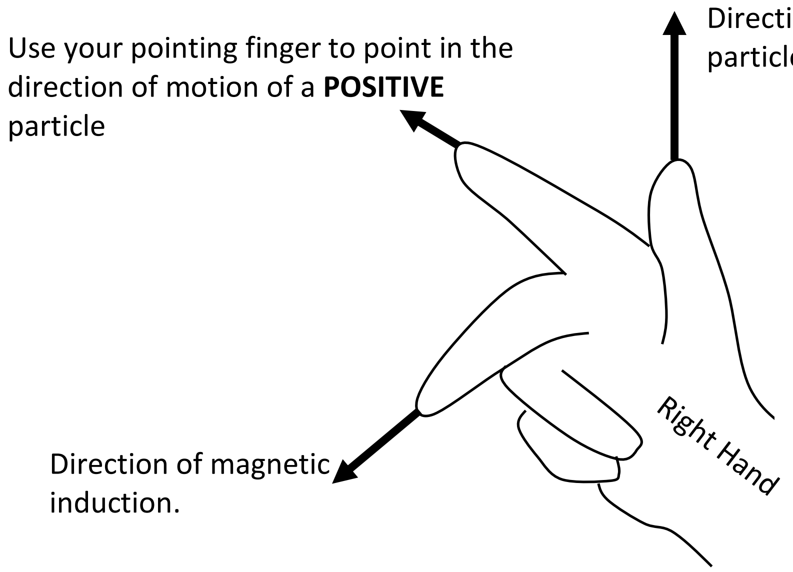
The solar wind flowing past the Earth interacts with the magnetic field distorting its shape. As the solar wind approaces the Earth it first interacts with the magnetic field at the bow shock where its direction is changed to flow around the Earth. The magnetopause marks the boundary between the space governed by the Earth's magnetic field called the magnetosphere, and the solar wind. Although the solar wind flows around the Earth some particles can get trapped by the Earth's magnetic field. This can occur in the trapping region near both poles.



2.4 I can solve problems involving charged particles in magnetic fields in terms of their; mass, velocity, charge, radius of their path and the magnetic induction of the magnetic field.

Direction of Force on a Charged Particle in a Magnetic Field
(Also see sections 2.9 to 2.13 in the Particle and Waves Higher Physics Notes)
A charged particle moving in a magnetic field experiences a force which is perpendicular to both its velocity and the magnetic field lines.

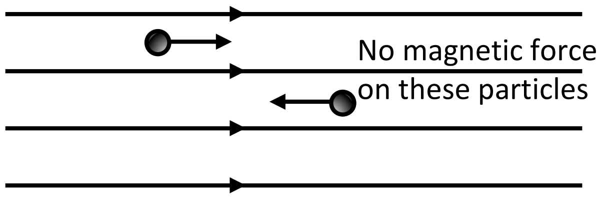




The direction of force on a positively charged particle in a magnetic field is given by the right-hand rule.
To find the direction of a negatively charged particle first find the direction of a positive particle then reverse this direction.

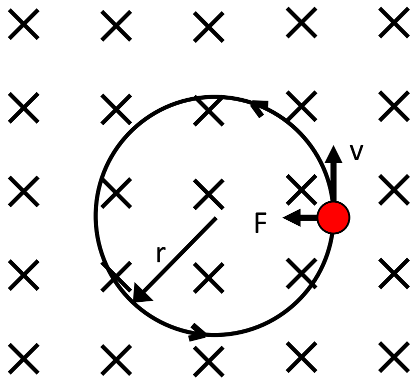
Particles Moving Parallel to the Magnetic Field Lines

When particles are moving parallel to the magnetic field lines the force on the particle is zero.



Charged Particles Moving Perpendicular to the Magnetic Field Lines

When charge particles are moving perpendicular to the magnetic field lines they follow circular paths (see section 2.13 in the Particle and Waves Higher Physics Notes).



From section 2.7 in the Rotational Motion and Astrophysics Notes the centripetal force on an object following circular motion is given by

$$F = \frac{mv^2}{r}$$

The centripetal force acting on a charged particle is provided by the magnetic field and is given by

$$F = qvB$$

Force (N)

Charge (C)

Magnitude of the component of the velocity perpendicular the magnetic field lines (ms^{-1})

Magnetic Induction (T)

This relationship is also covered in the magnetic induction section in the electromagnetism unit.

Example 1

An electron is moving at $2.0 \times 10^5 \text{ms}^{-1}$ in a uniform magnetic field perpendicular to the magnetic induction of 4.0mT. Calculate the force on the electron.

Solution 1

$F = ?$
$q = 1.6 \times 10^{-19}\text{C}$
$v = 2.0 \times 10^5 \text{ms}^{-1}$
$B = 4.0\text{mT} = 4.0 \times 10^{-3}\text{T}$

$F = qvB$
$F = 1.6 \times 10^{-19} \times 2.0 \times 10^5 \times 4.0 \times 10^{-3}$
$F = 1.3 \times 10^{-16}\text{N}$

Example 2

A proton is travelling in a circular path of radius 2.0m in magnetic field of magnetic induction 0.012T. Find the speed of the proton.

Solution 2

Equate the relationship $F = \dfrac{mv^2}{r}$ to $F = qvB$ and solve for v.

$m = 1.673 \times 10^{-27}\text{kg}$
$q = 1.6 \times 10^{-19}\text{C}$
$v = ?$
$B = 0.012\text{T}$
$r = 2.0\text{m}$

$$\frac{mv^2}{r} = qvB$$

$$v = \frac{qBr}{m}$$

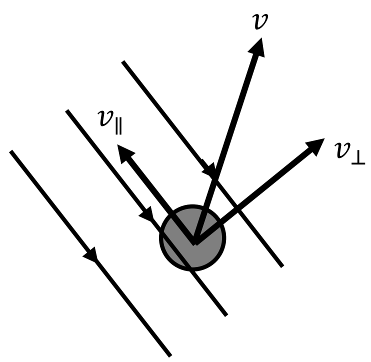
$$v = \frac{1.6 \times 10^{-19} \times 0.012 \times 2.0}{1.673 \times 10^{-27}}$$

$$v = 2.3 \times 10^6 \text{ms}^{-1}$$

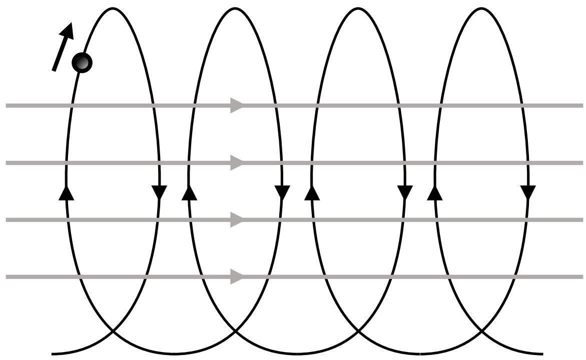
Quanta and Waves problem book pages 5 to 8, question 1 to 14.

Charged Particles Moving at an Angle to the Magnetic Field Lines

It is only the component of the velocity perpendicular to the magnetic field lines, $v_{\perp}$, which produces a force on a charged particle. This produces a centripetal force on the particle causing it to follow circular path. The component parallel to the magnetic field lines does not produce a force on the charged particle so this component of velocity will be constant.

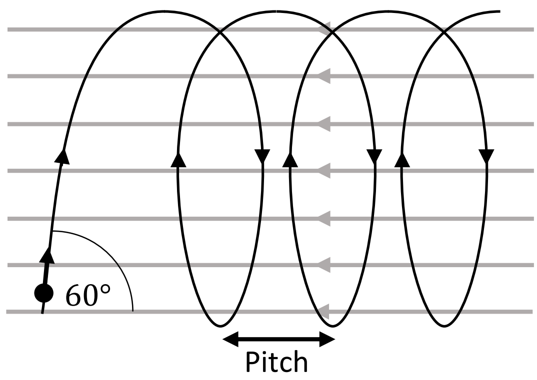


The combination of circular motion and constant linear velocity produces a helical path.



<u>Example</u>
A proton travelling at $2.3 \times 10^6 \mathrm{ms}^{-1}$ enters a uniform magnetic field of magnetic induction $200\mathrm{mT}$ at an angle of 60°. It follows a helical path shown in the diagram.
Find:

a. The component of the velocity parallel to *B*.
b. The component of the velocity perpendicular to *B*.
c. Radius of the helix
d. The period of rotation of the proton.
e. The pitch of the helix.



Solution

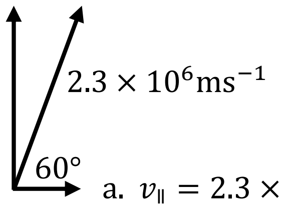
$m = 1.673 \times 10^{-27}\text{kg}$

$q = 1.6 \times 10^{-19}\text{C}$

$v = 2.3 \times 10^{6}\text{ms}^{-1}$

$B = 200\text{mT} = 200 \times 10^{-3}\text{T}$

b. $v_{\perp} = 2.3 \times 10^{6} \sin 60° = 1.99 \times 10^{6}\text{ms}^{-1}$



a. $v_{\parallel} = 2.3 \times 10^{6} \cos 60° = 1.15 \times 10^{6}\text{ms}^{-1}$

c.
$$\frac{mv^2}{r} = qvB$$
$$r = \frac{mv}{qB}$$
$$r = \frac{1.673 \times 10^{-27} \times 1.99 \times 10^{6}}{1.6 \times 10^{-19} \times 200 \times 10^{-3}}$$
$$r = 0.10\text{m} \quad (0.104\text{m})$$

d. Circumference, $\text{C} = 2\pi r$ and $d = v_{\perp}t$ where $v_{\perp}$ is the perpendicular component of the velocity.

$$t = \frac{2\pi r}{v_{\perp}}$$
$$t = \frac{2 \times \pi \times 0.104}{1.99 \times 10^{6}}$$
$$t = 3.3 \times 10^{-7}\text{s} \quad (3.28 \times 10^{-7}\text{s})$$

e. The pitch will be the distance travelled in one rotation of the helix.

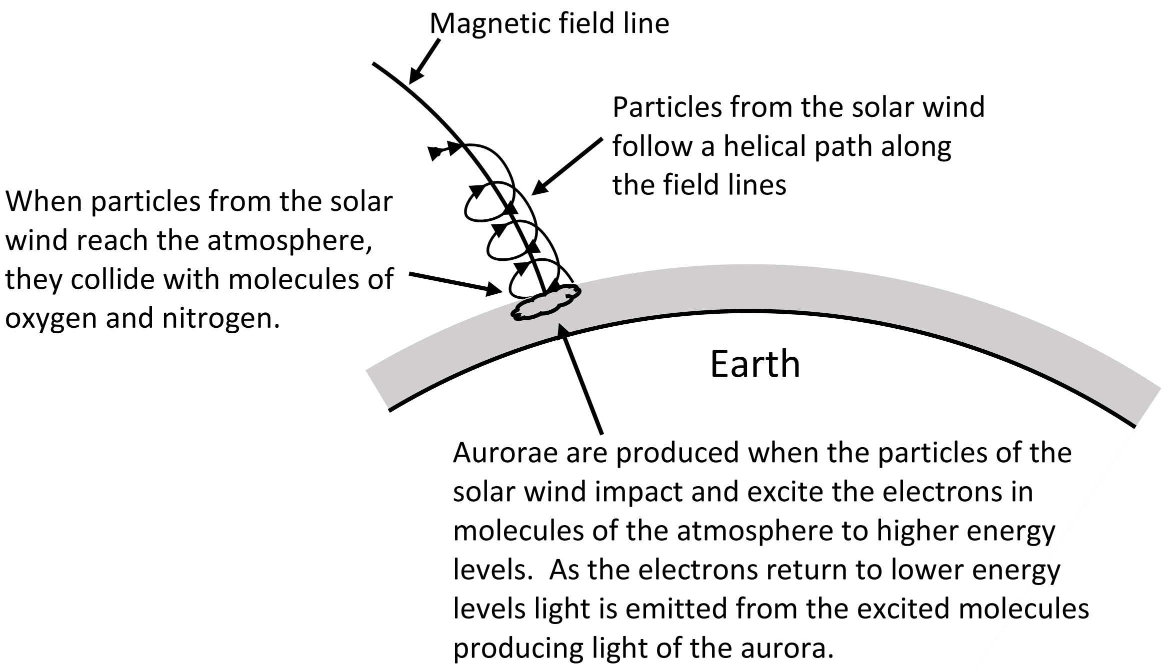
$d = v_{\parallel}t$ where $v_{\parallel}$ is the parallel component of the velocity.

$$d = 1.15 \times 10^{6} \times 3.28 \times 10^{-7}$$
$$d = 0.38\text{m}$$

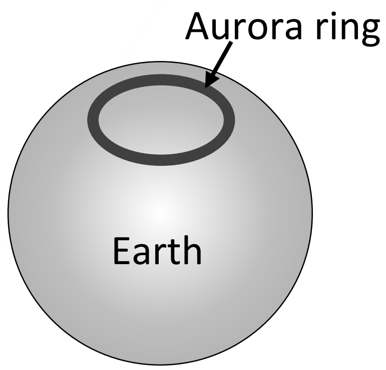
Quanta and Waves problem book pages 5 to 9, question 15 to 18.

2.5 I can explain how aurorae are produced in the upper atmosphere.

Aurorae are natural phenomena which produce displays of light in the upper atmosphere mostly in the Arctic and Antarctic regions.
Usually the solar wind is deflected around the Earth by the Earth's magnetic field. The flow of the solar wind can be disrupted and increased by the ejection of large amounts of material from the Sun e.g. a solar flare. This disruption allows some particles from the solar wind to interact with the Earth's magnetic field in the trapping region near the poles and reach the atmosphere.



