Advanced Higher Physics

Rotational Motion and Astrophysics

Notes

Name……………………………………………………….

Key Area Notes, Examples and Questions

Other Notes

Key Area: Kinematic Relationships

Previous Knowledge

- Know what is meant by the vector terms displacement, velocity and acceleration.
- Know the symbols and units for displacement, velocity and acceleration.
- Know that kinematics is the study of motion without reference to its causes.
- Know and can use notation for differentiation and integration.
- Know how to find first and second derivatives with respect to time.
- Know how to find indefinite and definite integrals.

Success Criteria

1.1 I can derive the kinematic equations of motion using calculus methods.

1.2 I can use calculus to calculate; instantaneous displacement, velocity and acceleration for straight line motion with constant or varying acceleration.

1.3 I can interpret graphs of motion for objects moving in a straight line.

1.4 I can calculate displacement, velocity and acceleration from a graph.

1.1 I can derive the kinematic equations of motion using calculus methods.

Calculus Review and Notation

$\boldsymbol{f(t)}$ means function f is a function of the variable t.

e.g. $f(t) = at + bt^2$ where *a* and *b* are constants.

$\boldsymbol{\frac{df}{dt}}$ is the first derivative of the function $f(t)$ and gives the gradient of this function. This is sometimes written as $f'(t)$ or $\dot{f}$.

e.g. $\frac{df}{dt} = \frac{d}{dt}(f(t)) = \frac{d}{dt}(at + bt^2) = a + 2bt$

$\frac{d}{dt}(f(t))$

This is an instruction which means differentiate the function $f(t)$.

$\boldsymbol{\frac{d^2f}{dt^2}}$ is the second derivative of the function $f(t)$. It gives the gradient of the first derivative $\frac{df}{dt}$. This is sometimes written as $f''(t)$ or $\ddot{f}$.

e.g. $\frac{d^2f}{dt^2} = \frac{d}{dt}\left(\frac{df}{dt}\right) = \frac{d}{dt}(a + 2bt) = 2b$

$\boldsymbol{\int f(t)dt}$ is an indefinite integral. This means integrate the function $f(t)$.

e.g. $\int (at + bt^2)dt = \frac{1}{2}at^2 + \frac{1}{3}bt^3 + c$

When the function $f(t)$ is drawn on a graph, integration gives the area between the function and the horizontal axis.

$\boldsymbol{\int_d^e f(t)dt}$ is a definite integral. This means integrate the function $f(t)$ and evaluate it between the limits *d and e* as shown.

e.g. $\int_0^1 3t\,dt = \left[\frac{3}{2}t^2\right]_0^1 = \left(\frac{3}{2} \times 1^2\right) - \left(\frac{3}{2} \times 0^2\right) = 1.5$

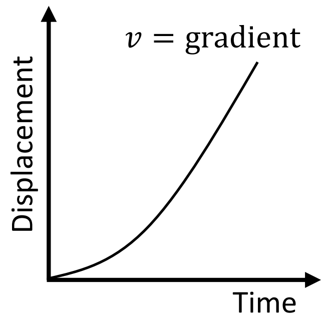
The kinematic relationships $v = u + at$ and $s = ut + \frac{1}{2}at^2$ can be derived using calculus from the definitions of velocity and acceleration.
These kinematic relationships assume that the acceleration is constant. Varying acceleration is dealt with in section 1.2. The other kinematic relationship $v^2 = u^2 + 2as$ can be derived from the above two relationships.

Definition of Velocity

Velocity is defined as

$$v = \frac{ds}{dt}$$

i.e. velocity is the gradient of the displacement time graph.

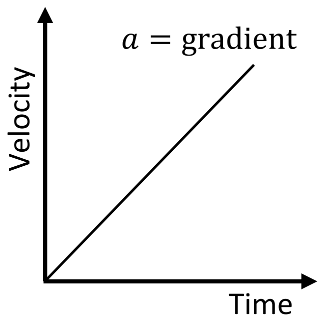


Definition of Acceleration

Acceleration is defined as

$$a = \frac{dv}{dt} = \frac{d^2s}{dt^2}$$

i.e. acceleration is the gradient of the velocity time graph.



Deriving $\boldsymbol{v = u + at}$

Acceleration is constant for the kinematic equations

$$\frac{dv}{dt} = a$$

Integrating this with respect to time

$$\int \frac{dv}{dt}.dt = \int a.dt$$

$v = at + c$ where c is a constant

At $t = 0$ the initial velocity is u. This can be substituted in the above relationship.

$$u = a \times 0 + c$$

$$c = u$$

So $v = u + at$

Deriving $\boldsymbol{s = ut + \frac{1}{2}at^2}$

From the definition of velocity

$$v = \frac{ds}{dt} = u + at$$

Integrating this with respect to time

$$\int \frac{ds}{dt}.\,dt = \int (u + at)dt$$

$$s = ut + \tfrac{1}{2}at^2 + c$$

At $t = 0$ the initial displacement is $s = 0$. This can be substituted into the above relationship.

$$s = u \times 0 + \tfrac{1}{2}a \times 0^2 + c$$

$$c = 0$$

So $\quad s = ut + \tfrac{1}{2}at^2$

Deriving $\boldsymbol{v^2 = u^2 + 2as}$

Taking $v = u + at$ and squaring both sides gives the

$$v^2 = (u + at)^2 = u^2 + 2uat + a^2t^2$$

Taking a factor 2a from the last two terms gives

$$v^2 = u^2 + 2a\left(ut + \tfrac{1}{2}at^2\right)$$

The term is brackets is the displacement, s. This leaves

$$v^2 = u^2 + 2as$$

1.2 I can use calculus to calculate; instantaneous displacement, velocity and acceleration for straight line motion with constant or varying acceleration.

Differentiation Example

The displacement of an object after t seconds is given by $s = 3 + 9.8t^2$

a. Using calculus find an expression for the velocity of the object.

b. Find the velocity of the object at 4.0s.

c. Show that the acceleration of the object is constant.

Differentiation Solution

a. As $v = \frac{ds}{dt}$ differentiate $s = 3 + 9.8t^2$ to find the expression for velocity.

$v = \frac{d}{dt}(3 + 9.8t^2)$

$v = 19.6t$

b. When $t = 4.0\text{s}$

$v = 19.6 \times 4.0$

$v = 78.4\text{ms}^{-1}$

c. $a = \frac{dv}{dt}$ differentiate $v = 19.6t$ to find the expression for acceleration.

$a = \frac{d}{dt}(19.6t)$

$a = 19.6\text{ms}^{-2}$ Which is a constant.

RMA Question Book Pages 4 and 5 Questions 1 to 7.

Integration Example

An object is dropped at $t = 0\text{s}$ from a height of 100m and accelerates down at 9.8ms^{-2}.

a. Using calculus find an expression for the velocity of the object.

b. Using calculus find an expression for the displacement of the object.

Integration Solution

a. $a = -9.8\text{ms}^{-1}$

From the definition of acceleration $a = \frac{dv}{dt}$

$$\frac{dv}{dt} = -9.8$$

Integrating with respect to time gives

$$\int \frac{dv}{dt} dt = \int -9.8\, dt$$

$$v = -9.8t + c$$

As the object is dropped from rest at $t = 0\text{s}$

$0 = -9.8 \times 0 + c$ Where c is a constant.

So $v = -9.8t$

b. From the definition of velocity $v = \frac{ds}{dt}$

$$\frac{ds}{dt} = -9.8t$$

Integrating with respect to time gives

$$\int \frac{ds}{dt} dt = \int -9.8t\, dt$$

$s = -\frac{1}{2} \times 9.8t^2 + d$ Where d is a constant.

$$s = -4.9t^2 + d$$

As the object is dropped from $s = 100\text{m}$ at $t = 0\text{s}$

$$100 = -4.9 \times (0)^2 + d$$

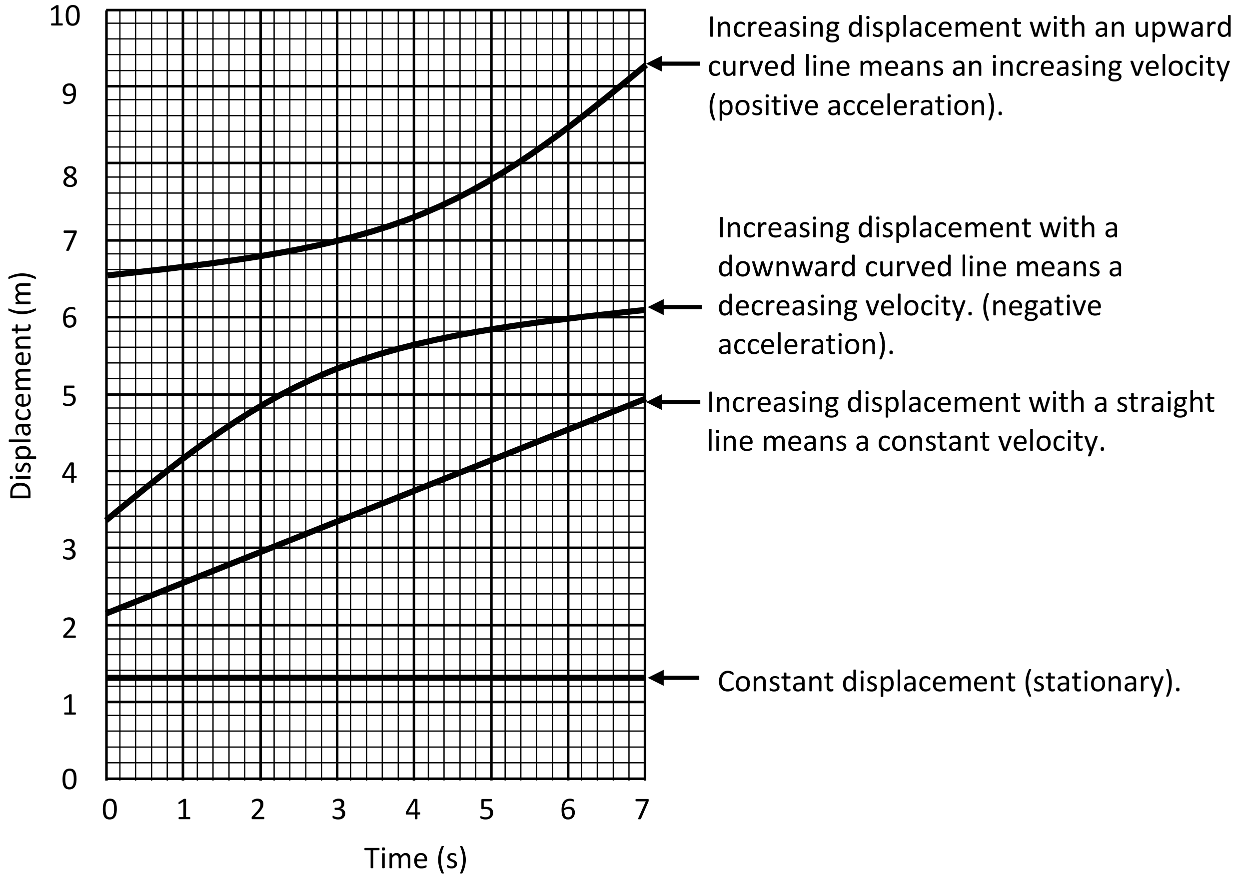
Which give $d = 100\text{m}$

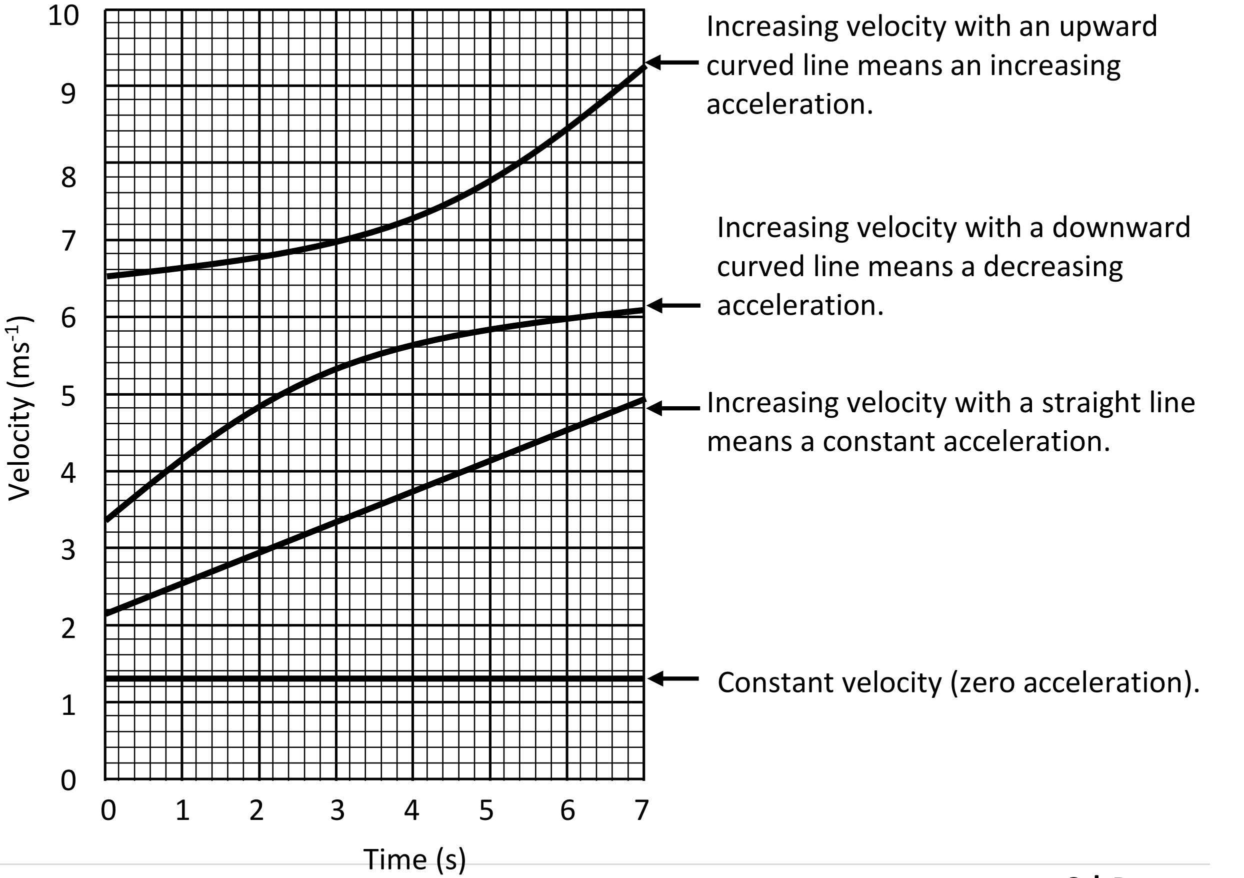
So $s = 100 - 4.9t^2$

RMA Question Book Page 5 Questions 8 and 9.