The aurorae tend to occur in a ring between 10° and 20° around the both poles. During solar storms, they can occur at lower latitudes.



Note
Aurora occur near both poles.
In the arctic they are called the **Northern Lights** or the **Aurora Borealis**.
In the Antarctic, they are called the **Aurora Australis.**

Quanta and Waves problem book page 10, question 18 and 19.

Key Area: Simple Harmonic Motion

Success Criteria

3.1 I can define simple harmonic motion (SHM).

3.2 I know the relationship Between SHM and Circular Motion.

3.3 I can derive the relationship $v = \pm\omega\sqrt{A^2 - y^2}$.

3.4 I can solve problems involving displacement, velocity, acceleration, angular frequency and period.

3.5 I can derive the relationship $E_k = \frac{1}{2}m\omega^2(A^2 - y^2)$

3.6 I can solve problems in simple harmonic motion involving kinetic and potential energy.

3.7 I know the effects of damping on simple harmonic motion and can explain the terms underdamping, critical damping and overdamping.

3.1 I can define simple harmonic motion (SHM).

Simple harmonic motion is a sinusoidal oscillation of an object. It occurs when the unbalanced restoring force is in the opposite direction and directly proportional to the displacement.

Negative sign shows *F* is in the opposite direction to *y*.

Restoring Force (N)

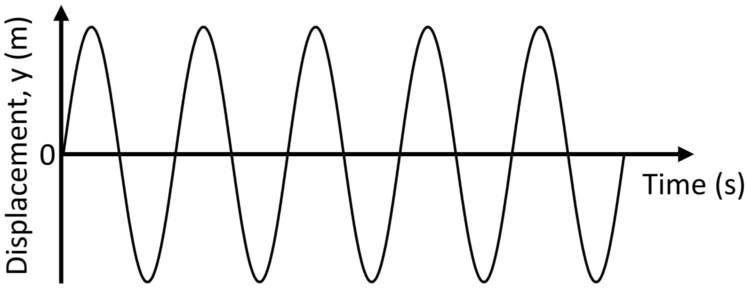
Displacement from the equilibrium position (m)

$$F = -ky$$

A constant which depends on the properties of the system (Nm^{-1})

This relationship is not given on the relationship sheet.

The oscillation of an object following SHM is a sine wave or a cosine wave.
For a particular system, the displacement may be non-zero at $t = 0$s



SHM Example – Mass on a spring.

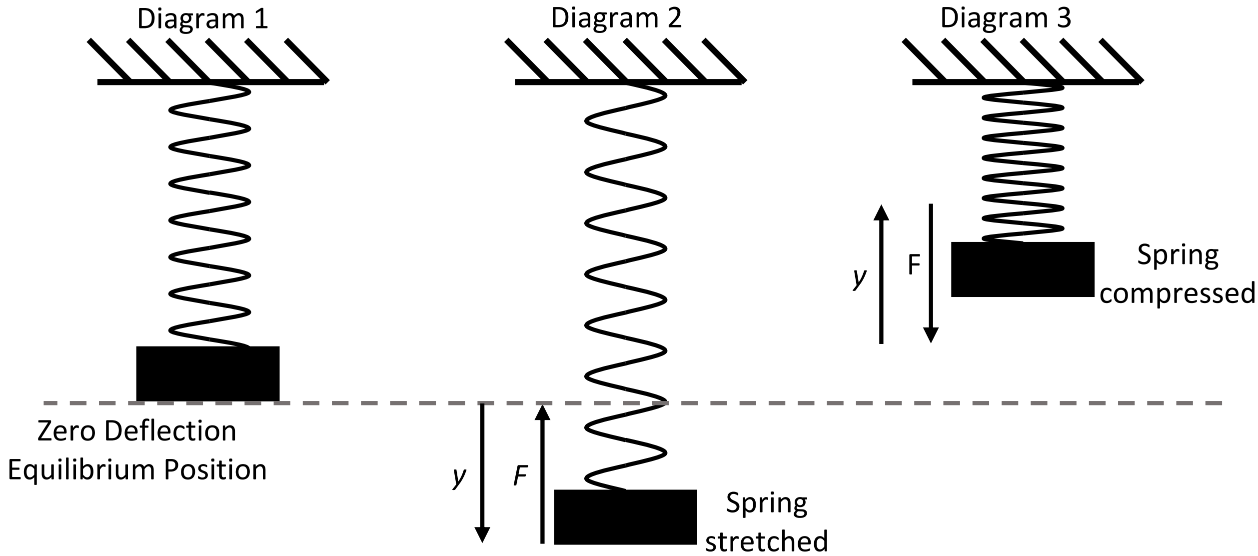
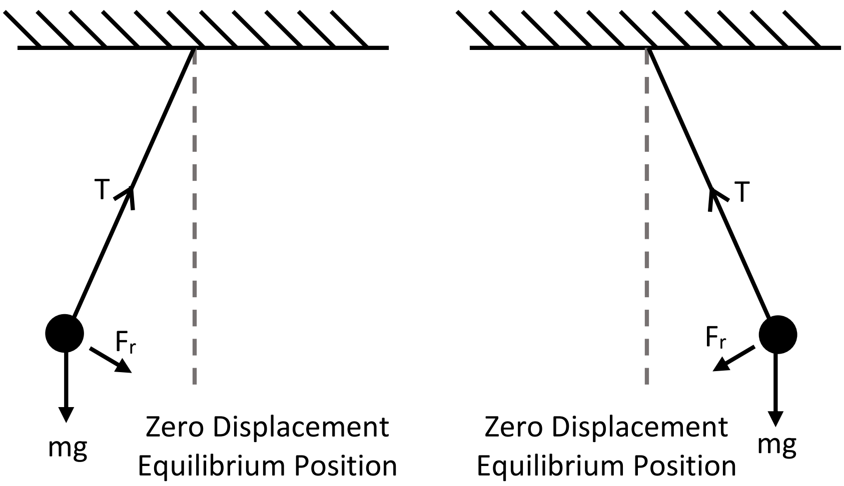


Diagram 1 shows a stationary mass suspended from a spring in its equilibrium position. The weight of the mass will balance the upward force produced by the spring.

In Diagram 2 the mass has been pulled downwards and released. The force produced by the spring will be greater than the weight so the restoring force will be upwards. As the force produced by a spring is proportional to its change its length the restoring force will be proportional to the displacement of the mass.

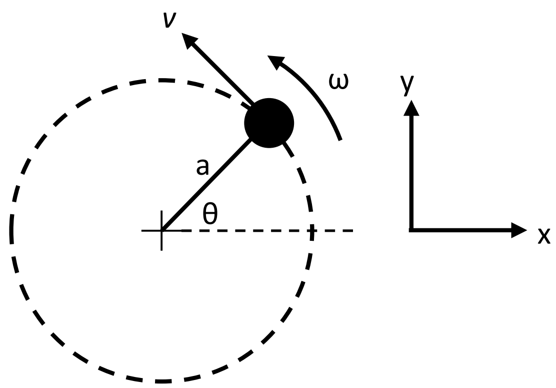
In diagram 3 the spring is compressed relative to the equilibrium position. The weight will be greater than the force produced by the spring so the restoring force will be downwards.

SHM Example – Pendulum oscillating through small angle
For small angles the restoring force is proportional to the displacement and acts in the opposite direction to the displacement. The displacement is measured from the vertical position along the arc followed by the bob.

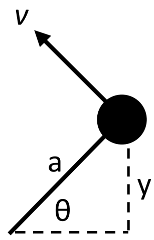


3.2 I know the relationship Between SHM and Circular Motion

The object below is rotating at a constant angular velocity.



Taking the y-component of the circular motion



$y = a \sin \theta$

Angular velocity is defined as

$$\omega = \frac{\theta}{t}$$

Then

$\theta = \omega t$

This can be substituted into the y-component giving

$y = a \sin \omega t$

Which also describes the displacement in SHM. The linear projection of circular motion gives simple harmonic motion.

3.3 I can derive the relationship $v = \pm\omega\sqrt{A^2 - y^2}$.

In simple harmonic motion the relationship for v can be found from the relationship for displacement by the following steps

Start with $y = A\cos\omega t$ (You could also start with $y = A\sin\omega t$)

Differentiate $v = \frac{dy}{dt} = -A\omega\sin\omega t$

Square both expressions

$$y^2 = A^2\cos^2\omega t$$
$$v^2 = A^2\omega^2\sin^2\omega t$$

rearrange for $\sin^2\omega t$ and $\cos^2\omega t$

$$\cos^2\omega t = \frac{y^2}{A^2}$$
$$\sin^2\omega t = \frac{v^2}{A^2\omega^2}$$

Add both expressions $\cos^2\omega t + \sin^2\omega t = \frac{y^2}{A^2} + \frac{v^2}{A^2\omega^2}$

Use the identity $\cos^2\theta + \sin^2\theta = 1$

$$\frac{y^2}{A^2} + \frac{v^2}{A^2\omega^2} = 1$$

Some rearrangement required to find the final expression for v.

$$\frac{v^2}{A^2\omega^2} = 1 - \frac{y^2}{A^2}$$
$$v^2 = A^2\omega^2\left(1 - \frac{y^2}{A^2}\right)$$
$$v^2 = \omega^2(A^2 - y^2)$$
$$v = \pm\omega\sqrt{A^2 - y^2}$$

3.4 I can solve problems involving displacement, velocity, acceleration, angular frequency and period.

In the advanced higher course we will only deal with systems where the displacement can be represented by a simple sine or cosine.
The following relationships are used to calculate displacement, velocity, acceleration, angular velocity and period

Displacement

Amplitude of the oscillation (m)

Angular Frequency (rad s^{-1})

Displacement (m)

$$y = A\sin\omega t$$

Time (s)

or

$$y = A\cos\omega t$$

Velocity

$$v = \pm\omega\sqrt{A^2 - y^2}$$

The + gives the speed in one direction and the – in the other direction.

Acceleration

$$a = \frac{d^2y}{dt^2} - \omega^2 y$$

Angular frequency

Frequency (Hz)

$$\omega = 2\pi f$$

Period

Period (s)

$$\underbrace{T = \frac{1}{f}} = \frac{2\pi}{\omega}$$

From National 5.
Not given on the relationships sheet.

Example

An object moves with simple harmonic motion given by the expression $y = 4.0 \sin 1.7t$ where y is measured in metres.

Find:

a. the amplitude of the motion.

b. the frequency.

c. the period of the motion.

d. the maximum speed of the object.

e. the magnitude of acceleration at $t = 1.2\text{s}$

Solution

a. Compare the given expression with the displacement relationship

$y = 4.0 \sin 1.7t$

$y = \ A \ \ \sin \omega t$

$A = 4.0\text{m}$

$\omega = 1.7 \text{ rad s}^{-1}$

b. $\omega = 2\pi f \quad \Rightarrow \quad f = \dfrac{\omega}{2\pi}$

$f = \dfrac{1.7}{2\pi}$

$f = 0.27\text{Hz} \quad (0.271\text{Hz})$

c. $T = \dfrac{1}{f}$

$T = \dfrac{1}{0.271} = 3.7\text{s}$

d. $v = \pm\omega\sqrt{A^2 - y^2}$ Maximum speed will occur when $y = 0\text{m}$

$v_{max} = \omega A$ No need for the $\pm$ term as speed is required not velocity.

$v_{max} = 1.7 \times 4.0 = 6.8\text{ms}^{-1}$

e. Use $y = 4.0 \sin 1.7t$ to find y at $t = 1.2\text{s}$

$y = 4.0 \sin(1.7 \times 1.2)$

$y = 3.57\text{m}$

Remember to set your calculator to radians.

$a = \omega^2 y$ negative not required as magnitude only is required.

$a = 0.271^2 \times 3.57$

$a = 0.26\text{ms}^{-2}$

Quanta and Waves problem book pages 10 and 11, question 1 to 6.

3.5 I can derive the relationship $E_k = \frac{1}{2}m\omega^2(A^2 - y^2)$

Start with the relationships	$E_k = \frac{1}{2}mv^2$ and $v = \pm\omega\sqrt{A^2 - y^2}$
Substitute the relationship for v into the relationship for E_k.	$E_k = \frac{1}{2}m\left(\omega\sqrt{A^2 - y^2}\right)^2$ No need for ± as speed is required $E_k = \frac{1}{2}m\omega^2(A^2 - y^2)$

3.6 I can solve problems in simple harmonic motion involving kinetic and potential energy.

Kinetic Energy (J)

$$E_k = \frac{1}{2}m\omega^2(A^2 - y^2)$$

Potential Energy (J)

$$E_p = \frac{1}{2}m\omega^2 y^2$$

Example

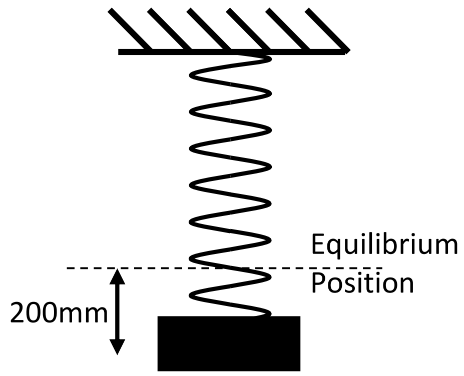
A 0.2kg mass on a spring is pulled downward by 200mm and released. It oscillates with a period of 0.80s.