1.3 I can interpret graphs of motion for objects moving in a straight line

The displacement time graph below shows how a stationary, a constant velocity and an accelerating object would appear.





1.4 I can calculate displacement, velocity and acceleration from a graph

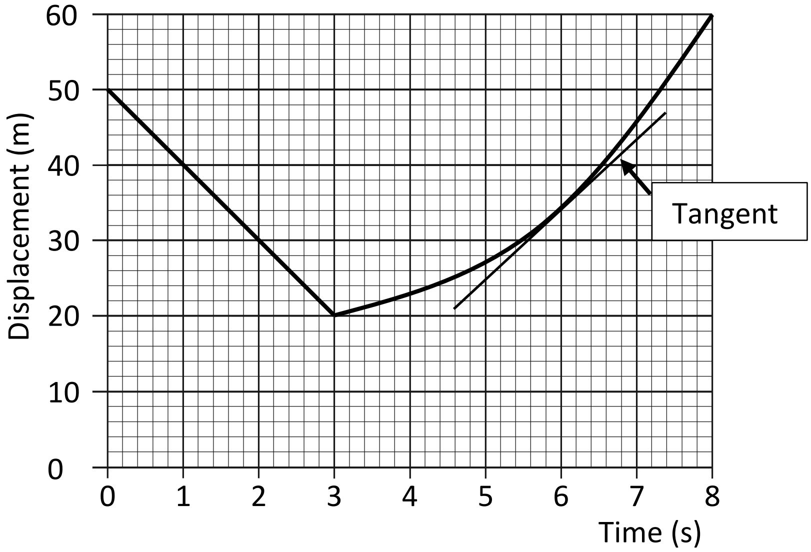
The velocity of an object can be obtained from the gradient of its displacement-time graph.

Example

The graph below shows the displacement time graph of an object. Find

a. Its velocity at 1s.

b. Its velocity a 6s.



Solution

a. At 1 second the velocity is given by the gradient of the straight line section of the graph between 0s and 3s.

$$v = \text{gradient} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{20 - 50}{3 - 0} = -10\text{ms}^{-1}$$

b. At 6 seconds the graph is curved. To find the velocity find the gradient of the tangent to the curve at 6s.

$$v = \text{gradient of tangent} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{47 - 21}{7.4 - 4.6} = 9.3\text{ms}^{-1}$$

The displacement and acceleration of an object can be obtained from its velocity time graph.

- Displacement is obtained by finding the area between the line on the graph and the time axis.
- Acceleration is given by the gradient of the line.

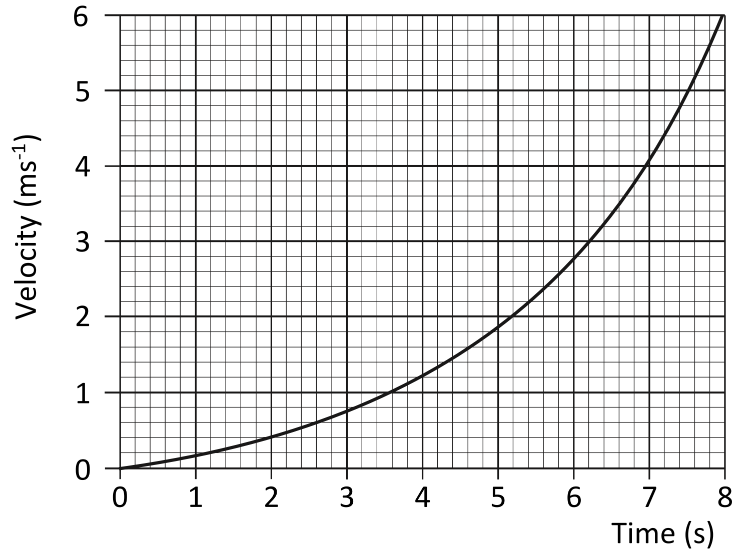
Example

The velocity-time graph shown below shows the motion of an object between 0s and 8s.

Find

a. Its acceleration at 5s

b. Its displacement at 3s.

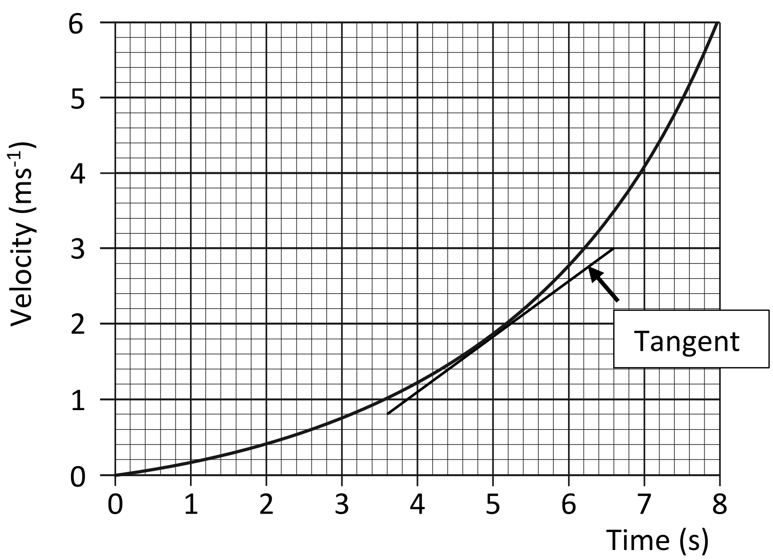


Solution

a. The acceleration is given by the gradient of the tangent to the curve at 5s.

$$a = \text{gradient of tangent} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$a = \frac{3.0 - 1.8}{6.6 - 3.6} = 0.4\text{ms}^{-2}$$



b.

The displacement is given by the area under the curve. This can be done by counting the number of whole boxes and adding the half the number of partial boxes.

$$s = \underbrace{16}_{\text{Number of whole boxes}} + \underbrace{\left(\frac{1}{2} \times 15\right)}_{\text{Number of partial boxes}} = 23.5\text{m}$$

RMA Question Book Page 6 Questions 10 and 11.

Key Area: Angular Motion

Success Criteria

2.1 I can use the radian as a measure of angular displacement.

2.2 I can convert between degrees and radians.

2.3 I can perform calculations involving angular displacement, angular velocity, angular acceleration and revolutions per minute.

2.4 I can solve problems involving angular velocity, period of rotation and tangential velocity.

2.5 I understand the terms angular acceleration, tangential acceleration and radial (centripetal) acceleration and the relationship between them.

2.6 I can derive the relatationships $a_r = \frac{v^2}{r}$ and $a_r = r\omega^2$ for the radial acceleration of a rotating object.

2.7 I can carry out calculations involving centripetal acceleration and centripetal force.

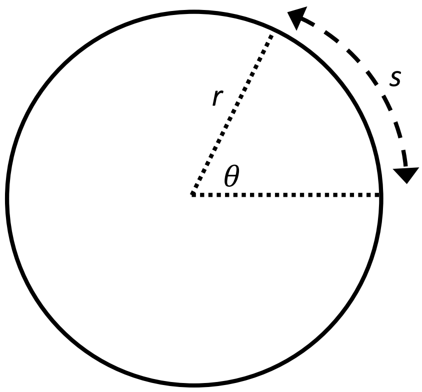
2.1 I can use the radian as a measure of angular displacement.

Definition of the radian

Given a circle with radius *r* where the angle θ subtends and arc of length *s*.

The definition of the radian is given by

$$\theta = \frac{s}{r}$$



The relationship between radians and degrees

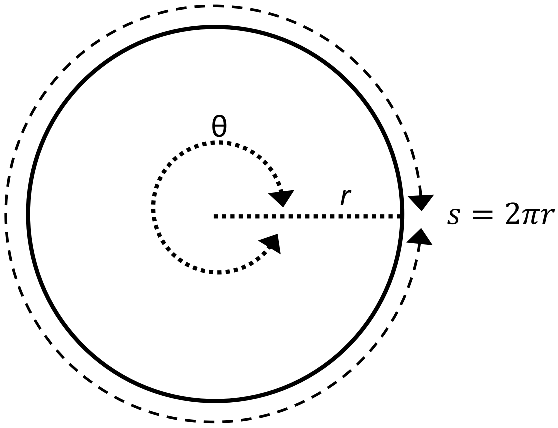
For a full circle

$s = \text{circmferance} = 2\pi r$

Angle in radians $\frac{s}{r} = \frac{2\pi r}{r} = 2\pi \text{ radians}$

Angle in degrees 360°

So $\quad 1° = \frac{2\pi}{360°} \text{ radians or } \frac{\pi}{180°} \text{ radians}$



Note

All angles used in the rotational motion relationships in Advanced Higher Physics should be converted to radians.

2.2 I can convert between degrees and radians.

Converting from degrees ⟺ radians

To convert degrees to radians multiply by $\frac{\pi}{180}$

To convert radians to degrees multiply by $\frac{180}{\pi}$

Note on units Angles measured in degrees have the unit symbol ° e.g. 45° Angles measured in radians have no units. For clarity angles in radians are usually followed by either "rad" or "radians". e.g. 1.3 rad or 1.3 radians.

<u>Example</u>

Convert

a. 45° to radians.

b. 1.0 radian to degrees.

<u>Solution</u>

a. $45 \times \frac{\pi}{180} = \frac{\pi}{4} \text{ rad} = 0.79 \text{ rad}$

b. $1.0 \times \frac{180}{\pi} = 57°$

RMA Question Book Page 7 Questions 1 and 2

2.3 I can perform calculations involving angular displacement, angular velocity, angular acceleration and revolutions per minute.

Rotational motion is described using rotational quantities in a similar way to the linear quantities used in the kinematic equations.

Linear Quantity	Symbol	Linear Units	Rotational Quantity	Symbol	Rotational Units
Displacement	*s*	Metres (m)	Angular displacement	θ	Radians (rad)
Velocity	*u, v*	Metres per second (ms^{-1})	Angular velocity	ω_0, ω	Radians per second (rad s^{-1})
Acceleration	*a*	Meters per second squared (ms^{-2})	Angular acceleration	α	Radians per second squared, (rad s^{-2})
Time	*t*	Seconds (s)	Time	t	Seconds (s)

The definitions of angular velocity and angular acceleration take the same form as those for linear velocity and linear acceleration.

Definition of linear velocity $v = \frac{ds}{dt}$

Definition of angular velocity $\omega = \frac{d\theta}{dt}$

Definition of linear acceleration $a = \frac{dv}{dt} = \frac{d^2s}{dt^2}$

Definition of angular acceleration $\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$

The rotational kinematic relationships derived from these definitions will take the same form as the linear kinematic relationships (suvat relationships).

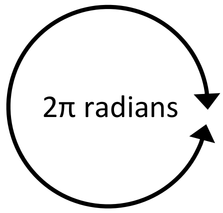
Linear Kinematic Relationship	Rotational Kinematic Relationship
$v = \frac{s}{t}$	$\omega = \frac{\theta}{t}$
$v = u + at$	$\omega = \omega_0 + \alpha t$
$s = ut + \frac{1}{2}at^2$	$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$
$v^2 = u^2 + 2as$	$\omega^2 = \omega_0^2 + 2\alpha\theta$

These two relationships are for constant linear/angular velocity. They are not on the relationship sheet

These rotational kinematic relationships can be used to solve problems of rotational motion.

Converting between revolutions per minute (rpm) and radians per second

Consider an object rotating at 1 rpm. As it completes one full revolution it moves through 2π radians in 60 seconds.



$$1 \text{ rpm} = \frac{2\pi}{60} \text{ rad s}^{-1}$$

To convert from revolutions per minute to radians per second multiply by $\frac{2\pi}{60}$

To convert from radians per second to revolutions per minute multiply by $\frac{60}{2\pi}$

<u>Example</u>
A car's wheels rotate through 600 radians in 10 seconds. Calculate their angular velocity.

<u>Solution</u>
$\theta = 600$ radians
$t = 10\text{s}$

$$\omega = \frac{\theta}{t}$$

$$\omega = \frac{600}{10} = 60 \text{ rad s}^{-1}$$

Example

The crankshaft in a car engine rotates at 2000rpm. Calculate the angular velocity of the crankshaft.

Solution

$$1\ \text{rpm} = \frac{2\pi}{60}\ \text{rad s}^{-1}$$

$$2000\text{rpm} = \frac{2\pi}{60} \times 2000 = 209.4\ \text{rad s}^{-1}\ \ (209.44\ \text{rad s}^{-1})$$

Example

The car engine now increases its angular velocity to 6000rpm in 1.2s.

a. Calculate the angular acceleration of the crankshaft.

b. How many revolutions does the engine takes during the 1.2s acceleration?

Solution

a.

$\omega_0 = 209.44\ \text{rad s}^{-1}$

$t = 1.2\text{s}$

$$\omega = 6000\ \text{rpm} = \frac{2\pi}{60} \times 6000 = 628.32\ \ \text{rad s}^{-1}$$

$$\omega = \omega_0 + \alpha t \quad \Longrightarrow \quad \alpha = \frac{\omega - \omega_0}{t}$$

$$\alpha = \frac{628.32 - 209.44}{1.2} = 350\ \text{rad s}^{-2}\ \ \ (349.067\ \text{rad s}^{-2})$$

b.

$$\omega^2 = \omega_0^2 + 2\alpha\theta \quad \Longrightarrow \quad \theta = \frac{\omega^2 - \omega^2}{2\alpha}$$

$$\theta = \frac{628.32^2 - 209.44^2}{2 \times 349.07}$$

$$\theta = 500\ \text{rad}\ \ \ (502.65\ \text{rad})$$

To convert to number or rotations divide by 2π

$$\text{Number of revolutions} = \frac{502.65}{2\pi} = 80\ \text{revolutions}$$

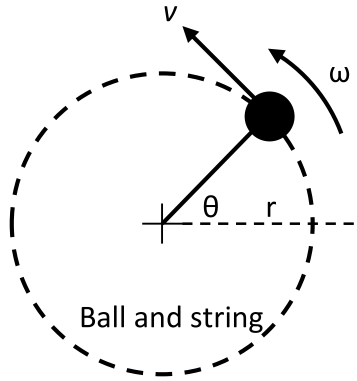
RMA Question Book pages 7 to 9 questions 3, 6 to 9.

2.4 I can solve problems involving angular velocity, period of rotation and tangential velocity.

Tangential velocity is the velocity in meters per second of the point on the rotating object being considered. E.g. in the diagram below of ball being swung around by a string the tangential velocity is the linear velocity the ball would have the instant the string was cut. If the angular velocity is constant the magnitude of the tangential velocity will be constant but its direction will be constantly changing.

Relationship between Tangential Velocity and Angular Velocity

The ball shown in the diagram is rotating with angular velocity ω and tangential velocity *v*. The time it takes for one revolution is the period, *T*.



$$\omega = \frac{Angle\ in\ one\ revolution}{Time\ for\ one\ revolution} = \frac{2\pi}{T}$$

$$v = \frac{Circumferance}{Time\ for\ one\ revolution} = \frac{2\pi r}{T} = \left(\frac{2\pi}{T}\right) r = \omega r$$

So $v = \omega r$

<u>Example</u>

The diagram above shows a ball being spun of the end of a string of length 20cm. The ball is rotating at 100rpm. Find the ball's tangential velocity.

<u>Solution</u>

$r = 20\text{cm} = 0.2\text{m}$

Converting rpm to radians per second

$\omega = 100 \times \frac{2\pi}{60} \times 0.2 = 2.09 \text{ rad s}^{-1}$

$v = \omega r$

$v = 2.09 \times 0.2 = 0.42\text{ms}^{-1}$

Example

The wheels on a bicycle are 0.66m in diameter and are rotating once every 0.80s.

Find.

a. The angular velocity of the wheels.

b. The velocity of the bicycle.

Solution

a. $\omega = \frac{2\pi}{T}$

$$\omega = \frac{2\pi}{0.80}$$

$$\omega = 7.9 \text{ rad s}^{-1} \ \ (7.85 \text{ rad s}^{-1})$$

b. $v = \omega r$

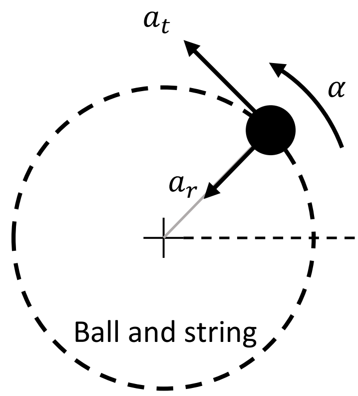
$$v = 7.85 \times \frac{0.66}{2}$$

$$v = 2.6\text{ms}^{-1} \ \ (2.59\text{ms}^{-1})$$

RMA Question Book page 7 questions 4 and 5.

2.5 I understand the terms angular acceleration, tangential acceleration and radial (centripetal) acceleration and the relationship between them.

Constant Period of Rotation
Consider a ball being swung around by a string with a **constant period of rotation *T*** as shown in the diagram.



- **Angular acceleration $\boldsymbol{\alpha}$** is the acceleration in radians per second squared of the ball. This is the rate of change of angular velocity.
  When $T = \text{constant}$ the magnitude of $\alpha = 0 \text{ rad s}^{-2}$.

- **Tangential acceleration $\boldsymbol{a_t}$** is the acceleration in meters per second squared of the point on the rotating object being considered. This is the magnitude of the rate of change of tangential velocity.
  When $T = \text{constant}$ the magnitude of $a_t = 0\text{ms}^{-2}$.

- **Radial acceleration (also known as centripetal acceleration) $\boldsymbol{a_r}$** is the acceleration of the ball towards the centre of rotation. This is measured in metres per second. This is the acceleration caused by the change in direction of the rotating object.
  When $T = \text{constant}$ the magnitude of $a_r = \text{constant}$.

Even when the rate of rotation is constant the radial acceleration is not zero as the direction of the ball's velocity is changing.

Changing Period of Rotation
Consider a ball being swung around but with the period of rotation is decreasing at a constant rate i.e. the rotation rate is increasing. In this case

Angular acceleration, $\alpha = \text{constant}$
Tangential acceleration, $a_t = \text{constant}$
Radial acceleration, $a_r - \text{increasing}$

Relationship between a_t and α

From the relationship $v = u + a_t t \implies a_t = \dfrac{v-u}{t}$

As $v = \omega r$ and $u = \omega_0 r$ then

$$a_t = \frac{v-u}{t} = \frac{\omega r - \omega_0 r}{t} = \left(\frac{\omega - \omega_0}{t}\right) r$$

As $\alpha = \dfrac{\omega - \omega_0}{t}$

Then $a_t = \alpha r$

<u>Example</u>

A flywheel 1.0m in diameter is accelerated from 5.0 rad s^{-1} to 10 rad s^{-1} in 4.0s.

a. Find the angular acceleration of the flywheel.

b. Find the tangential acceleration of the outside edge of the flywheel.

<u>Solution</u>

a.

$\omega_0 = 5.0 \text{ rad s}^{-1}$

$\omega = 10 \text{ rad s}^{-1}$

$t = 4.0 \text{ s}$

$$\alpha = \frac{\omega - \omega_0}{t}$$

$$\alpha = \frac{10 - 5.0}{4.0}$$

$$\alpha = 1.2 \text{ rad s}^{-1}$$

b.

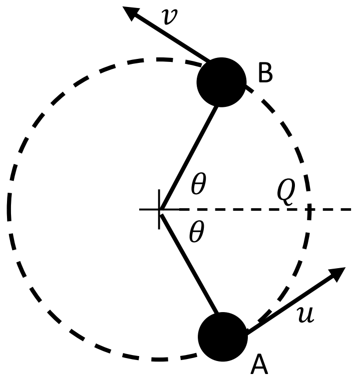
$$a_t = \alpha r$$

$$a_t = 1.2 \times \frac{1.0}{2}$$

$$a_t = 0.6\text{ms}^{-2}$$

2.6 I can derive the relatationships $a_r = \frac{v^2}{r}$ and $a_r = r\omega^2$ for the radial acceleration of a rotating object.

Consider an object rotating at a constant rate
So $\quad \omega = \text{constant and } \alpha = 0 \text{ rad s}^{-2}$



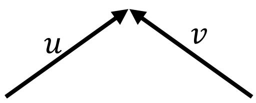
To find the radial acceleration at point Q first find an expression for the acceleration between points A and B. We then reduce the angle θ to zero to find the exact radial acceleration at Q.

The radial acceleration is given by

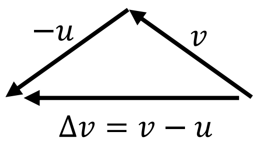
$$a_r = \frac{v-u}{t} = \frac{\Delta v}{\Delta t}$$

To calculate a_r it is necessary to find Δv and Δt.

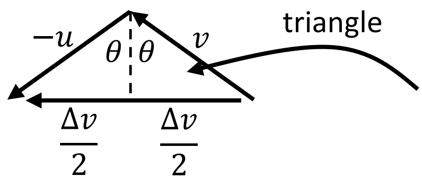
Finding Δv



Initial and final velocity vectors



The vectors $-u$ and v can be combined to give the change in velocity, Δv.



From this triangle

$$\sin\theta = \frac{\Delta v}{2v} \qquad v = |u| = |v|$$

$\Delta v = 2v \sin\theta \qquad$ Equation 1

Finding Δt

$$\Delta t = \frac{\text{distance}}{\text{speed}} = \frac{\text{arc length AB}}{\text{v}}$$

As arc length AB $= 2\text{r}\theta$

$$\Delta t = \frac{2r\theta}{v} \quad \text{Equation 2}$$

Finding a_r

Taking $a_r = \frac{\Delta v}{\Delta t}$ and substituting in equations 1 and 2 gives

$$a_r = \frac{2v\sin\theta}{\left(\frac{2r\theta}{v}\right)} = \frac{v^2 \sin\theta}{r\theta}$$

To find the centripetal acceleration we can reduce the angle θ and use the small angle approximation $\sin\theta = \theta$

$$a_r = \frac{v^2\theta}{r\theta}$$

$$a_r = \frac{v^2}{r}$$

To find this relationship in terms of angular velocity substitute in $v = \omega r$

$$a_r = \frac{v^2}{r} = \frac{\omega^2 r^2}{r}$$

$$a_r = \omega^2 r$$

2.7 I can carry out calculations involving centripetal acceleration and centripetal force.

The relationships $a_r = \frac{v^2}{r}$ $and\ a_r = \omega^2 r$ can be used to find the centripetal force using Newton's Second Law.

$$F = ma \quad \Rightarrow \quad a = \frac{F}{m}$$

This can be substituted into the relationships for centripetal acceleration a_r giving

$$\frac{F}{m} = \frac{v^2}{r} \quad \text{and} \quad \frac{F}{m} = \omega^2 r$$

These give

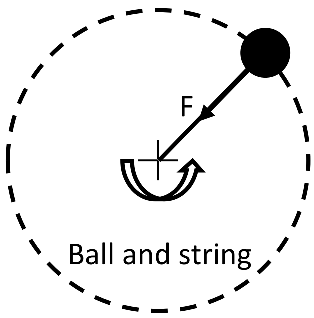
$$F = \frac{mv^2}{r} = m\omega^2 r$$

Example Horizontal Rotation

A 100g ball on a string of length 90cm is rotated at 100 revolutions per minute in a horizontal circle.

a. Calculate the tension in the string.

b. Sketch the path of the ball if the string broke.



Solution Horizontal Rotation

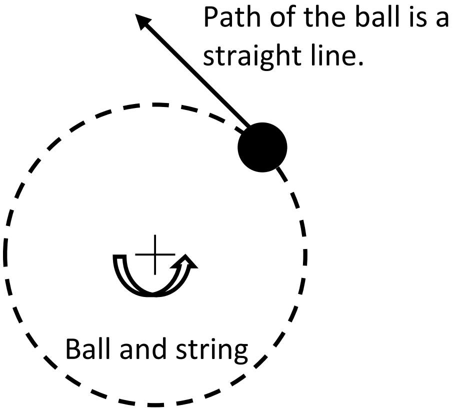
a.

$m = 100\text{g} = 0.100\text{kg}$ $\quad \omega = 100\text{rpm} = 100 \times \frac{2\pi}{60} = 10.47 \text{ rad s}^{-1}$

$r = 90\text{cm} = 0.90\text{m}$

$$F = m\omega^2 r$$
$$F = 0.100 \times 10.47^2 \times 0.90$$
$$F = 9.9\text{N}$$

b.

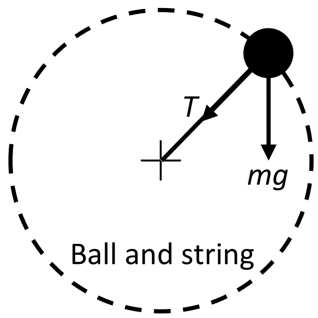


Example Vertical Rotation

A ball of mass 3.0kg is swining in a vertical circle on the end of a 50cm string. Calcualte the minimum angular speed to keep the motion circular.