Find:

a. an expression for the displacement, y, in mm.

b. The maximum kinetic energy.

c. The potential energy when the mass is 100mm below the equilibrium position.



Solution

a. Find ω.

$A = 200\text{mm} = 0.2\text{m}$
$T = 0.80\text{s}$

$$f = \frac{1}{T} = \frac{1}{0.80} = 1.25\text{Hz}$$

$$\omega = 2\pi f = 2\pi \times 1.25 = 7.85 \text{ rad s}^{-1}$$

At $t = 0$s the mass is at maximum displacement so the expression for amplitude must contain a cosine term

$y = A\cos\omega t$

Substituting in the values gives

$y = 200\cos 7.85\text{t}$

b. $$E_k = \frac{1}{2}m\omega^2(A^2 - y^2)$$

Maximum kinetic energy occurs at $y = 0\text{m}$.

$$E_{k,max} = \frac{1}{2}m\omega^2 A^2 = \frac{1}{2} \times 0.2 \times 7.85^2 \times 0.2^2 = 0.25\text{J}$$

c. $y = 100\text{mm} = 0.1\text{m}$

$$E_p = \frac{1}{2}m\omega^2 y^2$$

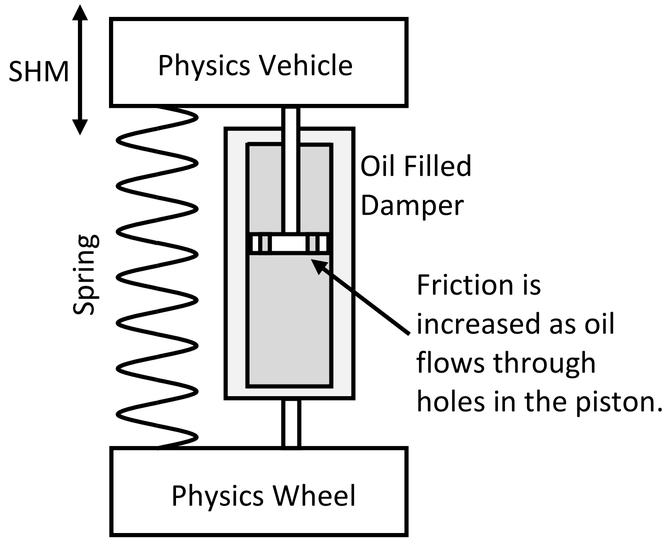
$$E_p = \frac{1}{2} \times 0.2 \times 7.85^2 \times 0.1^2 = 6.2 \times 10^{-2}\text{J}$$

Quanta and Waves problem book pages 11 to 13, question 7 to 10.

3.7 I know the effects of damping on simple harmonic motion and can explain the terms underdamping, critical damping and overdamping.

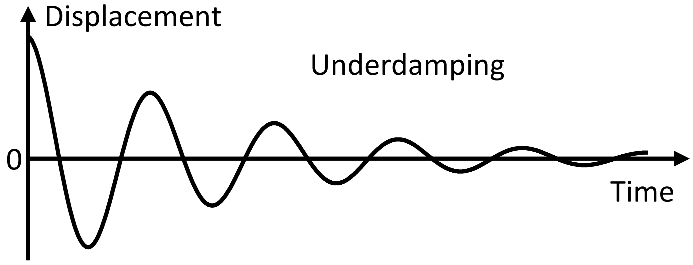
Damping is frequently used to reduce the amplitude of, or prevent, oscillations in systems where simple harmonic motion can occur. Damping increases the friction in the oscillating system which dissipates energy and so reduces the amplitude of oscillation.

An example of damping occurs in vehicle suspension systems. These use a spring and an oil filled damper to smooth out bumps in the road. Without damping a bump in the road would cause an oscillation in the vehicle which would continue for an extended period of time.

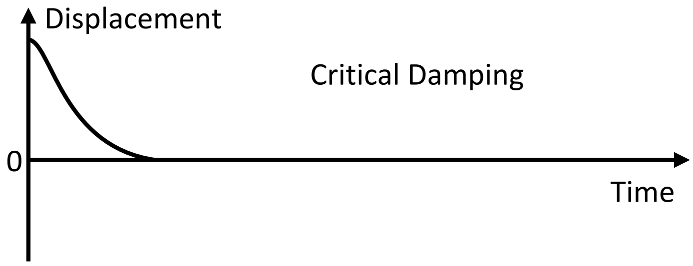


The graphs below show how damping causes the system to return to the equilibrium position after a disturbance.

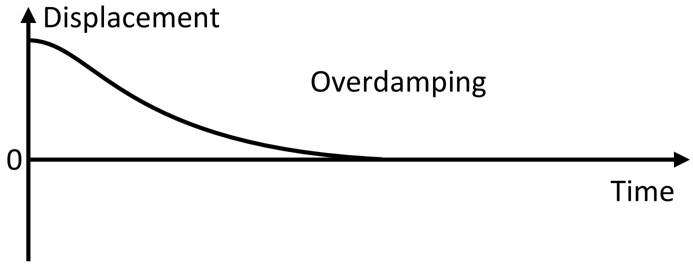
When there is insufficient damping in a system it will oscillate for an extended period of time before returning to the equilibrium position. This is called **underdamping.**



Damping of an oscillating system is usually set so that the system returns to the equilibrium position in the minimum time without oscillation. This is called **critical damping.**



When there is too much damping in the system, it returns to the equilibrium position over an extended period of time without oscillating. This is called **overdamping**.



Key Area: Waves

Success Criteria

4.1 I can use the relationship $E = kA^2$ to solve problems involving amplitude and energy of waves.

4.2 I can explain why the relationship $y = A\sin 2\pi\left(ft - \frac{x}{\lambda}\right)$ represents travelling wave.

4.3 I can solve problems involving travelling waves.

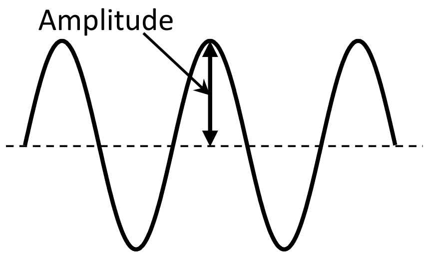
4.4 I understand the term phase difference (phase angle).

4.5 I understand what is meant by the superposition of waves.

4.6 I understand how stationary waves (standing waves) are formed; the term node and anti-node; and the relationship between nodes, anti-nodes and wavelength.

4.1 I can use the relationship $E = kA^2$ to solve problems involving amplitude and energy of waves.

The energy carried by the motion of a wave is proportional to the amplitude squared.



Energy (J)

Amplitude (m)

$$E = kA^2$$

A constant which depends on the properties of the medium e.g. density, viscosity, rigidity etc.

<u>Example</u>

Waves on a calm day approach a pier with an amplitude of 1.0m. On a stormy day, they have an amplitude of 4.0m. How many times greater is the energy of the waves on the stormy day?

<u>Solution</u>

Calm day

$$E_{calm} = kA^2 \quad \Rightarrow \quad k = \frac{E_{calm}}{A^2}$$

$$k = \frac{E_{calm}}{1.0^2} = E_{calm}$$

This expression for k can be substituted in to $E = kA^2$ to give

$$E = E_{calm}A^2$$

Stormy day

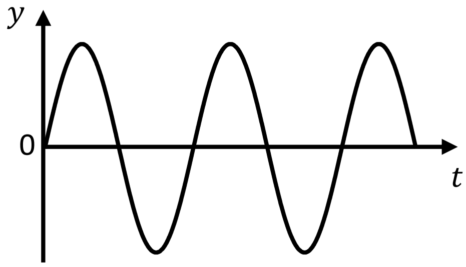
$$E_{stormy} = E_{calm} \times 4.0^2$$

$$E_{stormy} = 16E_{calm}$$

4.2 I can explain why the relationship $y = A\sin 2\pi\left(ft - \frac{x}{\lambda}\right)$ represents travelling wave.

Any travelling wave can be represented in two different ways.

Time representation
Imagine standing still in a swimming pool as waves travel past. If you plotted the displacement of the water's surface, y, against time at your position you would get the following graph.



The time representation of a wave shows how the displacement, y, varies with time, t, at a particular point in space.

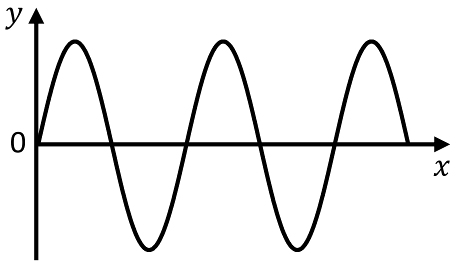
The relationship which describes is graph is

$y = A\sin\omega t$. i.e. SHM

As $\omega = 2\pi f$

Then $y = A\sin(2\pi ft)$ Where the term in brackets is in radians.

Spatial representation
Take a photograph of the same wave in the swimming pool. Draw some axes and you now have a graph of how displacement varies with distance along the pool, x, at a particular instant of time



Spatial representation of a wave shows how the displacement, y, varies with distance, x, at a particular instant of time.

The relationship which describes the graph is

$y = A\sin\left(\frac{2\pi x}{\lambda}\right)$ Where the term in brackets is in radians.

The relationships for the time and spatial representations of a wave can be combined to give the equation of a travelling wave.

$$y = A\sin 2\pi\left(ft - \frac{x}{\lambda}\right)$$

This represents a wave moving in the positive x direction. For a wave moving in the negative x direction the negative sign in the bracket is changed to a positive.

4.3 I can solve problems involving travelling waves.

Example 1

A travelling wave is represented by the expression $y = 4.0 \sin 2\pi(1.4t - 16x)$ where y and x are in metres and t is in seconds.

Find:

a. the amplitude

b. the frequency

c. the period

d. the wavelength

e. the speed

Solution

Compare the travelling wave relationship the one given

$$y = A \sin 2\pi \left(ft - \frac{x}{\lambda}\right)$$

$$y = 4.0 \sin 2\pi(1.4t - 16x)$$

a. $A = 4.0\text{m}$

b. $f = 1.4\text{Hz}$

c. $T = \frac{1}{f} = \frac{1}{1.4} = 0.71\text{s}$

d. $\lambda = \frac{1}{16} = 0.063\text{m} \quad (0.0625\text{m})$

e. $v = f\lambda$

$v = 1.4 \times 0.0625$

$v = 0.088\text{ms}^{-1}$

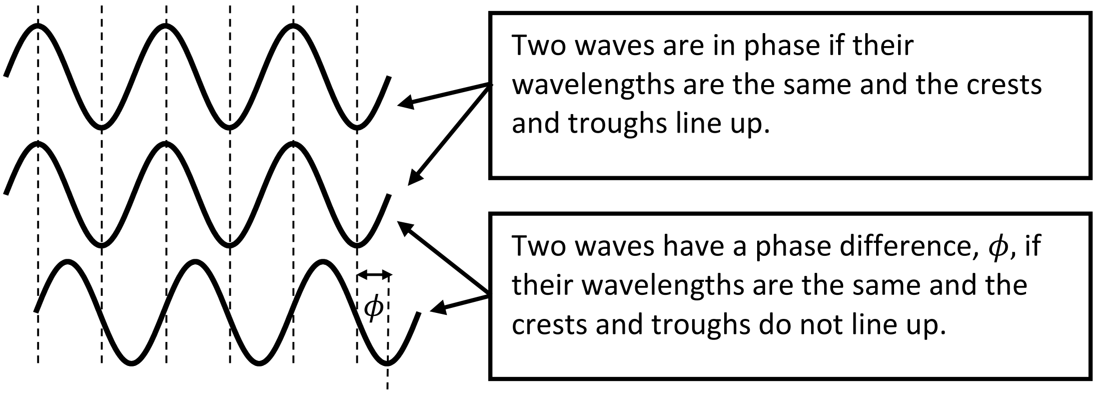
Quanta and Waves problem book pages 13 to 15, question 1 to 10.

Remember to set your calculator to radians.

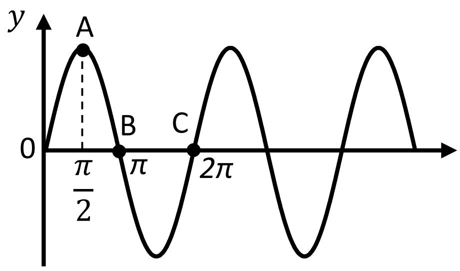
4.4 I understand the term phase difference (phase angle).

There are two meanings of phase difference (also called phase angle).

Phase difference between two waves
The phase difference between the two waves is the angular separation (in radians) of two peaks, two troughs or two other corresponding points on the wave.



Phase difference between points on the same wave
The phase difference between two points on a wave is the angular separation (in radians) of the two points.



The phase difference between A and B is $\pi - \frac{\pi}{2} = \frac{\pi}{2}$

The phase difference between A and C is $2\pi - \frac{\pi}{2} = \frac{3\pi}{4}$

The phase difference between B and C is $2\pi - \pi = \pi$

Phase difference can also be calculated using the equation

Phase difference (radians)

Distance between the two points (metres)

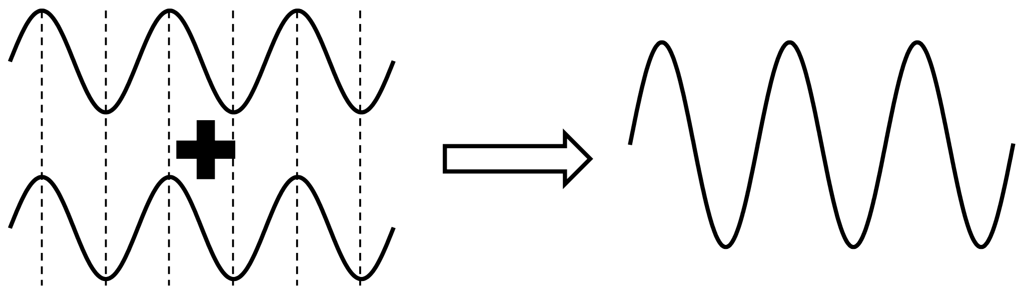
$$\phi = \frac{2\pi x}{\lambda}$$

Wavelength (metres)

Quanta and Waves problem book page 15, questions 11 to 13.