Solution Vertical Rotation

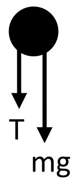
There are two forces acting on the ball; the tension, T,from the string and its weight, mg.



$m = 3.0\text{kg} \qquad \text{r} = 50\text{cm} = 0.50\text{m}$

$F = m\omega^2 r \quad \text{and} \quad \text{W} = \text{mg}$

At the top.



$F = m\omega^2 r = T + mg$

For minimum speed $T = 0\text{N}$

$m\omega^2 r = 0 + mg$

$$\omega = \sqrt{\frac{g}{r}}$$

$$\omega = \sqrt{\frac{9.8}{0.50}} = 4.4 \text{ rad s}^{-1}$$

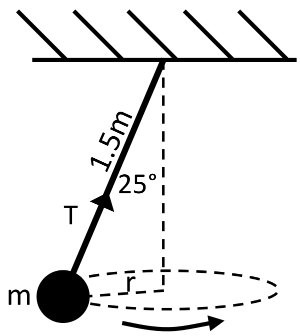
Example Conical Pendulum

A conical pendulum of length 1.5m makes an angle to the vertical of 25°. Find

a. The angular speed.

b. The period of rotation.

c. The tangential speed of the bob.



Solution Conical Pendulum

a. T is the tension in the string and F is the radial force.

Resolve the tension into horizontal and vertical components.

Vertical components	**Horizontal components**
$T\cos 25° = mg$	Centripetal Force, F
$T = \dfrac{mg}{\cos 25°}$	$F = m\omega^2 r = T\sin 25°$

Substituting T from the vertical component in the horizontal component gives

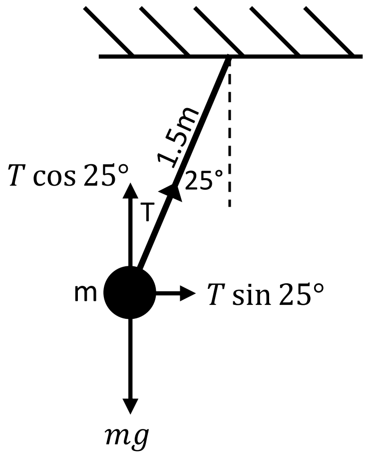
$$m\omega^2 r = \frac{mg}{\cos 25°} \times \sin 25°$$

Which simplifies to

$$\omega^2 r = g\tan 25°$$

Solving for ω gives

$$\omega = \sqrt{\frac{g\tan 25°}{r}}$$



Substituting in the values for g and r gives

$$\omega = \sqrt{\frac{9.8 \times \tan 25°}{1.5}}$$

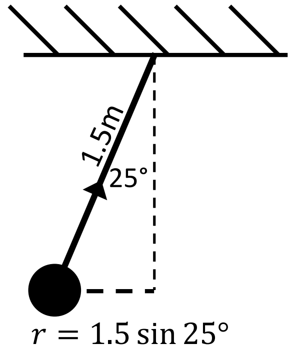
$$\omega = 1.7 \text{ rad s}^{-1} \ \ (1.75 \text{ rad s}^{-1})$$

b. $T = \dfrac{2\pi}{\omega}$ where T is the period

$$T = \frac{2\pi}{1.75} = 2.6\text{s}$$

c. $v = \omega r = 1.75 \times 1.5 \times \sin 25°$

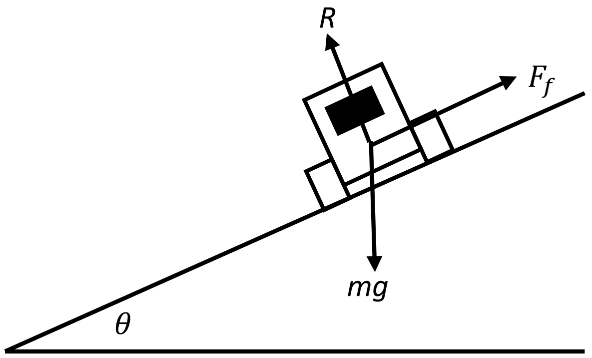
$$v = 1.1\text{ms}^{-1}$$



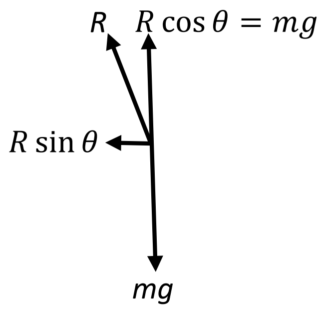
Example Banked Track

A car drives around a circular banked track at a radius of 50m at $10ms^{-1}$. Find the angle at which the frictional force (side thrust) is zero.

Solution Banked Track



When the car is stationary. The forces acting on the car are the weight (mg), the reaction force and the frictional force(F_f).



For there to be zero side force on the car when moving $F_f = 0\text{N}$. The only other horizontal component is the horizontal component of the reaction force. This must provide the centripetal force F_C required to maintain the circular motion.

From the diagram using the components of the reaction force

$$F_C = R\sin\theta \text{ and } R\cos\theta = mg \quad \Rightarrow \quad R = \frac{mg}{\cos\theta}$$

Thees combine to give

$$F_C = \frac{mg\sin\theta}{\cos\theta} = mg\tan\theta$$

From the relationship sheet

$$F_C = \frac{mv^2}{r}$$

Equating these gives

$$\frac{mv^2}{r} = mg\tan\theta$$

This simplifies to

$$\frac{v^2}{r} = g\tan\theta$$

$$\tan\theta = \frac{v^2}{gr}$$

$$\theta = \tan^{-1}\left(\frac{v^2}{gr}\right)$$

From the question $v = 10\text{ms}^{-1}, r = 50\text{m}$ and $\text{g} = 9.8\text{Nkg}^{-1}$

$$\theta = \tan^{-1}\left(\frac{10^2}{9.8 \times 50}\right)$$

$$\theta = 12°$$

RMA Question Book pages 9 and 10 questions 1 and 6.

Key Area: Rotational Dynamics

Success Criteria

3.1 I understand what is mean by the term torque and can use the relationship $T = Fr$ to solve problems involving torque, perpendicular force and radius.

3.2 I know that an unbalanced torque causes an angular acceleration.

3.3 I can define moment of inertial of an object.

3.4 I can select an appropriate relationship and calculate the moment of inertia of discrete masses, rods, discs, spheres and their combinations about a given axis.

3.5 I can use the relationships $T = Fr$ and $T = I\alpha$ to solve problems involving torque, perpendicular force, distance from the axis, angular acceleration and moment of inertia.

3.6 I can define the term angular momentum.

3.7 I can state the principle of conservation of angular momentum.

3.8 I can use the relationships $L = mvr = mr^2\omega = I\omega$ and $L = I\omega = \text{constant (no external torque)}$ to solve problems involving angular momentum, angular velocity, moment of inertia, tangential velocity, mass and its distance from the axis.

3.9 I can define rotational kinetic energy as $E = \frac{1}{2}I\omega^2$

3.10 I can solve problems involving potential energy, rotational kinetic energy, translational kinetic energy, angular velocity, linear velocity, moment of inertia and mass.

3.1 I understand what is mean by the term torque and can use the relationship $T = Fr$ to solve problems involving torque, perpendicular force and radius.

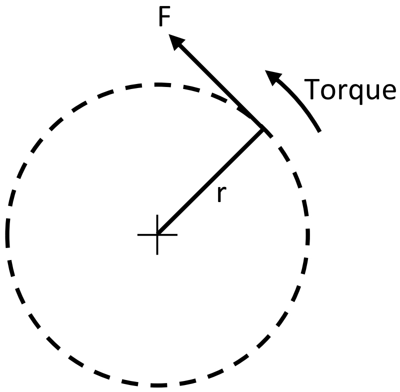
Torque is the turning effect produced by a force acting at a radius, *r*, from the centre of rotation.
It is defined as

$$T = Fr$$

Torque (Nm)

Force (N)

Radius (m)



RMA Question Book page 12 questions 3 to 5.

3.2 I know that an unbalanced torque causes an angular acceleration.

Force, mass and acceleration have equivalent quantities for rotational motion.

Linear Motion	Rotational Motion
Acceleration, *a*	Angular acceleration, *α*
Mass, *m*	Moment of inertia, *I*
Force, *F*	Torque, *T*

- In linear motion an unbalanced force causes a linear acceleration.
- In rotational motion an unbalanced torque causes an angular acceleration.

The relationship between unbalanced torque, moment of inertia and angular acceleration is given by the relationship

$$T = I\alpha$$

Torque (Nm)

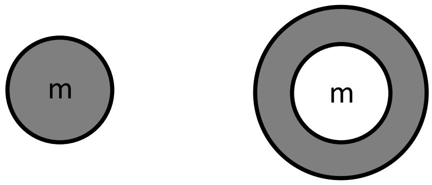
Moment of inertial (kgm^2)

Angular acceleration (radians s^{-2})

3.3 I can define moment of inertial of an object.

The moment of inertia of an object is a measure of its resistance to angular acceleration about a given axis. An object with a higher moment of inertial will resist angular acceleration more than an object with lower moment of inertia.
The moment of inertia of an object depends on the mass of the object **and** the distribution of the mass.

e.g. A solid cylinder and a hollow cylinder have the same mass. The moment of inertia of the hollow cylinder will be greater as most of the mass is at a greater radius. How the mass is distributed affects the moment of inertial.



3.4 I can select an appropriate relationship and calculate the moment of inertia of discrete masses, rods, discs, spheres and their combinations about a given axis.

The moments of inertia of object can be calculated using calculus methods but it is more common to use given formulae for different shapes. These formulae are given in the Additional Relationships Sheet you will receive when doing tests. This sheet is also at the end of these notes.

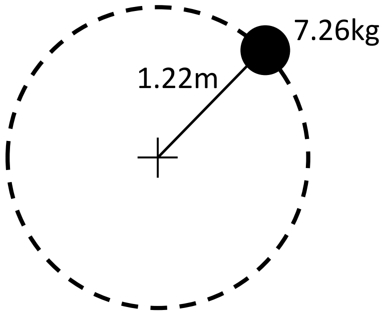
Frequently is not possible to find the exact moment of inertia using these formulae. However, they can, with careful choice of formulae, give a good approximation. See the examples of the point mass and thin ring below.

Moment of inertia symbol is I with units of kgm^2.

Shape	Moment of Inertia
Point mass	$I = mr^2$
Rod about centre	$I = \frac{1}{12}ml^2$
rod about end	$I = \frac{1}{3}ml^2$
disc about centre	$I = \frac{1}{2}mr^2$
sphere about centre	$I = \frac{2}{5}mr^2$

Example – Point Mass

In the Olympics, the throwing hammer consists of a 7.26kg mass on a thin wire 1.22m long. Calculate the moment of inertia of the hammer when being spun around.



Solution – Point Mass

The system approximates a point mass.

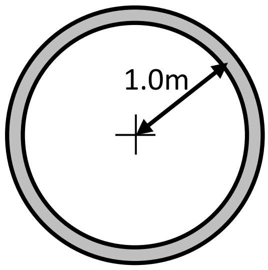
$$I = mr^2$$

$$I = 7.26 \times 1.22^2$$

$$I = 10.8\text{kgm}^2$$

Example – Thin Ring

Find the moment of inertia of a 1.0kg thin ring of radius 1.0m.



Solution

The ring can be approximated to many point masses at a radius of 1.0m.

$$I = mr^2$$

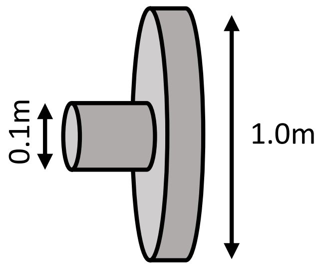
$$I = 1.0 \times 1.0^2$$

$$I = 1.0\text{kgm}^2$$

Notice that this could also be used for a thin cylinder as the length of the ring does not appear in the formula.

Example – Combining Moments of Inertia

A flywheel is assembled from an axle of mass 5.5kg and 0.1m in diameter together with a flywheel of mass 100kg and 1.0m in diameter. Find the moment of inertia of the assembled axle and flywheel.



Assembled flywheel

Solution – Combining Moments of Inertia

Both the axle and the flywheel section are discs so their moments of inertia can be calculated separately then added.

Axle

$$r = \frac{\text{Diameter}}{2} = \frac{0.1}{2} = 0.05\text{m}$$

$$m = 5.5\text{kg}$$

$$I = \frac{1}{2}mr^2$$

$$I = \frac{1}{2} \times 5.5 \times 0.05^2$$

$$I = 6.875 \times 10^{-3}\text{kgm}^2$$

Flywheel

$$r = \frac{\text{Diameter}}{2} = \frac{1.0}{2} = 0.5\text{m}$$

$$m = 100\text{kg}$$

$$I = \frac{1}{2}mr^2$$

$$I = \frac{1}{2} \times 100 \times 0.5^2$$

$$I = 12.5\text{kgm}^2$$

Total moment of Inertia

$$I_T = 6.875 \times 10^{-3} + 12.5$$

$$I_T = 13\text{kgm}^2$$

Notice that due to the r^2 term in the moment of inertia formula the smaller radius axle has almost no contribution to the overall moment of inertia.

RMA Question Book pages 11 and 12 questions 1 and 2.

3.5 I can use the relationships $T = Fr$ and $T = I\alpha$ to solve problems involving torque, perpendicular force, distance from the axis, angular acceleration and moment of inertia.

Example

A flywheel consisting of a disk of mass 2.5kg with radius 20cm and initial angular velocity of 8.0 rad s^{-1} is braked by a force until stationary in 5.0 seconds.

a. Calculate the moment of inertia of the disk

b. Find the angular acceleration during braking.

c. Find the magnitude of the braking force

Solution

a. $m = 2.5\text{kg}$

$$r = 20\text{cm} = 0.20\text{m}$$

$$I = \frac{1}{2}mr^2 = \frac{1}{2} \times 2.5 \times 0.20^2 = 0.05\text{kgm}^2$$

b. $\omega_0 = 8.0 \text{ rad s}^{-1}$

$\omega = 0 \text{ rad s}^{-1}$

$t = 5.0\text{s}$

$$\omega = \omega_0 + \alpha t \quad \Rightarrow \quad \alpha = \frac{\omega - \omega_0}{t}$$

$$\alpha = \frac{0 - 8.0}{5.0} = -1.6 \text{ rad s}^{-2}$$

c. $T = I\alpha \quad \text{and } T = Fr$

$$I\alpha = Fr$$

$$F = \frac{I\alpha}{r}$$

$$F = \frac{0.05 \times 1.6}{0.20} = 0.40\text{N}$$

RMA Question Book pages 13 and 15 questions 6 and 12.

3.6 I can define the term angular momentum

Angular momentum, *L*, is the product of moment of inertia, *I*, and angular velocity, ω. It is given by the relationship

$$L = I\omega$$

Angular momentum $(\mathrm{kgm^2s^{-1}})$

Moment of inertial $(\mathrm{kgm^2})$

Angular velocity (rad s^{-1})

The above relationship applies to any rotating object. When the object being considered is a point mass or thin ring the relationship $I = mr^2$ and $v = \omega r$ can be substituted for *I* in the above relationship to give

$$L = mvr = mr^2\omega = I\omega$$

3.7 I can state the principle of conservation of angular momentum.

Angular momentum is conserved when there are no external unbalanced torques.

$$L = I\omega = \text{constant} \quad \text{(for no external torques)}$$

3.8 I can use the relationships $L = mvr = mr^2\omega = I\omega$ and $L = I\omega = \textbf{constant (no external torque)}$ to solve problems involving angular momentum, angular velocity, moment of inertia, tangential velocity, mass and its distance from the axis.

<u>Example</u>
A turntable is rotating freely at 80rpm about a vertical axis. A small mass of 40g falls vertically onto the turntable and lands at a distance of 80mm from the central axis. The rotation of the turntable is reduced to 20rpm.
Find the moment of inertia of the turntable.

<u>Solution</u>

Before the mass is dropped

$$\omega_{Before} = 80 \times \frac{2\pi}{60} = \frac{8\pi}{3} \text{ rad s}^{-1}$$

$I_t = \text{moment of inertia of the turntable}$

$$L_{Before} = I_t\omega_{Before} = I_t \times \frac{8\pi}{3}$$

$$L_{Before} = \frac{8\pi I_t}{3}$$

After the mass is dropped

$$\omega_{After} = 20 \times \frac{2\pi}{60} = \frac{4\pi}{6} \text{ rad s}^{-1}$$

$m = 40\text{g} = 0.040\text{kg}$

$r = 80\text{mm} = 0.080\text{m}$

$$L_{After} = (I_t + mr^2)\omega_{after}$$

$$L_{After} = (I_t + 0.040 \times 0.080^2) \times \frac{4\pi}{6}$$

$$L_{After} = (I_t + 0.000256) \times \frac{4\pi}{6}$$

As angular momentum is conserved

$$L_{Before} = L_{after}$$

$$\frac{8\pi I_t}{3} = (I_t + 0.000256) \times \frac{4\pi}{6}$$

$$\frac{8\pi I_t}{3} \times \frac{6}{4\pi} = I_t + 0.000256$$

$$4I_t = I_t + 0.000256$$

$$I_t(4 - 1) = 0.000256$$

$$I_t = 8.5 \times 10^{-5}\text{kgm}^2$$

RMA Question Book pages 15 and 16 questions 2, 3, 4, 5.

3.9 I can define rotational kinetic energy as $E = \frac{1}{2} I\omega^2$

$$E = \frac{1}{2} I\omega^2$$

Rotational kinetic energy (J)

Angular velocity (rad s^{-1})

Moment of inertial (kgm^2)

3.10 I can solve problems involving potential energy, rotational kinetic energy, translational kinetic energy, angular velocity, linear velocity, moment of inertia and mass.

Example 1

A unicyclist pedals at 60rpm along a flat surface. Use the information below to calculate the total kinetic energy of unicyclist and unicycle.

Mass of unicycle $= 10.0\text{kg}$
Mass of the unicyclist = 70kg
Moment of inertia of the wheels and pedals $= 0.63\ \text{kgm}^2$
Diameter of the unicyle wheel $= 0.5\text{m}$

Solution 1

Angular velocity, $\omega = 60 \times \frac{2\pi}{60} = 6.28\ \text{rad s}^{-1}$

Linear velocity, $v = \omega r = 6.28 \times \frac{0.5}{2} = 1.57\ \text{ms}^{-1}$

Linear kinetic energy

$E = \frac{1}{2} m v^2$

$E = \frac{1}{2} \times (10.0 + 70) \times 1.57^2$

$E = 98.60\text{J}$

Rotational kinetic energy

$E = \frac{1}{2} I\omega^2$

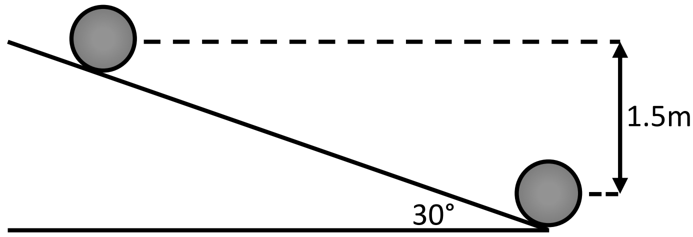
$E = \frac{1}{2} \times 0.63 \times 6.28^2$

$E = 12.42\text{J}$

Total kinetic energy $= 98.60 + 12.42 = 110\text{J}$ (to 2 significant figures)

Example 2

A solid 1.0kg ball of radius 0.10m starts from rest and rolls down a 1.5m high ramp, sloping at 30° to the horizontal. The linear speed of the ball at the bottom of the slope 1.5ms^{-1}. Find the moment of inertia of the ball.



Solution 2

At the top of the slope the gravitational potential energy of the ball is given by

$E = mgh$

$E = 1.0 \times 9.8 \times 1.5$

$E = 14.7\text{J}$

This gravitational potential energy will be converted to kinetic energy at the bottom of the slope. The total kinetic energy will also be given by

$E = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = 14.7\text{J}$

$v = \omega r \quad \Rightarrow \quad \omega = \frac{v}{r}$ which can be subsituted into the expression for kinetic energy

$E = \frac{1}{2}mv^2 + \frac{1}{2}\frac{v^2 I}{r^2}$

$E = \frac{1}{2}v^2\left(m + \frac{I}{r^2}\right)$

This can be rearranged to give

$I = r^2\left(\frac{2E}{v^2} - m\right)$

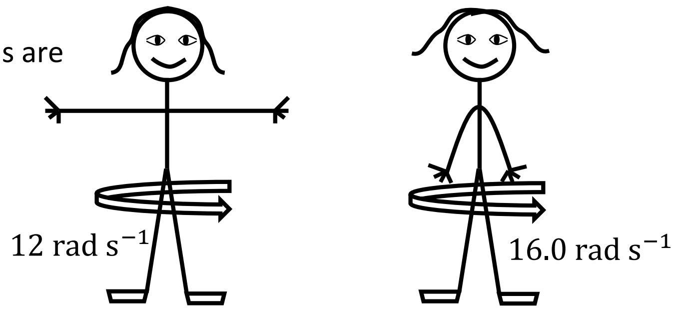
Substituting in $E = 14.7\text{J}$ and the other values for *r*, *m* and *v* gives

$I = 0.10^2\left(\frac{2 \times 14.7}{1.5^2} - 1.0\right)$

$I = 0.12\text{kgm}^2$

Example 3

An ice skater is spinning at an angular velocity of $12\ \mathrm{rad\ s^{-1}}$ with her arms held out. When her arms are brought to her side her angular velocity increases to $16.0\ \mathrm{rad\ s^{-1}}$.



a. Explain why the skater's angular velocity increases.

b. State what happens to the skater's kinetic energy as her angular velocity increases.

Solution 3

a. There are no external torques so angular momentum is conserved. When the skater brings her hands to her side her moment of inertia decreases. Her angular velocity increases as $L = I\omega$.

b. Kinetic energy increases.

RMA Question Book
page15 question 1
pages 17 and 18 questions 6 to 8.

Key Area: Gravitation

Success Criteria

4.1 I can define gravitational field strength.

4.2 I can sketch field lines and field line patterns around a planet and a planet–moon system.

4.3 I can use the relationship $F = \frac{GMm}{r^2}$ to carry out calculations involving gravitational force, masses and their separation.

4.4 I can define gravitational field strength.

4.5 I can derive gravitational field strength from Newton's Law of Gravitation.

4.6 I can carry out calculations involving the period of satellites in circular orbit, masses, orbit radius and satellite speed.

4.7 I can define the terms "gravitational potential energy" and "gravitational potential".

4.8 I can use the relationships for gravitational potential energy and gravitational potential to solve problems.

4.9 I can describe a gravitational potential well.

4.10 I know that the energy required to move a mass between two points in a gravitational field is independent of the path taken.

4.11 I can define escape velocity.

4.12 I can derive the relationship $\mathrm{v} = \sqrt{\frac{2\mathrm{GM}}{\mathrm{r}}}$ which gives escape velocity

4.13 I can use the relationship for escape velocity to solve problems involving mass, distance and escape velocity.

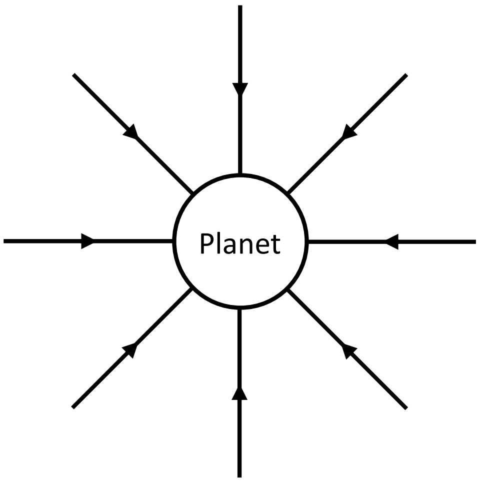
4.14 I understand the relevance of escape velocity to explain the low incidence of helium in the Earth's atmosphere and why small astronomical bodies have no atmosphere.

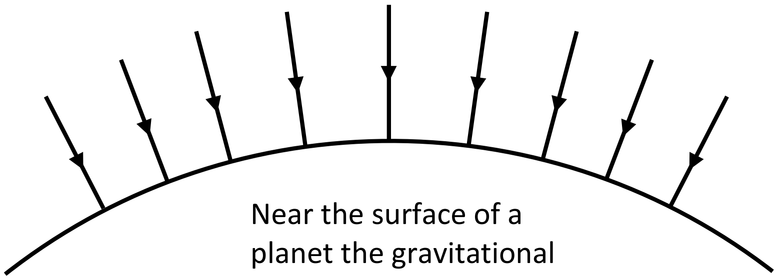
4.1 I can define gravitational field strength.

Gravitational field strength is defined as the gravitational force acting on a unit mass. The units of gravitational field strength are Newtons per kilogram (Nkg^{-1}).

4.2 I can sketch field lines and field line patterns around a planet and a planet–moon system.

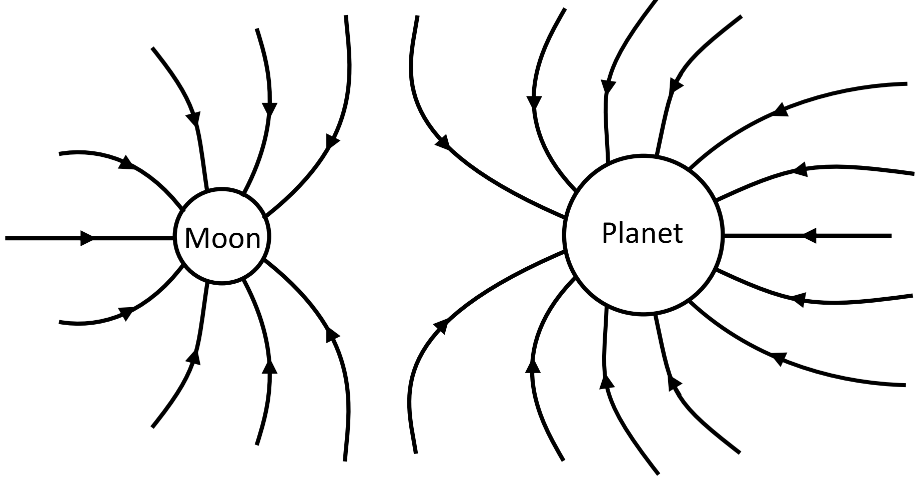
The gravitational field lines around a mass indicate the direction of the force on mass in the gravitational field. Closer field lines indicate higher gravitational field strength.