4.5 I understand what is meant by the superposition of waves.

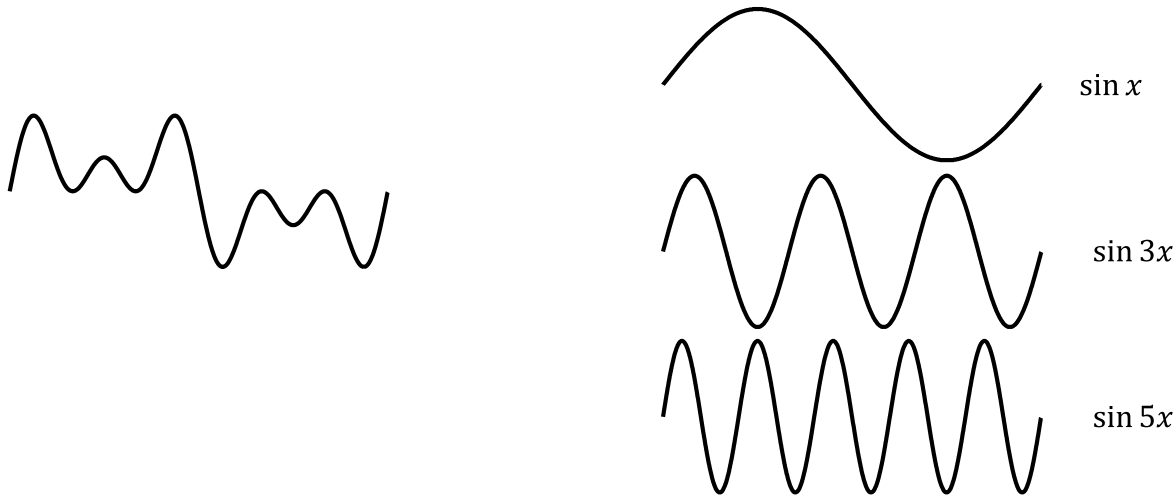
Superposition of waves is when two or more waves overlap at a point. The overall displacement is the sum of the displacements at that point.



Superposition Example - Fourier Analysis

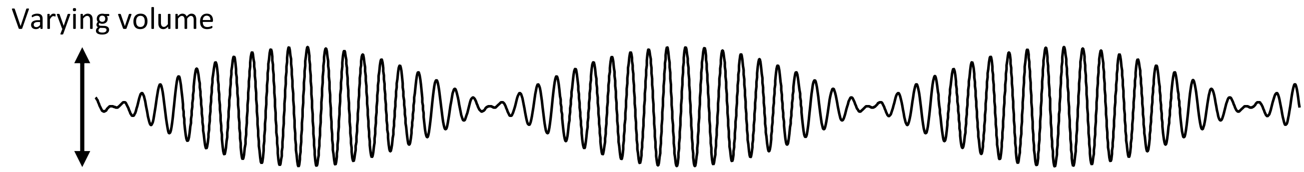
All waves (or any periodic motion), regardless of how complicated, can be broken down into a series of sine and cosine waves. This is called Fourier Analysis.

In the example below the complicated wave on the left is the superposition of the three waves on the right.



Superposition Example – Audio Beats

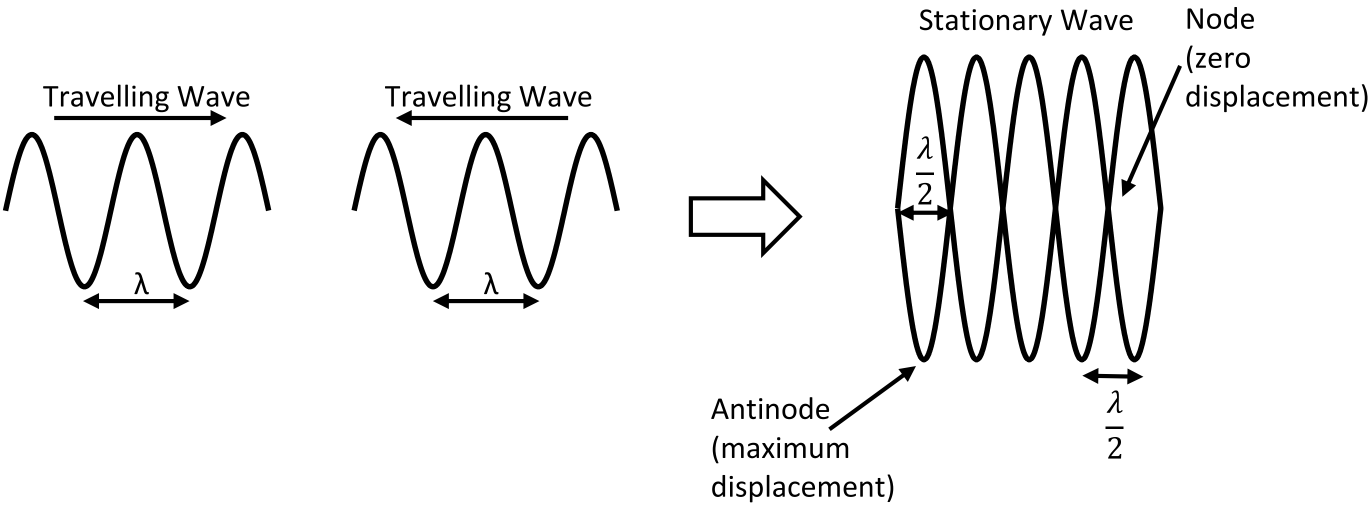
Audio beats occur when two sounds of slightly different frequency are played together. The superposition of the two waves causes a variation in volume at a frequency corresponding to the difference in frequency between the two sounds.



Audio beats can be used when tuning musical instruments. A reference source e.g. a tuning fork can be played with the instrument to be tuned. The tuning is adjusted until the audio beats can no longer be heard.

4.6 I understand how stationary waves are formed; the term node and anti-node and the relationship between nodes; anti-nodes and wavelength.

When two waves of the same wavelength are travelling in opposite directions they form a stationary wave (sometimes called a standing wave). The profile of a stationary wave does not move.



Node A point of zero displacement on a stationary wave.
The distance between two adjacent points of zero displacement. This is equal to half a wavelength.
Antinode A point of maximum displacement on a stationary wave.
The distance between two adjacent points of maximum displacement. This is equal to half a wavelength.

Stationary waves are frequently caused by the interference of a wave and its reflection.

Quanta and Waves problem book page 15 and 16, questions 14 to 16.

Key Area: Interference

Success Criteria

5.1 I understand the term coherent.

5.2 I understand the terms geometric path length, optical path length and optical path difference.

5.3 I know the conditions for constructive and destructive interference.

5.4 I can explain what it meant by division of amplitude.

5.5 I understand the phase changes which occur when a light ray is reflected or transmitted.

5.6 I can explain and solve problems involving thin film interference.

5.7 I can explain and solve problems involving wedge fringes.

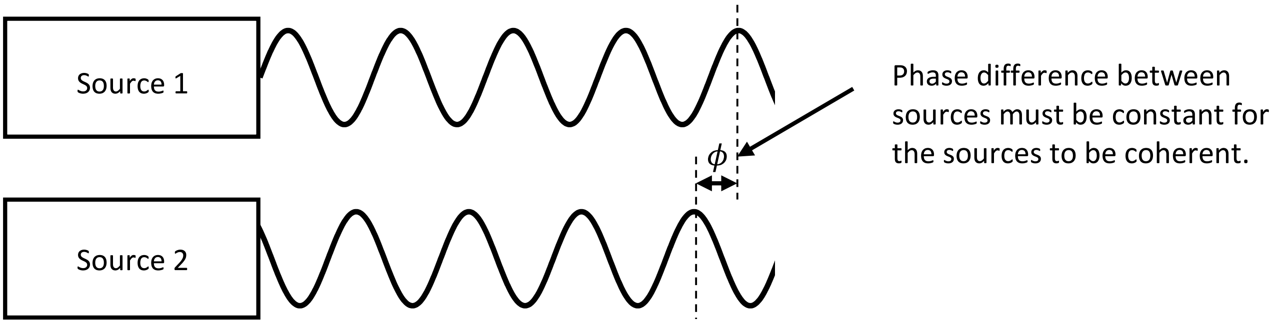
5.8 I can explain how anti-reflective coatings (blooming) on lenses reduce reflections, derive the relationship $d = \frac{\lambda}{4n}$ and solve problems using this relationship.

5.9 I can explain what it meant by division of wavefront.

5.10 I can use the relationship $\Delta x = \frac{\lambda D}{d}$ to solve problems involving division of wavefront.

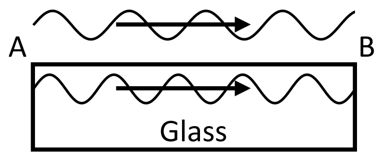
5.1 I understand the term coherent.

Two sources are coherent when they have the same frequency, travel in the same direction and have a constant phase difference.



5.2 I understand the terms geometric path length, optical path length and optical path difference.

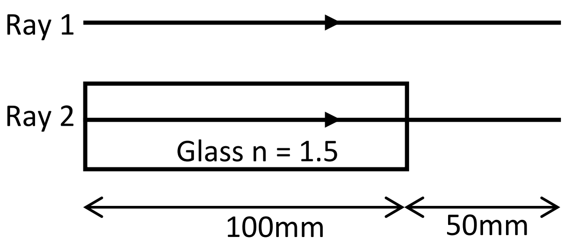
When light travels through a material its speed and wavelength are changed. To take these changes into account when considering interference optical path length must be used.



Geometric path length is the physical distance travelled by the wave the distance between A and B.
Optical path length = Geometric path length × Refractive index
Optical path difference is the difference in optical path length between two rays of light.

<u>Example</u>
The diagram shows two light rays. For each ray find;

a. the geometric path length.
b. the optical path length.
c. the optical path difference.



<u>Solution</u>

a. The for both rays the distance travelled and geometric path length is 150mm.

b. Optical path length = geometric path length × Refractive index
Ray 1 optical path length = 150 × 1 = 150mm (n = 1 for air).
Ray 2 optical path length = (100 × 1.5) + (50 × 1) = 200mm

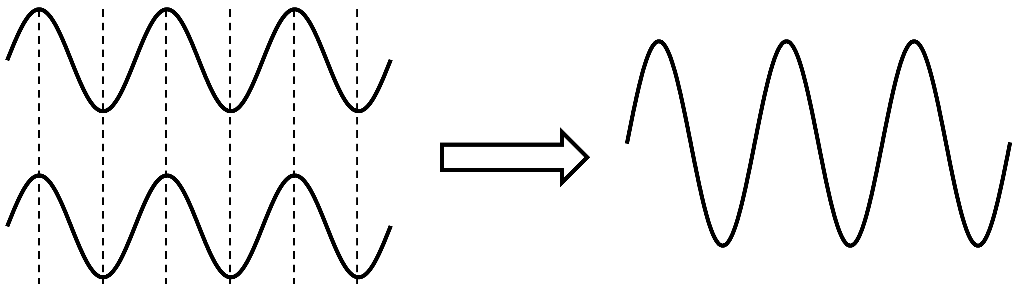
c. Optical path difference = 200 – 150 = 50mm

Quanta and Waves problem book page 16 and 17, questions 1 and 2.

5.3 I know the conditions for constructive and destructive interference.

Also, see section 5 in the Higher Physics Particle and Waves Notes.

Constructive Interference
When two waves cross and their crests and troughs line up to produce a larger amplitude wave. Constructive interference is at a maximum when the phase difference is zero or a multiple of whole wavelengths.

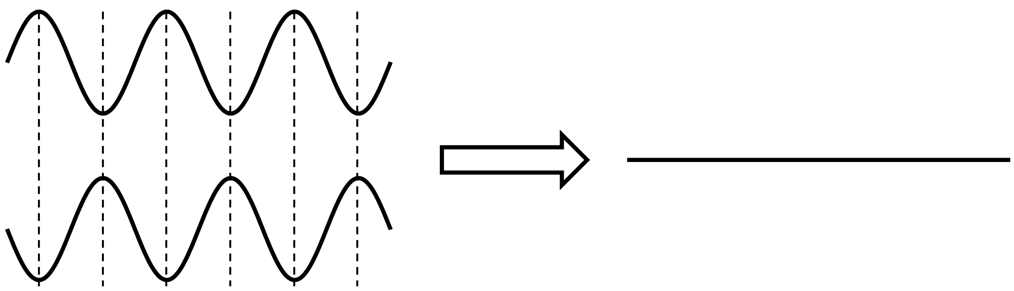


Number of wavelengths
$m = 0, 1, 2, \ldots$

$$\text{Optical path difference} = \mathrm{m}\lambda$$

Wavelength (metres)

Destructive Interference
When two waves cross and their crests line up with troughs the waves cancel each other. Destructive interference is at a maximum when the phase difference is half a wavelength or an odd multiple of half wavelengths.



Number of wavelengths
$m = 0, 1, 2, \ldots$

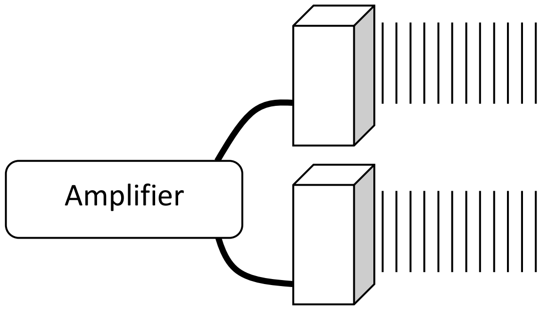
$$\text{Optical path difference} = \left(\mathrm{m} + \frac{1}{2}\right)\lambda$$

Wavelength (metres)

Interference occurs between all waves. However, if the waves are not coherent the changing phase relationship between them cancels out the constructive and destructive interference over more than one period. Interference effects are usually only apparent when interference occurs between waves from coherent sources.

For constructive or destructive interference to occur the interfering waves must be coherent.

Coherent sound waves can be produced by driving two speakers from the same source.

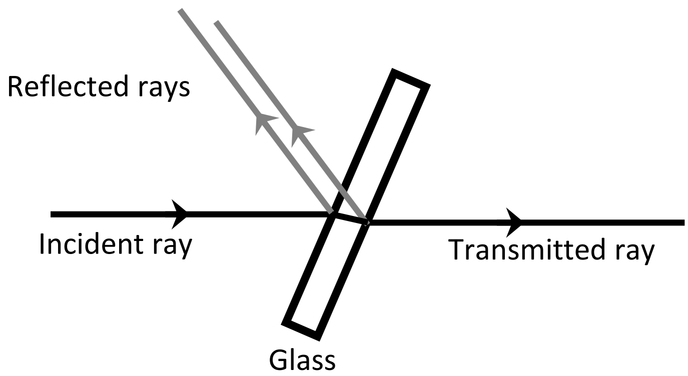


Electromagnetic radiation is produced by electrons transitioning between energy levels. As the timing of electron transitions is a random process the phase of the light produced will vary. This means that two separate light sources will not be coherent and so will not produce interference.

In order to obtain interference from light a single source is split into two separate rays. There are two ways of splitting a single source, division of amplitude and division of wavefront.

5.4 I can explain what it meant by division of amplitude.

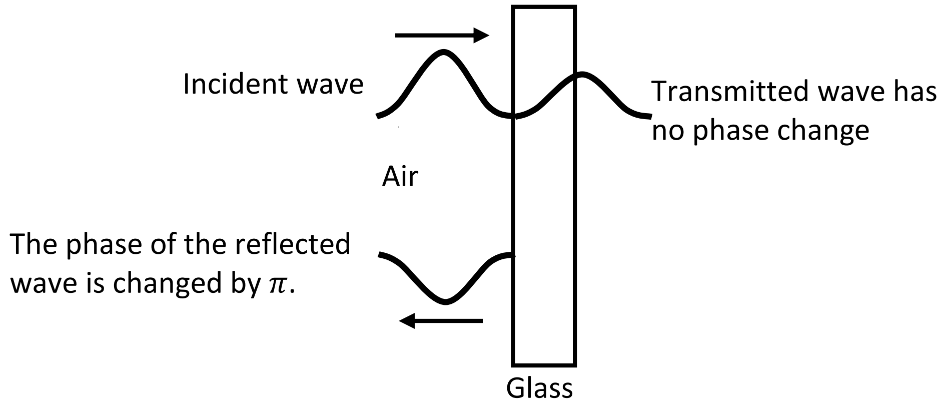
Division of amplitude occurs when an incident ray is divided into two parts by partial reflection. When light passes from one medium to another there is usually a ray that is transmitted through the material and a ray which is reflected. The amplitude of the reflected ray is usually of much lower in intensity than the transmitted ray.



5.5 I understand the phase changes which occur when a light ray is reflected or transmitted.

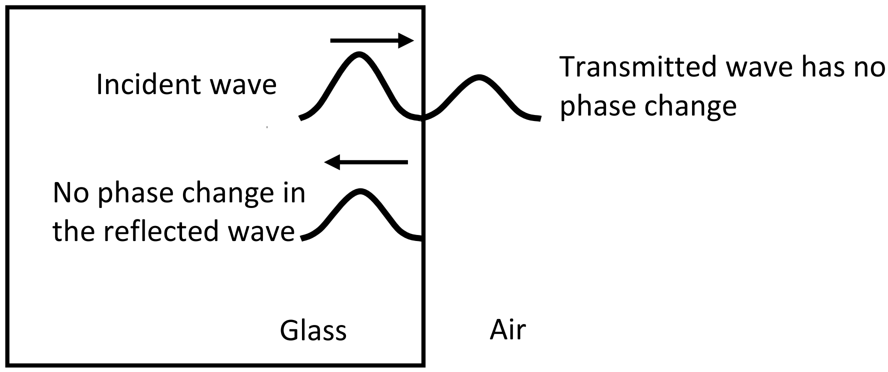
Ray moving from a low refractive index to a high refractive index.

- Transmitted ray – no phase change
- Reflected ray – phase change of π i.e. half a wavelength.



Ray moving from a high refractive index to a low refractive index.

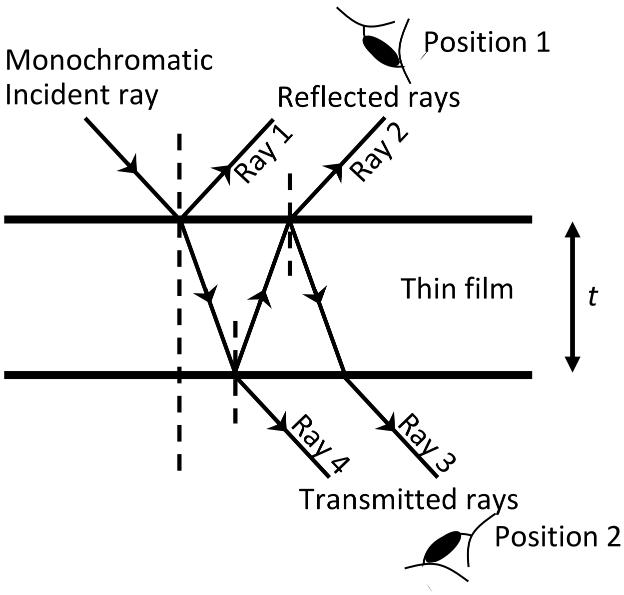
- Transmitted ray – no phase change
- Reflected ray – no phase change.



Quanta and Waves problem book page 17, question 3.

5.6 I can explain and solve problems involving thin film interference.

When light is incident on a thin film (e.g. oil on water) an interference pattern of dark and light bands (called fringes) is produced. This is caused by division of amplitude. This occurs as the ray passes through and is reflected from the surfaces of the film.
In the diagram, the path difference between ray 1 and ray 2 is $2t$. There is also a phase change in ray 1 due to reflection. This will cause destructive interference and formation of a dark band. Further interference be will occur between subsequent reflected rays to produce alternate dark and light bands.
The path difference between ray 3 and ray 4 is also $2t$. There is no phase change so constructive interference will occur giving light band. Further interference between the subsequent transmitted rays will occur producing the dark and light bands.



The angle of incidence is exaggerated in this diagram

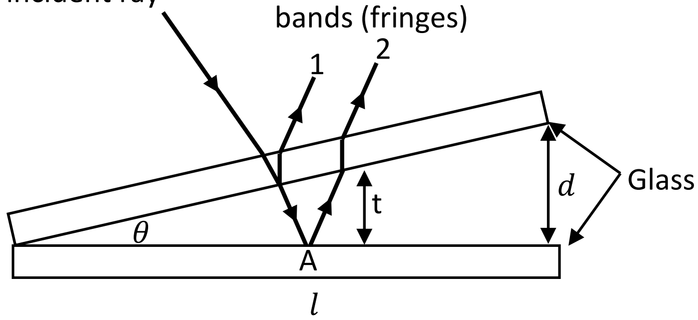
When viewed from position 1 the interference of the reflected rays will produce dark and light bands
When viewed from position 2 the transmitted rays will form similar bands.

When white light is shone on a thin film, interference between the different component colours will produce coloured bands.

Quanta and Waves problem book page 18, questions 4 to 6.

5.7 I can explain and solve problems involving wedge fringes.

When two pieces of flat glass are placed together with one edge slightly wedged apart a series of fringes caused by interference is produced.

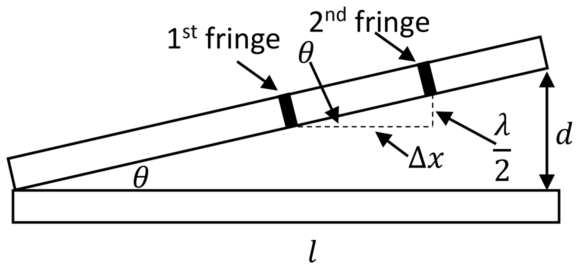


The angle of incidence and θ are exaggerated in this diagram.

The optical path difference between rays 1 and 2 is $2t$. As there is a phase change of π on reflection at A, destructive interference (dark fringe) occurs when

$$2t = m\lambda$$

For the next dark fringe the optical path difference must increase by λ so the thickness must increase by $\frac{\lambda}{2}$.



$$\tan\theta = \frac{\lambda}{2\Delta x}$$

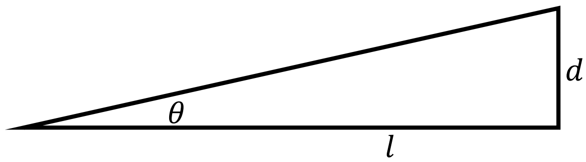
$$\Delta x = \frac{\lambda}{2\tan\theta}$$

Where Δx is the fringe spacing.

For a wedge of length l and spacing d

$$\tan\theta = \frac{d}{l}$$

Substituting this into the equation for Δx gives



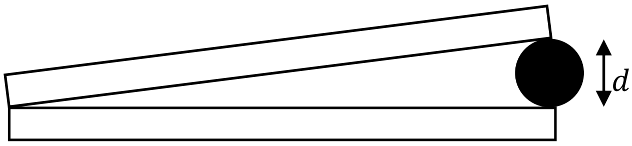
$$\Delta x = \frac{\lambda}{2\tan\theta} = \frac{\lambda l}{2d}$$

$$\Delta x = \frac{\lambda l}{2d}$$

You do not have to know this derivation.

Example

A wedge is formed between two glass plates 120mm long by placing a thin wire along one edge. Light of 589nm is used to illuminate the glass. Fringes are formed with a spacing of 1.1mm. Calculate the thickness of the wire.



Solution

$l = 120\text{mm} = 0.120\text{m}$
$\lambda = 589\text{nm} = 589 \times 10^{-9}\text{m}$
$\Delta x = 1.1\text{mm} = 1.1 \times 10^{-3}\text{m}$
$d = ?$

$$\Delta x = \frac{\lambda l}{2d} \quad \Rightarrow \quad d = \frac{\lambda l}{2\Delta x}$$

$$d = \frac{589 \times 10^{-9} \times 0.120}{2 \times 1.1 \times 10^{-3}}$$

$$d = 3.2 \times 10^{-5}\text{m}$$

Quanta and Waves problem book pages 19 and 20, questions 10 to 13.

5.8 I can explain how anti-reflective coatings (blooming) on lenses reduce reflections, derive the relationship $d = \frac{\lambda}{4n}$ and solve problems using this relationship.

Lenses can be coated to prevent unwanted reflections so all the light will pass through the lens. This can only be done exactly for one wavelength of light but as the visible spectrum is a narrow range of frequencies partial cancellation of reflections occur for all visible wavelengths.

The coating must a have a refractive index between glass and air

$$n_{air} < n_{coating} < n_{glass}$$

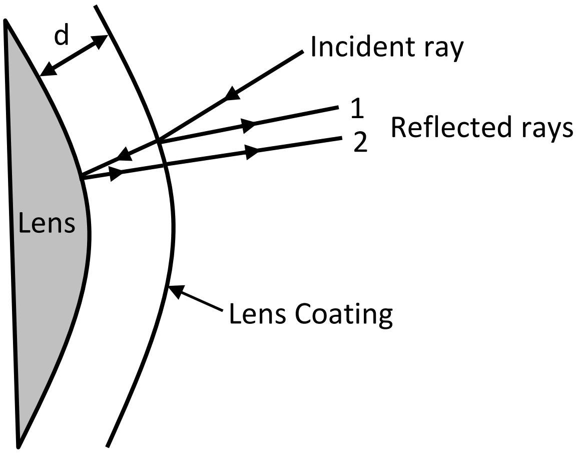
A common coating material is magnesium fluoride which has a refractive index of 1.2, is hard and transparent to visible radiation.

Complete removal of reflections is only achieved for one wavelength. This is usually set between yellow and green. Reflections of other colours at the red and blue ends of the spectrum are only partially reduced giving a purple colour any reflected light.

Deriving $\boldsymbol{d = \frac{\lambda}{4n}}$

Both rays reflected from the lens coating and the lens have a phase change of π so the relative phase change between rays 1 and 2 is zero.

To reduce reflections the thickness of the coating must produce destructive interference in the reflected rays.
The reflected ray 2 travels further than ray 1 as it passes through the coating twice.



$$\text{Optical path in coating} = \text{optical path difference} = 2nd$$

Where n is the refractive index of the coating.

For destructive interference

$$\text{Optical path difference} = \frac{\lambda}{2}$$

$$2nd = \frac{\lambda}{2}$$

$$d = \frac{\lambda}{4n}$$

<u>Example</u>

Calculate the thickness of magnesium fluoride lens coating (refractive index 1.2) required to eliminate reflection of yellow light of wavelength 570nm.