Near the surface of a planet the gravitational field lines are almost parallel.

The gravitational field lines around a planet and moon are distorted by the nearby mass.



$\mathbf{4.3}$ I can use the relationship $\boldsymbol{F = \frac{GMm}{r^2}}$ to carry out calculations involving gravitational force, masses and their separation.

This relationship is the same as the one you were using during the higher course.
This gives the gravitational force produced between two masses is called Newton's Law of Gravitation

$$F = \frac{GMm}{r^2} \text{ Where } \begin{cases} F \text{ is the gravitational force between the two objects in Newtons} \\ M \text{ and } m \text{ are the masses of the objects in kilograms.} \\ r \text{ is the distance between the centre of mass of the objects in metres.} \\ G = 6.67 \times 10^{-11}\,\mathrm{m^3kg^{-1}s^{-2}} \text{ Universal constant of gravitation.} \end{cases}$$

<u>Example</u>
Using the data given calculate the mean gravitational force between the Earth and the Moon.

<u>Data</u>
Mass of the Earth $= 6.0 \times 10^{24}\mathrm{kg}$
Mass of the Moon $= 7.3 \times 10^{22}\mathrm{kg}$
Mean Earth to Moon distance = $3.84 \times 10^{8}\mathrm{m}$
$G = 6.67 \times 10^{-11}\mathrm{m^3kg^{-1}s^{-2}}$

<u>Solution</u>
Substitute all the values into Newton's Law of Universal Gravitation.

$$F = \frac{Gm_1m_2}{r^2} = \frac{6.67 \times 10^{-11} \times 6.0 \times 10^{24} \times 7.3 \times 10^{22}}{(3.84 \times 10^8)^2} = 2.0 \times 10^{20}\mathrm{N}$$

RMA Question Book page 19 questions 1 to 3

4.4 I can define gravitational field strength

Gravitational field strength is defined as the gravitational force acting on a unit mass.

4.5 I can derive gravitational field strength from Newton's Law of Gravitation.

At the surface of a planet the gravitational force on an object of mass, m, is given by

$$F = w = mg$$

where, g, is the gravitational field strength.

Newton's law of Gravitation gives the gravitational force as

$$F = \frac{GMm}{r^2}$$

where, M, is the mass of the planet

Equating both these relationships gives

$$mg = \frac{GMm}{r^2}$$

Which simplifies to give

$$g = \frac{GM}{r^2}$$

<u>Example</u>

The relationships $F = \frac{GMm}{r^2}$ and $\mathrm{w} = mg$ both give the gravitational force on an object on the surface of the Earth. Show that $g = \frac{GM}{r^2} = 9.8\mathrm{Nkg^{-1}}$

<u>Solution</u>

Where G is the Universal gravitation constant $= 6.67 \times 10^{-11}\mathrm{m^3kg^{-1}s^{-2}}$

M is the mass of the Earth $= 6.0 \times 10^{24}\mathrm{m}$

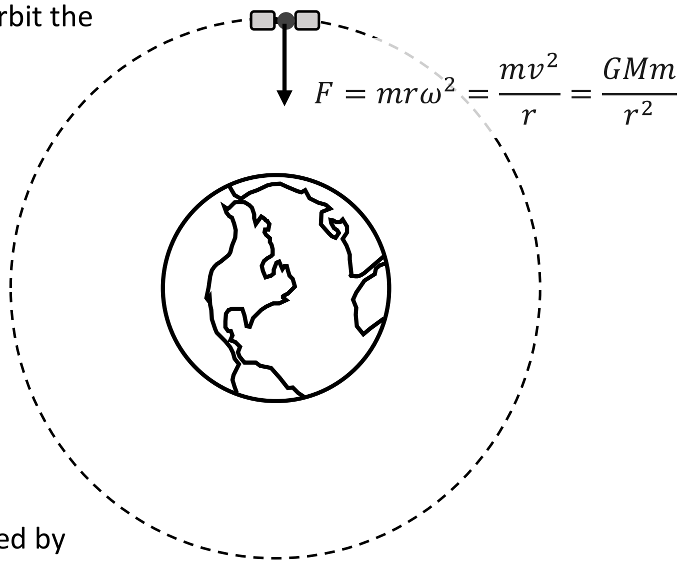
r is the radius of the Earth $= 6.4 \times 10^{6}\mathrm{m}$

Solution

$$g = \frac{GM}{r^2} = \frac{6.67 \times 10^{-11} \times 6.0 \times 10^{24}}{(6.4 \times 10^6)^2} = 9.8\mathrm{Nkg^{-1}}$$

RMA Question Book page 19 questions 4 to 6

4.6 I can carry out calculations involving the period of satellites in circular orbit, masses, orbit radius and satellite speed.



When a satellite is moving in a circular orbit the relationships

$F = mr\omega^2$ and $F = \frac{mv^2}{r}$

give the centripetal force required to keep the satellite in orbit.

This force is the gravitational force at the height of the satellite so this force is also given by

$$F = \frac{GMm}{r^2}$$

Problems involving satellites can be solved by equating one of the centripetal force relationships to Newton's Law of Gravitation.

$$F = mr\omega^2 = \frac{mv^2}{r} = \frac{GMm}{r^2}$$

<u>Example</u>

Find

a. the period of the international space station which orbits at an altitude of 400km.

b. the tangential speed of the international space station.

<u>Solution</u>

a.

Step 1 find *ω*

$F = mr\omega^2$ and $F = \frac{GMm}{r^2}$

$$mr\omega^2 = \frac{GMm}{r^2}$$

This rearranges to

$$\omega^2 = \frac{GMm}{mrr^2}$$

Which simplifies to

$$\omega = \sqrt{\frac{GM}{r^3}}$$

Step 2 find the period

Using the relationship $\omega = \frac{2\pi}{T}$ (It's not on the relationship sheet. See section 2.4) and the above relationship for ω gives

$$\frac{2\pi}{T} = \sqrt{\frac{GM}{r^3}}$$

This rearranges to give

$$T = 2\pi\sqrt{\frac{r^3}{GM}}$$

Using the data sheet at the end of these notes

$G = 6.67 \times 10^{-11}\mathrm{m^3kg^{-1}s^{-2}}$

Mass of the Earth, $M = 6.0 \times 10^{24}\mathrm{kg}$

$$\left.\begin{array}{l}\text{Altitude} = 400\text{km} = 0.4 \times 10^6\text{m} \\ \text{Radius of the Earth} = 6.4 \times 10^6\text{m}\end{array}\right\} \quad r = 6.4 \times 10^6 + 0.4 \times 10^6 = 6.8 \times 10^6\text{m}$$

$$T = 2\pi\sqrt{\frac{(6.8 \times 10^6)^3}{6.67 \times 10^{-11} \times 6.0 \times 10^{24}}}$$

$T = 5600\text{s}$ to 2 significant figures (5569s)

b.

$v = \omega r$ and $\omega = \frac{2\pi}{T}$

$$v = \frac{2\pi r}{T}$$

$$v = \frac{2\pi \times 6.8 \times 10^6}{5569}$$

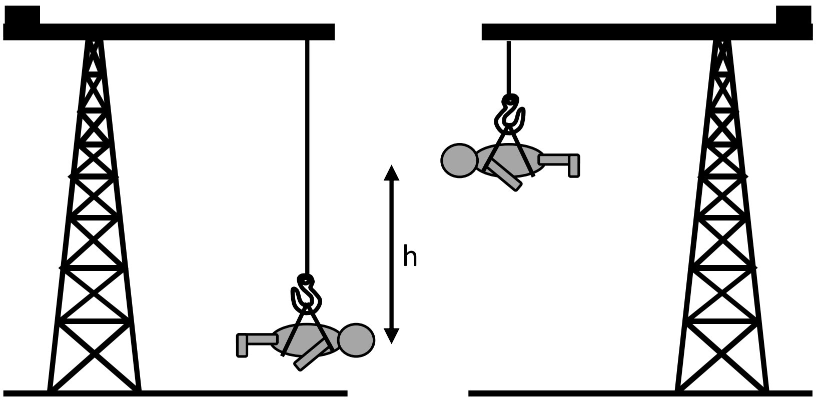
$$v = 7700\text{ms}^{-1} \quad (7672\text{ms}^{-1})$$

RMA Question Book pages 19 and 20 questions 7 to 13.

4.7 I can define the terms “gravitational potential energy” and “gravitational potential”.

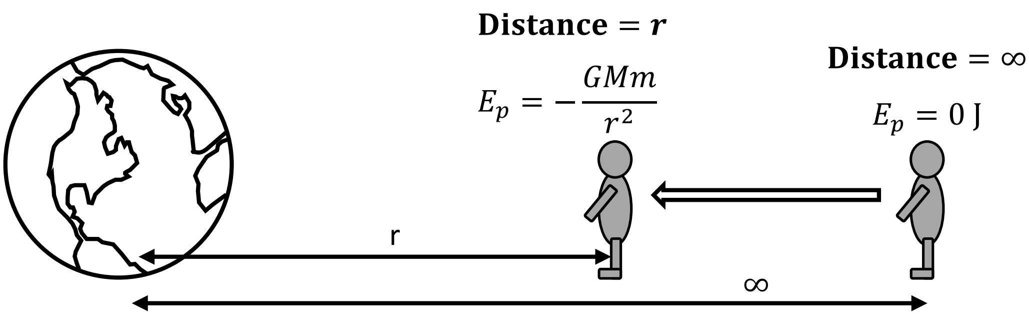
Gravitational Potential Energy Where *g* is Constant.

Near the surface of the Earth (or other planets) the gravitational field strength, *g,* will not vary significantly over small changes in height, so can be taken as constant. The relationship $E = mgh$ can be used to give the change in gravitational potential energy. Where h is the change in height.



When the gravitational field strength cannot be regarded as constant, gravitational potential energy must take into account the variation of g with distance.

Gravitational Potential Energy General Case
Gravitational potential energy is taken as zero at infinity.



When an object of mass m is moved to a distance r from the centre of the mass of a planet/star etc. its gravitational potential energy is given by

Universal constant of gravitation $(\mathrm{m^3kg^{-1}s^{-2}})$

Mass of planet/star etc. (kg)

Mass object (kg)

$$E_p = -\frac{GMm}{r}$$

Gravitational potential energy (J)

Distance from the centre of planet/star etc. (m)

As the object is moved through a distance by the gravitational force, the gravitational potential energy is the work done by a moving mass going from infinity to the distance *r*. As gravitational potential energy is lost as it moves from infinity to r the it will be less than zero. i.e. gravitational potential energy is negative.

Gravitational Potential
Gravitational potential, *V*, of a point in space is defined as the work done in moving **a unit mass** from infinity to that point.

Universal constant of gravitation ($\mathrm{m^3kg^{-1}s^{-2}}$)

Mass of planet/star etc. (kg)

$$V_p = -\frac{GM}{r}$$

Gravitational potential ($\mathrm{Jkg^{-1}}$)

Distance from the centre of planet/star etc. (m)

4.8 I can use the relationships for gravitational potential energy and gravitational potential to solve problems.

<u>Example – Gravitational Potential Energy</u>
A 1000kg satellite is launched into low orbit at a height of 150km.
Find
a. its gravitational potential energy before launch.
b. its gravitational potential energy while in orbit.
c. the change in the satellite's potential energy.

<u>Solution – Gravitational Potential Energy</u>
a.

$\text{Mass of the Earth} = M = 6.0 \times 10^{24}\text{kg}$

$\text{Mass of the satellite} = m = 1000\text{kg}$

$G = 6.67 \times 10^{-11}\mathrm{m^3kg^{-1}s^{-2}}$

$r = 6.4 \times 10^6\text{m}$

$$E_p = -\frac{GMm}{r} = -\frac{6.67 \times 10^{-11} \times 6.0 \times 10^{24} \times 1000}{6.4 \times 10^6}$$

$E_p = -6.3 \times 10^{10}\text{J} \quad (-6.25 \times 10^{10}\text{J})$

b. $r = 6.4 \times 10^6 + 150 \times 10^3 = 6.55 \times 10^6$

$$E_p = -\frac{6.67 \times 10^{-11} \times 6.0 \times 10^{24} \times 1000}{6.55 \times 10^6}$$

$E_p = -6.1 \times 10^{10}\text{J} \quad (-6.11 \times 10^{10}\text{J})$

c. $\Delta E_p = -6.25 \times 10^{-11} - (-6.11 \times 10^{10})$

$\Delta E_p = 1.4 \times 10^{9}\text{J} \quad (1.43 \times 10^{9}\text{J})$

Example – Gravitational Potential

Find the gravitational potential at the mean radius of the Moon orbit.

Solution – Gravitational Potential

From the data sheet

$r = 3.84 \times 10^{8}\text{m}$

$G = 6.67 \times 10^{-11}\text{m}^3\text{kg}^{-1}\text{s}^{-2}$

$M = \text{mass of the Earth} = 6.0 \times 10^{24}\text{kg}$

$$V_p = -\frac{GM}{r}$$

$$V_p = -\frac{6.67 \times 10^{-11} \times 6.0 \times 10^{24}}{3.84 \times 10^{8}}$$

$$V_p = 1.0 \times 10^{6}\text{Jkg}^{-1}$$

Example – Satellite Orbits

A 1000kg satellite is orbiting at 200km above the Earth's surface. A rocket on the satellite fires and moves the rocket into and orbit at an altitude of 250km. Find the energy required for this manoeuvre.