<u>Solution</u>
$n = 1.2$
$\lambda = 570\text{nm} = 570 \times 10^{-9}\text{m}$

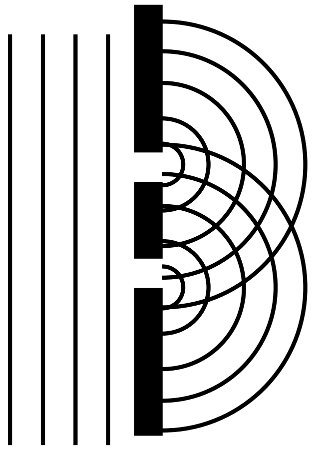
$$d = \frac{\lambda}{4n} = \frac{570 \times 10^{-9}}{4 \times 1.2} = 1.2 \times 10^{-7}\text{m}$$

Quanta and Waves problem book pages 18 and 19, questions 7 to 9.

5.9 I can explain what it meant by division of wavefront.

The light from a single source can be split using a double slit. The light exiting each of the slits will be coherent as they are from a single source. The light source must be a point or line source as light coming from different parts of an extended source will not have a constant phase difference.

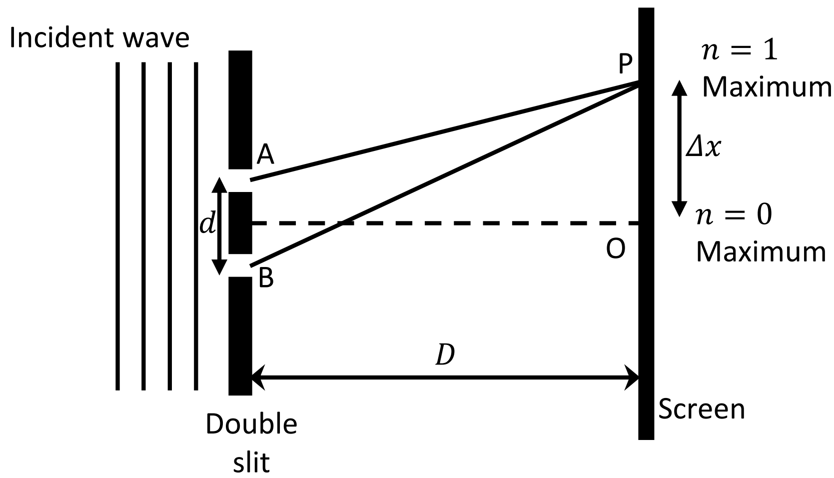
Incident wave
One wavefront



The wavefront is split by two slits which produces two coherent wavefronts.

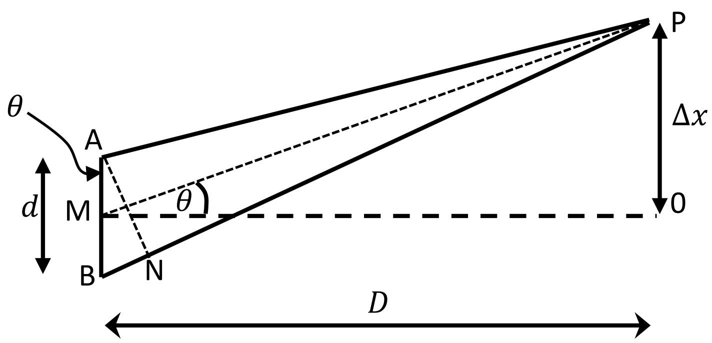
5.10 I can use the relationship $\Delta x = \frac{\lambda D}{d}$ to solve problems involving division of wavefront.

Young's Slits
Young's slits will produce an interference pattern on the screen. The relationship between the geometry and wavelength of light is derived below.



This derivation is only valid when $\Delta x \ll D$

Point N chosen so that $\mathrm{AP} = \mathrm{NP}$.
This gives BN as the optical path difference.



For constructive interference

at P, $BN = \lambda$

For small angles $\angle BNA = 90°$ and $\angle BAN = \theta$

From triangle PMO $\tan\theta = \frac{\Delta x}{D}$

From triangle ABN $\sin\theta = \frac{\lambda}{d}$

You do not need to know this derivation.

For small angles $\sin\theta = \tan\theta = \theta$

$$\theta = \frac{\Delta x}{D} \quad \text{and} \quad \theta = \frac{\lambda}{d}$$

$$\frac{\Delta x}{D} = \frac{\lambda}{d}$$

$$\Delta x = \frac{\lambda D}{d}$$

White light consists of many different wavelengths. As $\Delta x \propto \lambda$ each wavelength will have different fringe spacing. So, white light will be separated into its component wavelengths.

<u>Example</u>
Two parallel slits have a separation of 0.1mm. They are illuminated by a monochromatic light source and interference pattern is observed on a screen which is 4.0m from the double slits. The fringe separation is observed to be 24.4 mm. Calculate the wavelength of the light used.

<u>Solution</u>

$\Delta x = 24.4\text{mm} = 24.4 \times 10^{-3}\text{m}$
$D = 4.0\text{m}$
$d = 0.1\text{mm} = 0.1 \times 10^{-3}\text{m}$

$$\Delta x = \frac{\lambda D}{d} \Rightarrow \lambda = \frac{\Delta x d}{D}$$

$$\lambda = \frac{\Delta x d}{D} = \frac{24.4 \times 10^{-3} \times 0.1 \times 10^{-3}}{4.0}$$

$$\lambda = 6.1 \times 10^{-7}\text{m}$$

Quanta and Waves problem book pages 20 and 22, questions 13 to 17.

Key Area: Polarisation

Success Criteria

6.1 I can explain what is mean by the polarisation of transverse waves.

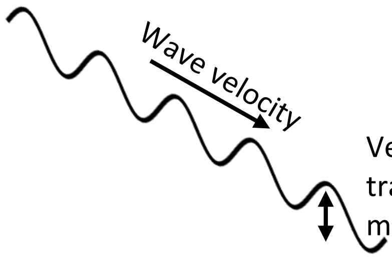
6.2 I can describe how electromagnetic waves can be polarised by a polariser.

6.3 I can; state that polarisation can be produced by reflection, derive the relationship $n = \tan i_p$, explain and solve problems involving the Brewster Angle.

6.4 I can describe an example of the use of polarisers.

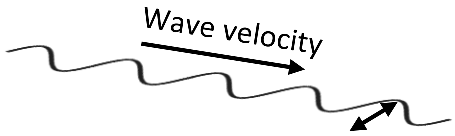
6.1 I can explain what is mean by the polarisation of transverse waves.

In a transverse wave the motion of the wave is perpendicular to the direction of its travel. This means that transverse motion can be orientated at different angles in space.
The orientation of the transverse motion in space is represented by a single arrow.

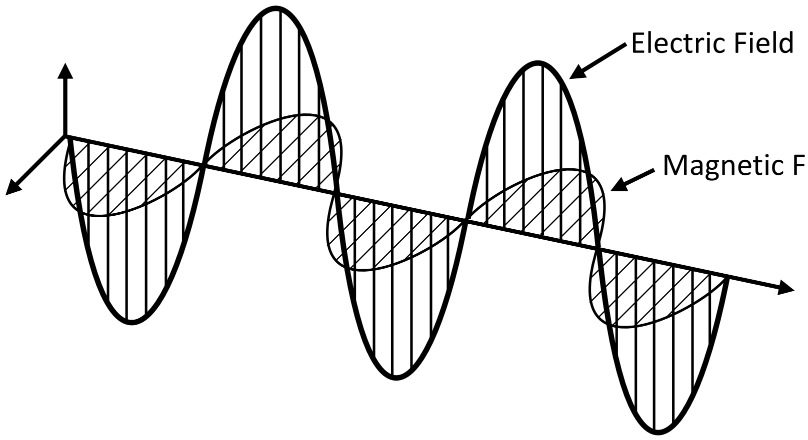


Vertically polarised wave

Horizontally polarised wave



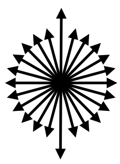
Electromagnetic Waves
Electromagnetic waves consist of an electric field and a magnetic field at 90° to the electric field.



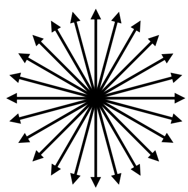
When referring to the polarisation of electromagnetic waves it is the orientation of the electric field which is used e.g. the wave above is vertically polarised.

Drawing Polarised Waves
A source of electromagnetic waves consists of many different waves. They can all be polarised in the same direction, some polarised or completely random unpolarised waves.

Polarised Waves
All the electric fields line up in one plane.



Partially Polarised Waves
Some of the electric fields of the waves are randomly orientated. A greater proportion is orientated in one direction.



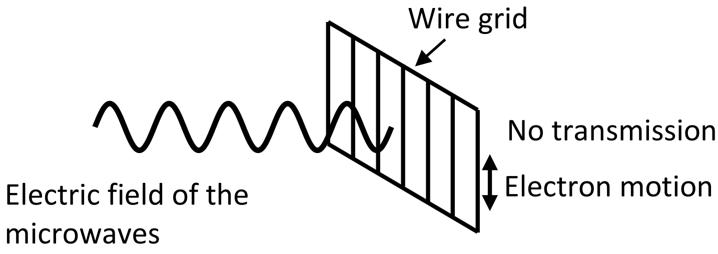
Unpolarised Waves
The electric fields are orientated in random directions.

6.2 I can describe how electromagnetic waves can be polarised by a polariser.

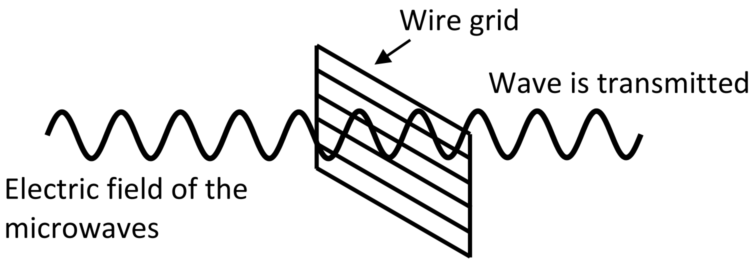
Polarising filters consist on a very fine grid of conducting material where the grid spacing is of similar size to the incident waves. They can be used to block electromagnetic radiation of a particular polarisation.

<u>Microwaves</u>

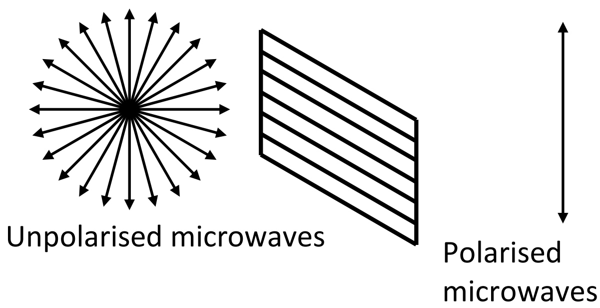
If the electric field is parallel to the wires, the electrons in the wire will be moved up and down by the field. Moving charge is an electric current. The resistance of the wire will dissipate the energy in the current. There will be no transmission.



If the electric field is perpendicular to the wires the electrons will only move a very short distance across the wire. There will be almost no current and or significant dissipation of energy. The wave will be transmitted.

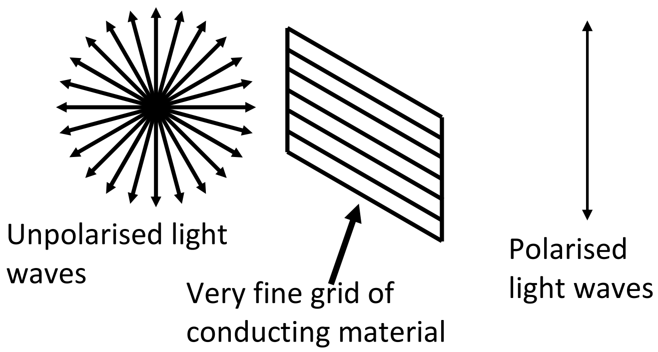


If unpolarised microwaves are incident on a wire grid only the waves with components of their electric fields perpendicular to the grid will be transmitted. The polariser selects polarised waves from unpolarised waves.



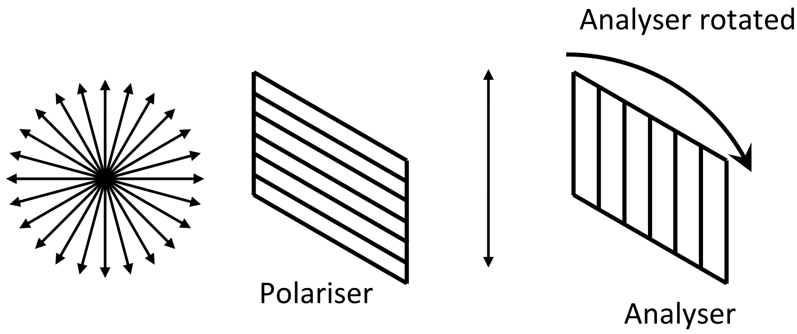
<u>Light waves</u>

Light (or any other electromagnetic radiation) can also be polarised by a grid of conducting material with spacing approximately equal to the wavelength.



Polariser and Analyser

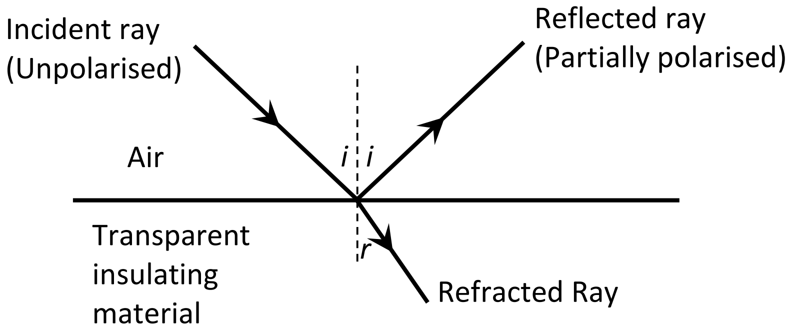
Polarised light is produced by a polarising filter. If this light is then passed through another polariser (the analyser) then the transmission can be stopped by rotating the analyser so that the grid lies parallel the electric field of the polarised light.



Some chemicals can change the polarisation angle of light passing through them. Some of the properties of these chemicals in solution can be determined by placing them between the polariser and analyser then rotating the analyser to extinguish the light. The relative angle between polariser and analyser gives the polarisation properties of the chemical in the solution.

6.3 I can; state that polarisation can be produced by reflection, derive the relationship $n = \tan i_p$, explain and solve problems involving the Brewster Angle.

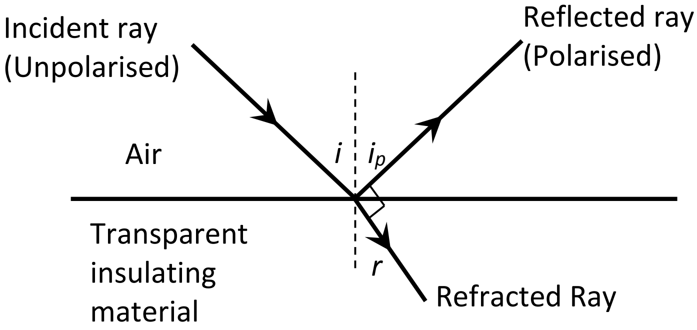
When electromagnetic radiation is reflected from a surface it can be polarised. The component of the wave's electric field parallel to a transparent insulating material (e.g. water and glass) surface is reflected to a greater extent than the perpendicular component. The reflected ray will then be partially polarised.



The Brewster Angle
At the Brewster Angle (of incidence) the reflected ray is completely polarised. The polarising angle occurs when the reflected and refracted rays are at 90°.

From the definition of refractive index

$n = \dfrac{\sin i_p}{\sin r}$ As $i = i_p$ for reflection



Summing the angles along the normal line gives

$$180° = i_p + 90° + r$$

Which simplifies to

$$r = 90° - i_p$$

Taking the sine of both sides gives

$$\sin r = \sin(90° - i_p) = \cos i_p$$

So $$n = \frac{\sin i}{\sin r} = \frac{\sin i_p}{\cos i_p}$$

$$n = \tan i_p$$

This relationship can be used to solve problems involving the Brewster Angle.

Example
Calculate the polarising angle for a material with refractive index 1.5.

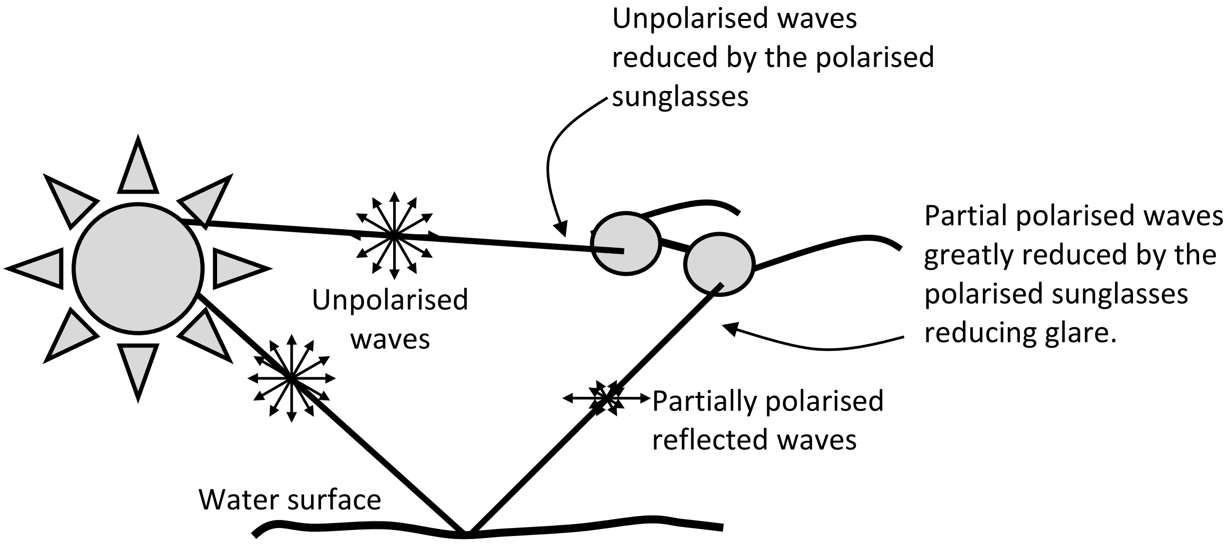
Solution
$n = 1.5$

$n = \tan i_p$
$i_p = \tan^{-1} n$
$i_p = \tan^{-1} 1.5 = 56°$

Quanta and Waves problem book pages 22 and 23, questions 1 to 6.

6.4 I can describe an example of the use of polarisers.

Polarising Sunglasses
Polarising sunglasses can remove glare (reflections) from water and other reflective surfaces. The sunglasses reduce the intensity of unpolarised light. They reduce the intensity of reflected light much more as the reflected rays are partially polarised. The grid lines in the polarising sunglasses are orientated to remove the polarised light in the reflected rays.



Quantities, Units and Multiplication Factors

Quantity	Quantity Symbol	Unit	Unit Abbreviation
charge	Q	coulomb	C
displacement	y	metre	m
energy	E	Joule	J
force	F	newton	N
frequency	f	hertz	Hz
magnetic induction	B	Tesla	T
mass	m	kilogram	kg
momentum	p	kilogram metre per second	kgms^{-1}
radius	r	metre	m
speed/velocity	v	metre per second	ms^{-1}
time	t	second	s
wavelength	λ	metre	m

Prefix Name	Prefix Symbol	Multiplication Factor
Pico	p	$\times 10^{-12}$
Nano	n	$\times 10^{-9}$
Micro	µ	$\times 10^{-6}$
Milli	m	$\times 10^{-3}$
Kilo	k	$\times 10^{3}$
Mega	M	$\times 10^{6}$
Giga	G	$\times 10^{9}$
Tera	T	$\times 10^{12}$

Relationships required for Physics Advanced Higher

$$v = \frac{ds}{dt}$$

$$a = \frac{dv}{dt} = \frac{d^2s}{dt^2}$$

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$v^2 = u^2 + 2as$$

$$\omega = \frac{d\theta}{dt}$$

$$\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$$

$$\omega = \omega_o + \alpha t$$

$$\theta = \omega_o t + \frac{1}{2}\alpha t^2$$

$$\omega^2 = \omega_o{}^2 + 2\alpha\theta$$

$$s = r\theta$$

$$v = r\omega$$

$$a_t = r\alpha$$

$$a_r = \frac{v^2}{r} = r\omega^2$$

$$F = \frac{mv^2}{r} = mr\omega^2$$

$$T = Fr$$

$$T = I\alpha$$

$$L = mvr = mr^2\omega$$

$$L = I\omega$$

$$E_K = \frac{1}{2}I\omega^2$$

$$F = G\frac{Mm}{r^2}$$

$$V = -\frac{GM}{r}$$

$$v = \sqrt{\frac{2GM}{r}}$$

$$\text{apparent brightness, } b = \frac{L}{4\pi r^2}$$

$$\text{Power per unit area} = \sigma T^4$$

$$L = 4\pi r^2 \sigma T^4$$

$$r_{Schwarzschild} = \frac{2GM}{c^2}$$

$$E = hf$$

$$\lambda = \frac{h}{p}$$

$$mvr = \frac{nh}{2\pi}$$

$$\Delta x \, \Delta p_x \geq \frac{h}{4\pi}$$

$$\Delta E \, \Delta t \geq \frac{h}{4\pi}$$

$$F = qvB$$

$$\omega = 2\pi f$$

$$a = \frac{d^2y}{dt^2} = -\omega^2 y$$

$$y = A\cos\omega t \quad \text{or} \quad y = A\sin\omega t$$

$$v = \pm\omega\sqrt{(A^2 - y^2)}$$

$$E_K = \frac{1}{2}m\omega^2(A^2 - y^2)$$

$$E_P = \frac{1}{2}m\omega^2 y^2$$

$$y = A\sin 2\pi(ft - \frac{x}{\lambda})$$

$$E = kA^2$$

$$\phi = \frac{2\pi x}{\lambda}$$

$$\text{optical path difference} = m\lambda \quad \text{or} \quad \left(m + \frac{1}{2}\right)\lambda$$

where $m = 0, 1, 2....$

$$\Delta x = \frac{\lambda l}{2d}$$

$$d = \frac{\lambda}{4n}$$

$$\Delta x = \frac{\lambda D}{d}$$

$$n = \tan i_P$$

$$F = \frac{Q_1 Q_2}{4\pi\varepsilon_o r^2}$$

$$E = \frac{Q}{4\pi\varepsilon_o r^2}$$

$$V = \frac{Q}{4\pi\varepsilon_o r}$$

$$F = QE$$

$$V = Ed$$

$$F = IlB\sin\theta$$

$$B = \frac{\mu_o I}{2\pi r}$$

$$c = \frac{1}{\sqrt{\varepsilon_o \mu_o}}$$

$$t = RC$$

$$X_C = \frac{V}{I}$$

$$X_C = \frac{1}{2\pi fC}$$

$$\mathcal{E} = -L\frac{dI}{dt}$$

$$E = \frac{1}{2}LI^2$$

$$X_L = \frac{V}{I}$$

$$X_L = 2\pi fL$$

$$\frac{\Delta W}{W} = \sqrt{\left(\frac{\Delta X}{X}\right)^2 + \left(\frac{\Delta Y}{Y}\right)^2 + \left(\frac{\Delta Z}{Z}\right)^2}$$

$$\Delta W = \sqrt{\Delta X^2 + \Delta Y^2 + \Delta Z^2}$$

$d = \bar{v}t$

$s = \bar{v}t$

$v = u + at$

$s = ut + \frac{1}{2}at^2$

$v^2 = u^2 + 2as$

$s = \frac{1}{2}(u + v)t$

$W = mg$

$F = ma$

$E_W = Fd$

$E_P = mgh$

$E_K = \frac{1}{2}mv^2$

$P = \frac{E}{t}$

$p = mv$

$Ft = mv - mu$

$F = G\frac{Mm}{r^2}$

$t' = \frac{t}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}$

$l' = l\sqrt{1 - \left(\frac{v}{c}\right)^2}$

$f_o = f_s\left(\frac{v}{v \pm v_s}\right)$

$z = \frac{\lambda_{observed} - \lambda_{rest}}{\lambda_{rest}}$

$z = \frac{v}{c}$

$v = H_0 d$

$E_W = QV$

$E = mc^2$

$E = hf$

$E_K = hf - hf_0$

$E_2 - E_1 = hf$

$T = \frac{1}{f}$

$v = f\lambda$

$d\sin\theta = m\lambda$

$n = \frac{\sin\theta_1}{\sin\theta_2}$

$\frac{\sin\theta_1}{\sin\theta_2} = \frac{\lambda_1}{\lambda_2} = \frac{v_1}{v_2}$

$\sin\theta_c = \frac{1}{n}$

$I = \frac{k}{d^2}$

$I = \frac{P}{A}$

path difference $= m\lambda$ or $\left(m + \frac{1}{2}\right)\lambda$ where $m = 0, 1, 2...$

random uncertainty $= \frac{\text{max. value - min. value}}{\text{number of values}}$

$V_{peak} = \sqrt{2}V_{rms}$

$I_{peak} = \sqrt{2}I_{rms}$

$Q = It$

$V = IR$

$P = IV = I^2R = \frac{V^2}{R}$

$R_T = R_1 + R_2 + ...$

$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + ...$

$E = V + Ir$

$V_1 = \left(\frac{R_1}{R_1 + R_2}\right)V_S$

$\frac{V_1}{V_2} = \frac{R_1}{R_2}$

$C = \frac{Q}{V}$

$E = \frac{1}{2}QV = \frac{1}{2}CV^2 = \frac{1}{2}\frac{Q^2}{C}$

Additional Relationships

Circle

$circumference = 2\pi r$

$area = \pi r^2$

Sphere

$area = 4\pi r^2$

$volume = \frac{4}{3}\pi r^3$

Trigonometry

$$\sin\theta = \frac{opposite}{hypotenuse}$$

$$\cos\theta = \frac{adjacent}{hypotenuse}$$

$$\tan\theta = \frac{opposite}{adjacent}$$

$$\sin^2\theta + \cos^2\theta = 1$$

Moment of inertia

point mass
$I = mr^2$

rod about centre
$I = \frac{1}{12}ml^2$

rod about end
$I = \frac{1}{3}ml^2$

disc about centre
$I = \frac{1}{2}mr^2$

sphere about centre
$I = \frac{2}{5}mr^2$

Table of standard derivatives

$f(x)$	$f'(x)$
$\sin ax$	$a\cos ax$
$\cos ax$	$-a\sin ax$

Table of standard integrals

$f(x)$	$\int f(x)dx$
$\sin ax$	$-\frac{1}{a}\cos ax + C$
$\cos ax$	$\frac{1}{a}\sin ax + C$