Solution – Satellite Orbits

The total energy of the satellite = kinetic energy + gravitational potential energy.

Finding the kinetic energy

The requires all three relationships below

$$F = \frac{mv^2}{r}\ ,\ F = \frac{GMm}{r^2}\ \text{ and }\ E_k = \frac{1}{2}mv^2$$

Equating the relationships for force gives

$$\frac{mv^2}{r} = \frac{GMm}{r^2}$$

Which can be solved for v^2

$$v^2 = \frac{GM}{r}$$

This can then be used in the kinetic energy relationship

$$E_k = \frac{1}{2}mv^2 = \frac{GMm}{2r}$$

Finding Potential Energy

This is given by

$$E_p = -\frac{GMm}{r}$$

Total energy is given by

$$E_T = E_k + E_p = \frac{GMm}{2r} - \frac{GMm}{r}$$

Multiply the top and bottom of the E_p term by 2 gives

$$E_T = \frac{GMm}{2r} - \frac{2GMm}{2r}$$

Which simplifies to

$$E_T = -\frac{GMm}{2r}$$

From the data sheet
$r_{200\text{km}} = 6.4 \times 10^6 + 2.00 \times 10^5 = 6.6 \times 10^6 \text{m}$
$r_{250\text{km}} = 6.4 \times 10^6 \times 2.50 \times 10^5 = 6.65 \times 10^6 \text{m}$
$G = 6.67 \times 10^{-11} \text{m}^3\text{kg}^{-1}\text{s}^{-2}$
$M = \text{mass of the Earth} = 6.0 \times 10^{24} \text{kg}$
$m = 1000\text{kg}$

At an altitude of 200km

$$E_T = -\frac{GMm}{2r} = \frac{6.67 \times 10^{-11} \times 6.0 \times 10^{24} \times 1000}{2 \times 6.6 \times 10^6} = -3.032 \times 10^{10} \text{J}$$

At an altitude of 250km

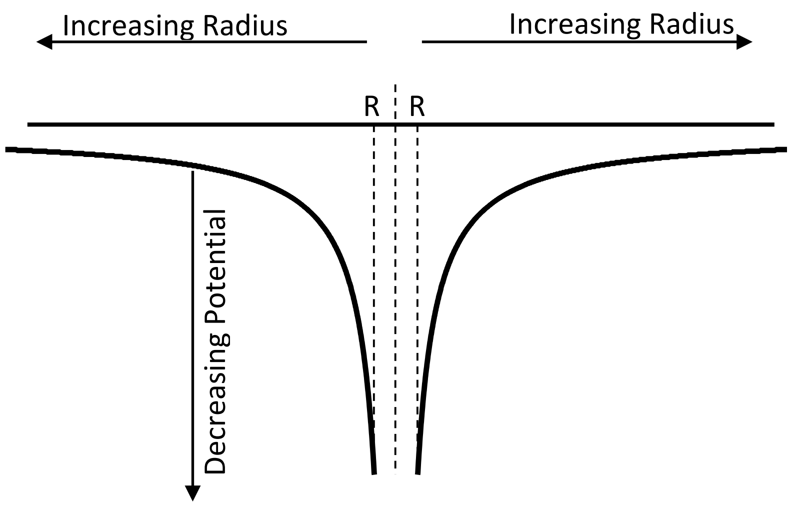
$$E_T = -\frac{GMm}{2r} = \frac{6.67 \times 10^{-11} \times 6.0 \times 10^{24} \times 1000}{2 \times 6.65 \times 10^6} = -3.009 \times 10^{10} \text{J}$$

Energy required $= -3.009 \times 10^{10} - (-3.032 \times 10^{10}) = 2.3 \times 10^8 \text{J}$

RMA Question Book pages 20 and 21 questions 14 to 19.

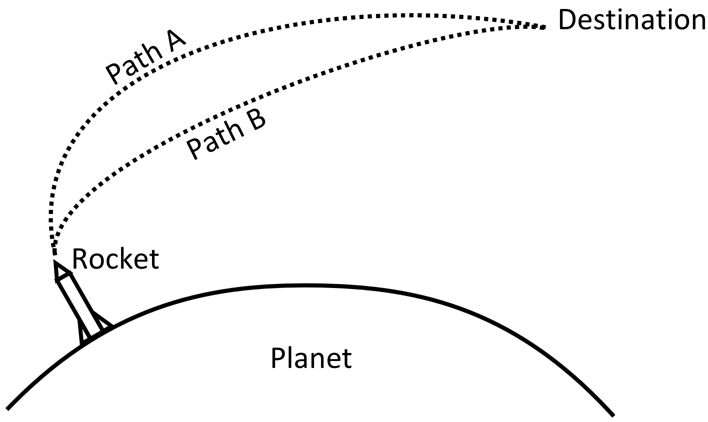
4.9 I can describe a gravitational potential well

A plot of gravitational potential against radius around a planet of radius, R is shown.
An object moving towards the planet will "fall" down the well and be captured by the planet unless is has sufficient kinetic energy to escape.



4.10 I know that the energy required to move a mass between two points in a gravitational field is independent of the path taken

The energy required to move a mass between two points in a gravitational field is given by the difference in gravitational potential energy between these points.
If rocket is launched to a destination in a gravitational field both paths A and B would result in the same difference in potential energy. The energy required by the rocket is independent of the path taken.



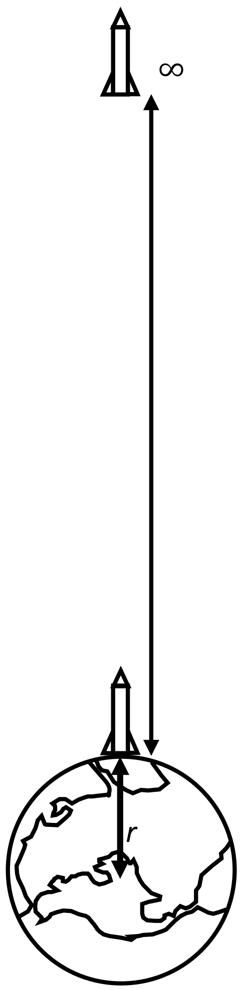
4.11 I can define escape velocity

There are two equivalent ways to define escape velocity.

- Escape velocity is the minimum velocity required to allow a mass to escape a gravitational field.
- Escape velocity is the minimum velocity required to achieve zero kinetic energy and maximum (zero) potential energy at an infinite distance.

This means that an unpowered object launched at escape velocity or above from a surface of a planet will not fall back down to the surface. It will continue on indefinitely.
Although escape velocity is the usual term it is actually a speed not a velocity.

4.12 I can derive the relationship $\mathbf{v} = \sqrt{\frac{\mathbf{2GM}}{\mathbf{r}}}$ which gives escape velocity



This derivation requires the relationship for gravitational potential energy and kinetic energy.

$$E_p = -\frac{GMm}{r} \quad \text{and} \quad E_k = \frac{1}{2}mv^2$$

An object launched at the escape velocity it will reduce in velocity as it moves out through the gravitational field. Kinetic energy is transformed into gravitational potential energy, reaching zero velocity and zero potential energy at an infinite distance. The total energy of the object at all times will be zero.

$$\text{So total energy} = \frac{1}{2}mv^2 - \frac{GMm}{r} = 0\text{J}$$

This rearranges to

$$\frac{1}{2}mv^2 = \frac{GMm}{r}$$

The *m* term cancels and the 2 can be moved to the right-hand side giving

$$v^2 = \frac{2GM}{r}$$

Taking the square root leaves

$$v = \sqrt{\frac{2GM}{r}}$$

Where *r* is the distance from the centre of the planet from which the object is launched. This can be the surface or from orbit.

4.13 I can use the relationship for escape velocity to solve problems involving mass, distance and escape velocity.

Example

Find the escape velocity of the Earth.

Solution

From the data sheet

$r = 6.4 \times 10^{6}\mathrm{m}$

$G = 6.67 \times 10^{-11}\mathrm{m^{3}kg^{-1}s^{-2}}$

$M = 6.0 \times 10^{24}\mathrm{kg}$

$$v = \sqrt{\frac{2GM}{r}}$$

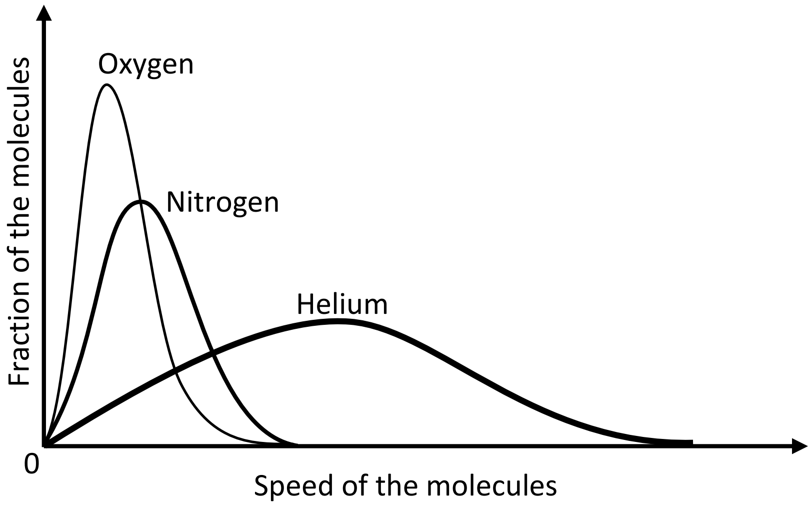
$$v = \sqrt{\frac{2 \times 6.67 \times 10^{-11} \times 6.0 \times 10^{24}}{6.4 \times 10^{6}}}$$

$$v = 11000\mathrm{ms^{-1}}$$

RMA Question Book pages 21 and 22 questions 20 to 23.

4.14 I understand the relevance of escape velocity to explain; the low incidence of helium in the Earth's atmosphere; why small astronomical bodies have no atmosphere.

The temperature of a gas is a measure of the average kinetic energy of its molecules. However, the speeds of the molecules are distributed over a range determined by the molecular mass and the temperature. The distribution of molecular speeds for oxygen, nitrogen and helium at the same temperature is shown below. Due to its lower molecular mass helium has more molecules at higher speeds than nitrogen or oxygen.



The Low Incidence of Helium in the Earth's Atmosphere
High in the Earth's atmosphere it is more likely that some helium will reach escape velocity and be lost from the atmosphere. Over time almost all the helium will be lost. The distribution of speeds for oxygen and nitrogen ensure that almost no molecules reach escape velocity. This means Earth has an abundance of nitrogen, oxygen, argon, carbon dioxide and water vapour which have a relatively high molecular mass. There are almost no low molecular mass molecules such as hydrogen and helium.

Small Astronomical Bodies
Small astronomical bodies e.g. The Earth's Moon have a low escape velocity. This means that all gases will have molecules with sufficient speed to escape into space. This means that small planets, moons, asteroids etc. will not have an atmosphere.

Key Area: General Relativity

Previous Knowledge

Frames of reference.
Light travels at a constant speed in a vacuum.
Time dilation.
Length contraction.

Success Criteria

5.1 I can describe what is meant by an inertial frame of reference and a non-inertial frame of reference.

5.2 I can state the “equivalence principle” of General Relativity

5.3 I know that acceleration and gravitational fields causes time to slow down.

5.4 I can draw and interpret spacetime diagrams.

5.5 I know that general relativity leads to the interpretation that mass curves spacetime, and that gravity arises from the curvature of spacetime.

5.6 I understand what is meant by “a black hole”

5.7 I understand what is meant by the terms “event horizon” and “Schwarzschild radius”.

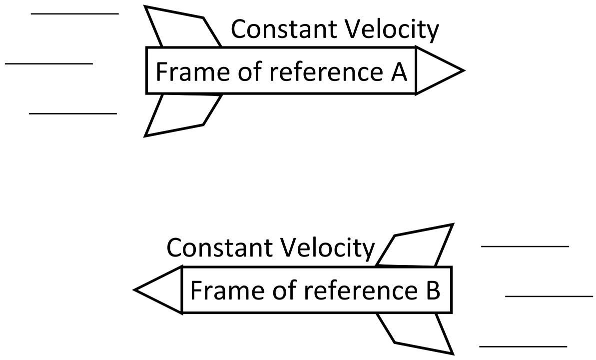
5.8 I know that time appears to be “frozen” at the event horizon of a black hole.

5.9 I can use the relationship $r = \frac{2GM}{c^2}$ to solve problems relating to the Schwarzschild radius of a black hole.

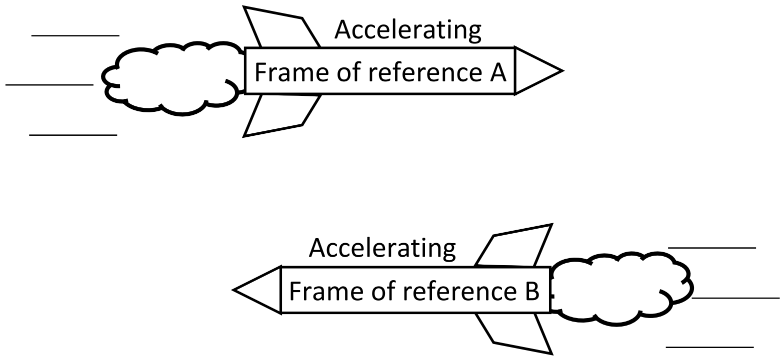
5.1 I can describe what is meant by an inertial frame of reference and a non-inertial frame of reference.