Electron Arrangements of Elements

Key

Atomic number Symbol Electron arrangement Name

Transition Elements

Group 1 (1)	Group 2 (2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	Group 3 (13)	Group 4 (14)	Group 5 (15)	Group 6 (16)	Group 7 (17)	Group 0 (18)
1 **H** 1 Hydrogen																	2 **He** 2 Helium
3 **Li** 2, 1 Lithium	4 **Be** 2, 2 Beryllium											5 **B** 2, 3 Boron	6 **C** 2, 4 Carbon	7 **N** 2, 5 Nitrogen	8 **O** 2, 6 Oxygen	9 **F** 2, 7 Fluorine	10 **Ne** 2, 8 Neon
11 **Na** 2, 8, 1 Sodium	12 **Mg** 2, 8, 2 Magnesium											13 **Al** 2, 8, 3 Aluminium	14 **Si** 2, 8, 4 Silicon	15 **P** 2, 8, 5 Phosphorus	16 **S** 2, 8, 6 Sulphur	17 **Cl** 2, 8, 7 Chlorine	18 **Ar** 2, 8, 8 Argon
19 **K** 2, 8, 8, 1 Potassium	20 **Ca** 2, 8, 8, 2 Calcium	21 **Sc** 2, 8, 9, 2 Scandium	22 **Ti** 2, 8, 10, 2 Titanium	23 **V** 2, 8, 11, 2 Vanadium	24 **Cr** 2, 8, 13, 1 Chromium	25 **Mn** 2, 8, 13, 2 Manganese	26 **Fe** 2, 8, 14, 2 Iron	27 **Co** 2, 8, 15, 2 Cobalt	28 **Ni** 2, 8, 16, 2 Nickel	29 **Cu** 2, 8, 18, 1 Copper	30 **Zn** 2, 8, 18, 2 Zinc	31 **Ga** 2, 8, 18, 3 Gallium	32 **Ge** 2, 8, 18, 4 Germanium	33 **As** 2, 8, 18, 5 Arsenic	34 **Se** 2, 8, 18, 6 Selenium	35 **Br** 2, 8, 18, 7 Bromine	36 **Kr** 2, 8, 18, 8 Krypton
37 **Rb** 2, 8, 18, 8, 1 Rubidium	38 **Sr** 2, 8, 18, 8, 2 Strontium	39 **Y** 2, 8, 18, 9, 2 Yttrium	40 **Zr** 2, 8, 18, 10, 2 Zirconium	41 **Nb** 2, 8, 18, 12, 1 Niobium	42 **Mo** 2, 8, 18, 13, 1 Molybdenum	43 **Tc** 2, 8, 18, 13, 2 Technetium	44 **Ru** 2, 8, 18, 15, 1 Ruthenium	45 **Rh** 2, 8, 18, 16, 1 Rhodium	46 **Pd** 2, 8, 18, 18, 0 Palladium	47 **Ag** 2, 8, 18, 18, 1 Silver	48 **Cd** 2, 8, 18, 18, 2 Cadmium	49 **In** 2, 8, 18, 18, 3 Indium	50 **Sn** 2, 8, 18, 18, 4 Tin	51 **Sb** 2, 8, 18, 18, 5 Antimony	52 **Te** 2, 8, 18, 18, 6 Tellurium	53 **I** 2, 8, 18, 18, 7 Iodine	54 **Xe** 2, 8, 18, 18, 8 Xenon
55 **Cs** 2, 8, 18, 18, 8, 1 Caesium	56 **Ba** 2, 8, 18, 18, 8, 2 Barium	57 **La** 2, 8, 18, 18, 9, 2 Lanthanum	72 **Hf** 2, 8, 18, 32, 10, 2 Hafnium	73 **Ta** 2, 8, 18, 32, 11, 2 Tantalum	74 **W** 2, 8, 18, 32, 12, 2 Tungsten	75 **Re** 2, 8, 18, 32, 13, 2 Rhenium	76 **Os** 2, 8, 18, 32, 14, 2 Osmium	77 **Ir** 2, 8, 18, 32, 15, 2 Iridium	78 **Pt** 2, 8, 18, 32, 17, 1 Platinum	79 **Au** 2, 8, 18, 32, 18, 1 Gold	80 **Hg** 2, 8, 18, 32, 18, 2 Mercury	81 **Tl** 2, 8, 18, 32, 18, 3 Thallium	82 **Pb** 2, 8, 18, 32, 18, 4 Lead	83 **Bi** 2, 8, 18, 32, 18, 5 Bismuth	84 **Po** 2, 8, 18, 32, 18, 6 Polonium	85 **At** 2, 8, 18, 32, 18, 7 Astatine	86 **Rn** 2, 8, 18, 32, 18, 8 Radon
87 **Fr** 2, 8, 18, 32, 18, 8, 1 Francium	88 **Ra** 2, 8, 18, 32, 18, 8, 2 Radium	89 **Ac** 2, 8, 18, 32, 18, 9, 2 Actinium	104 **Rf** 2, 8, 18, 32, 32, 10, 2 Rutherfordium	105 **Db** 2, 8, 18, 32, 32, 11, 2 Dubnium	106 **Sg** 2, 8, 18, 32, 32, 12, 2 Seaborgium	107 **Bh** 2, 8, 18, 32, 32, 13, 2 Bohrium	108 **Hs** 2, 8, 18, 32, 32, 14, 2 Hassium	109 **Mt** 2, 8, 18, 32, 32, 15, 2 Meitnerium									

Lanthanides	57 **La** 2, 8, 18, 18, 9, 2 Lanthanum	58 **Ce** 2, 8, 18, 20, 8, 2 Cerium	59 **Pr** 2, 8, 18, 21, 8, 2 Praseodymium	60 **Nd** 2, 8, 18, 22, 8, 2 Neodymium	61 **Pm** 2, 8, 18, 23, 8, 2 Promethium	62 **Sm** 2, 8, 18, 24, 8, 2 Samarium	63 **Eu** 2, 8, 18, 25, 8, 2 Europium	64 **Gd** 2, 8, 18, 25, 9, 2 Gadolinium	65 **Tb** 2, 8, 18, 27, 8, 2 Terbium	66 **Dy** 2, 8, 18, 28, 8, 2 Dysprosium	67 **Ho** 2, 8, 18, 29, 8, 2 Holmium	68 **Er** 2, 8, 18, 30, 8, 2 Erbium	69 **Tm** 2, 8, 18, 31, 8, 2 Thulium	70 **Yb** 2, 8, 18, 32, 8, 2 Ytterbium	71 **Lu** 2, 8, 18, 32, 9, 2 Lutetium
Actinides	89 **Ac** 2, 8, 18, 32, 18, 9, 2 Actinium	90 **Th** 2, 8, 18, 32, 18, 10, 2 Thorium	91 **Pa** 2, 8, 18, 32, 20, 9, 2 Protactinium	92 **U** 2, 8, 18, 32, 21, 9, 2 Uranium	93 **Np** 2, 8, 18, 32, 22, 9, 2 Neptunium	94 **Pu** 2, 8, 18, 32, 24, 8, 2 Plutonium	95 **Am** 2, 8, 18, 32, 25, 8, 2 Americium	96 **Cm** 2, 8, 18, 32, 25, 9, 2 Curium	97 **Bk** 2, 8, 18, 32, 27, 8, 2 Berkelium	98 **Cf** 2, 8, 18, 32, 28, 8, 2 Californium	99 **Es** 2, 8, 18, 32, 29, 8, 2 Einsteinium	100 **Fm** 2, 8, 18, 32, 30, 8, 2 Fermium	101 **Md** 2, 8, 18, 32, 31, 8, 2 Mendelevium	102 **No** 2, 8, 18, 32, 32, 8, 2 Nobelium	103 **Lr** 2, 8, 18, 32, 32, 9, 2 Lawrencium

DATA SHEET

COMMON PHYSICAL QUANTITIES

Quantity	*Symbol*	*Value*	*Quantity*	*Symbol*	*Value*
Gravitational acceleration on Earth	g	$9{\cdot}8\ \mathrm{m\,s^{-2}}$	Mass of electron	m_e	$9{\cdot}11 \times 10^{-31}$ kg
Radius of Earth	R_E	$6{\cdot}4 \times 10^{6}$ m	Charge on electron	e	$-1{\cdot}60 \times 10^{-19}$ C
Mass of Earth	M_E	$6{\cdot}0 \times 10^{24}$ kg	Mass of neutron	m_n	$1{\cdot}675 \times 10^{-27}$ kg
Mass of Moon	M_M	$7{\cdot}3 \times 10^{22}$ kg	Mass of proton	m_p	$1{\cdot}673 \times 10^{-27}$ kg
Radius of Moon	R_M	$1{\cdot}7 \times 10^{6}$ m	Mass of alpha particle	m_α	$6{\cdot}645 \times 10^{-27}$ kg
Mean Radius of Moon Orbit		$3{\cdot}84 \times 10^{8}$ m	Charge on alpha particle		$3{\cdot}20 \times 10^{-19}$ C
Solar radius		$6{\cdot}955 \times 10^{8}$ m	Planck's constant	h	$6{\cdot}63 \times 10^{-34}$ J s
Mass of Sun		$2{\cdot}0 \times 10^{30}$ kg	Permittivity of free space	ε_0	$8{\cdot}85 \times 10^{-12}\ \mathrm{F\,m^{-1}}$
1 AU		$1{\cdot}5 \times 10^{11}$ m	Permeability of free space	μ_0	$4\pi \times 10^{-7}\ \mathrm{H\,m^{-1}}$
Stefan-Boltzmann constant	σ	$5{\cdot}67 \times 10^{-8}\ \mathrm{W\,m^{-2}\,K^{-4}}$	Speed of light in vacuum	c	$3{\cdot}0 \times 10^{8}\ \mathrm{m\,s^{-1}}$
Universal constant of gravitation	G	$6{\cdot}67 \times 10^{-11}\ \mathrm{m^3\,kg^{-1}\,s^{-2}}$	Speed of sound in air	v	$3{\cdot}4 \times 10^{2}\ \mathrm{m\,s^{-1}}$

REFRACTIVE INDICES

The refractive indices refer to sodium light of wavelength 589 nm and to substances at a temperature of 273 K.

Substance	*Refractive index*	*Substance*	*Refractive index*
Diamond	2·42	Glycerol	1·47
Glass	1·51	Water	1·33
Ice	1·31	Air	1·00
Perspex	1·49	Magnesium Fluoride	1·38

SPECTRAL LINES

Element	*Wavelength*/nm	*Colour*
Hydrogen	656	Red
	486	Blue-green
	434	Blue-violet
	410	Violet
	397	Ultraviolet
	389	Ultraviolet
Sodium	589	Yellow

Element	*Wavelength*/nm	*Colour*
Cadmium	644	Red
	509	Green
	480	Blue

Lasers

Element	*Wavelength*/nm	*Colour*
Carbon dioxide	9550, 10590	Infrared
Helium-neon	633	Red

PROPERTIES OF SELECTED MATERIALS

Substance	*Density*/ $\mathrm{kg\,m^{-3}}$	*Melting Point*/ K	*Boiling Point*/K	*Specific Heat Capacity*/ $\mathrm{J\,kg^{-1}\,K^{-1}}$	*Specific Latent Heat of Fusion*/ $\mathrm{J\,kg^{-1}}$	*Specific Latent Heat of Vaporisation*/ $\mathrm{J\,kg^{-1}}$
Aluminium	$2{\cdot}70 \times 10^{3}$	933	2623	$9{\cdot}02 \times 10^{2}$	$3{\cdot}95 \times 10^{5}$	....
Copper	$8{\cdot}96 \times 10^{3}$	1357	2853	$3{\cdot}86 \times 10^{2}$	$2{\cdot}05 \times 10^{5}$	....
Glass	$2{\cdot}60 \times 10^{3}$	1400	....	$6{\cdot}70 \times 10^{2}$	....	....
Ice	$9{\cdot}20 \times 10^{2}$	273	....	$2{\cdot}10 \times 10^{3}$	$3{\cdot}34 \times 10^{5}$	....
Glycerol	$1{\cdot}26 \times 10^{3}$	291	563	$2{\cdot}43 \times 10^{3}$	$1{\cdot}81 \times 10^{5}$	$8{\cdot}30 \times 10^{5}$
Methanol	$7{\cdot}91 \times 10^{2}$	175	338	$2{\cdot}52 \times 10^{3}$	$9{\cdot}9 \times 10^{4}$	$1{\cdot}12 \times 10^{6}$
Sea Water	$1{\cdot}02 \times 10^{3}$	264	377	$3{\cdot}93 \times 10^{3}$	....	....
Water	$1{\cdot}00 \times 10^{3}$	273	373	$4{\cdot}19 \times 10^{3}$	$3{\cdot}34 \times 10^{5}$	$2{\cdot}26 \times 10^{6}$
Air	1·29	....	....	....	....	....
Hydrogen	$9{\cdot}0 \times 10^{-2}$	14	20	$1{\cdot}43 \times 10^{4}$	....	$4{\cdot}50 \times 10^{5}$
Nitrogen	1·25	63	77	$1{\cdot}04 \times 10^{3}$	....	$2{\cdot}00 \times 10^{5}$
Oxygen	1·43	55	90	$9{\cdot}18 \times 10^{2}$	....	$2{\cdot}40 \times 10^{4}$

The gas densities refer to a temperature of 273 K and a pressure of $1{\cdot}01 \times 10^{5}$ Pa.