In the Special Relativity section in the Higher Physics course you learned about frames of reference. You would only have considered frames of reference which were moving at a constant relative velocity.
Now you have to consider relativity in the context of accelerating frames of reference.

Inertial frames of reference are frames of reference where the frames have a constant relative velocity. Special relativity only applies to inertial frames of reference.



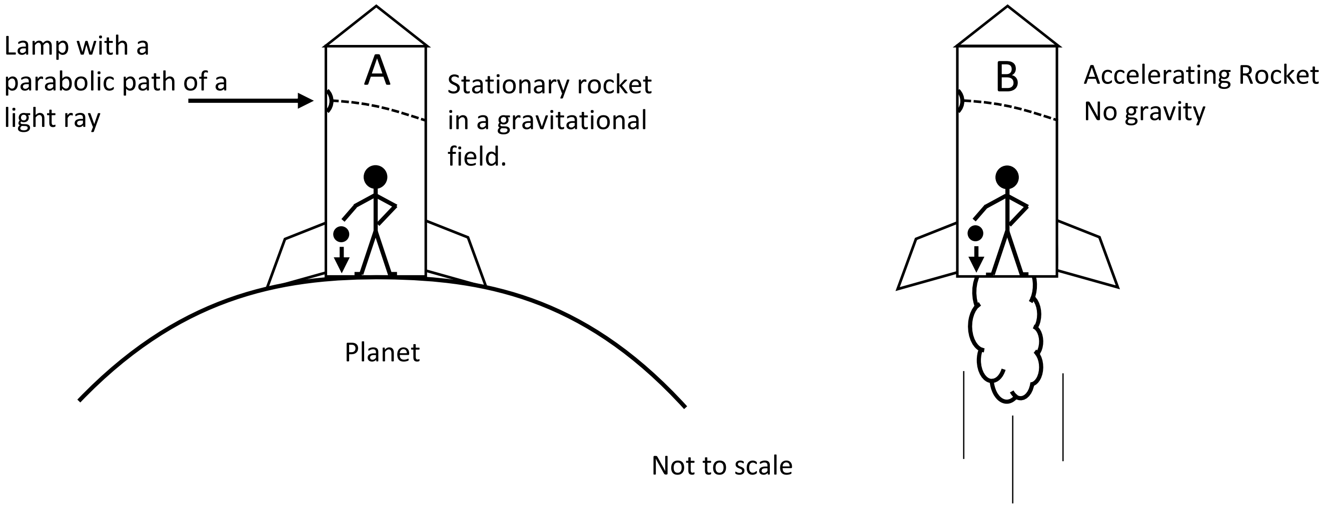
Non-inertial frames of reference are frames of reference where they are accelerating or in a gravitational field. General relativity is required to describe non-inertial frames of reference.



RMA Question Book page 23 questions 1 to 4

5.2 I can state the "equivalence principle" of general relativity

The equivalence principle states that the effects of gravity are exactly equivalent to the effects of acceleration.



Consider two rockets with no windows so the occupant cannot see outside. In each rocket the occupant will feel a force pressing their feet on the floor but they will not be able to tell which capsule they are in.

The equivalence principle means that any experiment in these rockets gives the same result. For example, when either person drops a ball it will fall to the floor of their rocket in the same way.

Both rockets have the same large magnitude of force acting on them. Rocket A has a gravitational force, Rocket B an accelerating force.

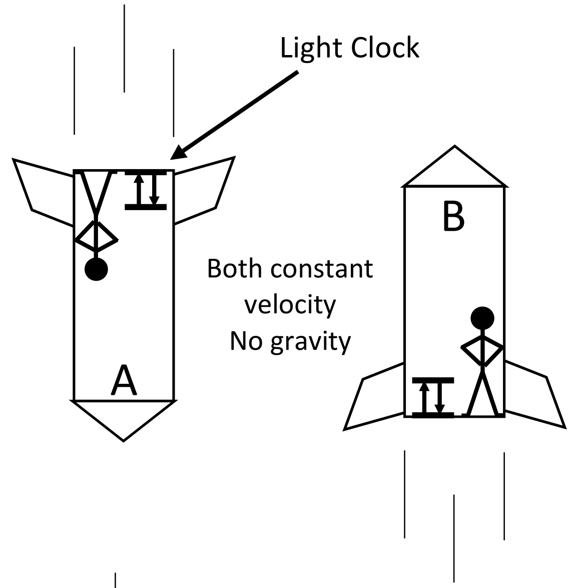
If a beam of light is shone horizontally from one side of the rocket will strike the other wall of the rocket slightly lower due to

- the gravitational force (A) causing a downward curvature.
- the upward acceleration of the rocket (B) during the travel time of the light ray.

An astronaut in deep space, far from any other gravitating matter, will feel weightless but so too would a person (who might also be an astronaut) in orbit around the Earth freely falling in a uniform gravitational field. In both these cases if an object (e.g. the astronaut's spanner) were released from rest it would remain near to the person's hands, in accordance with Newton's first law. The effects are the same in both cases since both situations are (locally) inertial frames.

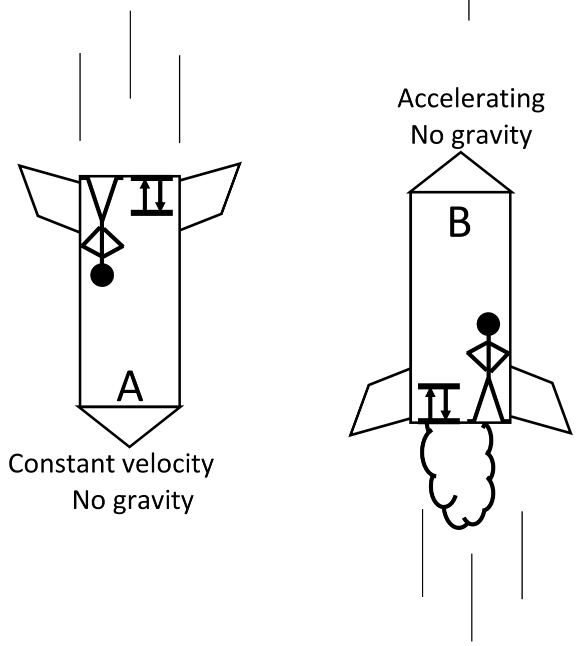
5.3 I know that acceleration and gravitational fields causes time to slow down.

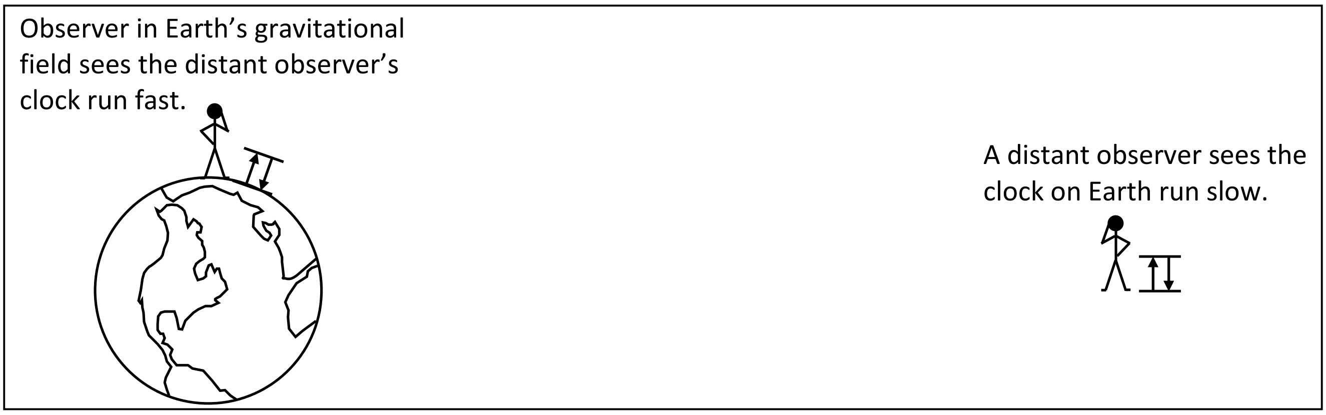
In the Higher Physics course you learned about time dilation. When moving at a constant relative velocity an observer in rocket A would observe time slowed down in rocket B. Symmetrically an observer in rocket B would observe time slowed down in rocket A.
This occurs when both frames are inertial.



When dealing with non-inertial frames the situation is no longer symmetrical. The observer in rocket A sees time slowed in rocket B. However, the observer in rocket B sees time sped up in rocket A.

Acceleration causes time to slow down. As the equivalence principle shows that the effects of acceleration and gravity are the same then **gravity also causes time to slow down.**

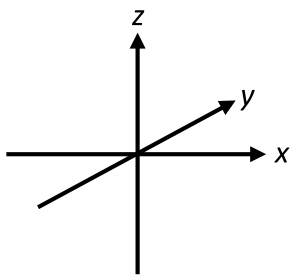




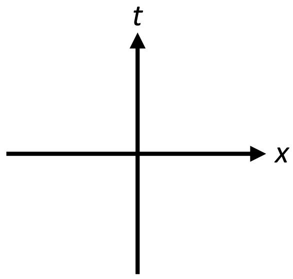
RMA Question Book page 23 questions 5 and 7

5.4 I can draw and interpret spacetime diagrams.

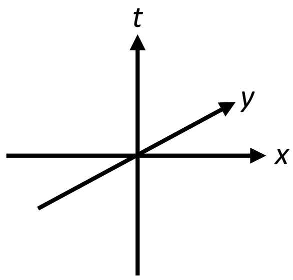
When representing space, we frequently use three axes x, y and z. These axes are perpendicular to each other. Time is then treated separately from space.



With special and general relativity time and space are represented together as a four dimensional spacetime. This representation consists of three space axes (*x, y* and *z)* and one axis of time (*t*).
It is not possible to draw all these axes on paper so we simplify by just drawing the *x*-axis and the time axis.



A second space axis can also be drawn if necessary but not all four axes.

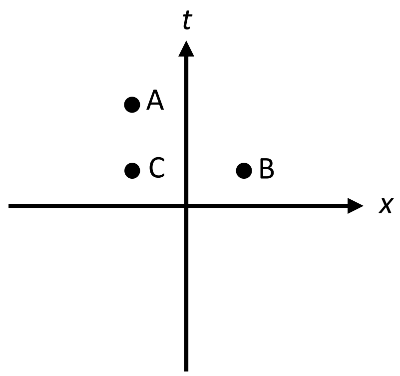


The origin on the *t*-axis represents the present time i.e. now. Positive values of time are in the future and negative values of time are in the past.
Positive and negative values along the *x*-axis represent the object or event's position in space.

Representing objects and Events on a spacetime diagram

Lines on spacetime diagram are called **world lines.**

If the object drawn on the spacetime diagram is moving freely then the line is referred to as a **geodesic.** These lines represent the shortest distance between two points is spacetime.



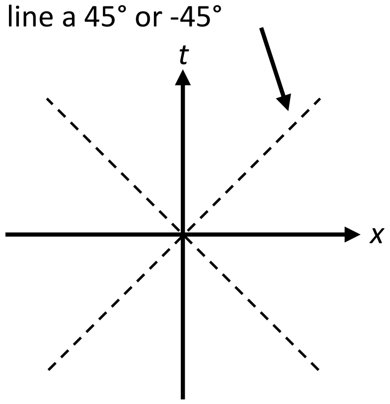
An Event

This is represented as a point on the space time diagram.

- Event A occurs at a different time and at a different place to Event B.
- Event C occurs at the same place as Event A but at a different time.
- Event C occurs at the same time as Event B but at a different place.

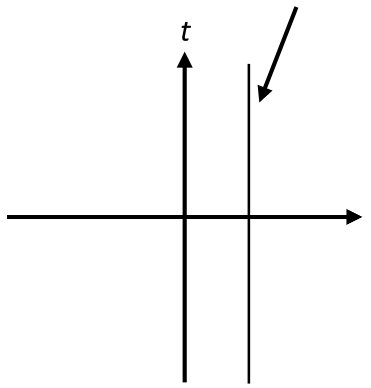
Light Rays

The scales on spacetime diagrams are scaled so that the world line of a light ray appears as a line a 45° or -45°



Stationary Object

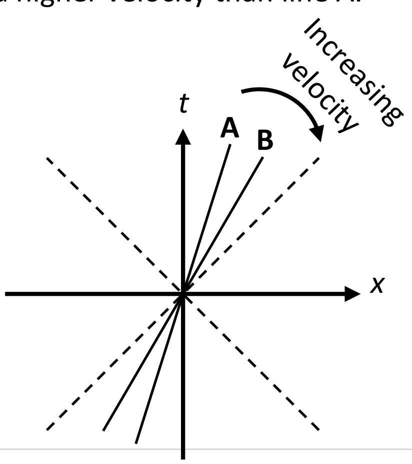
When an object is stationary in space it is still moving forward in time. This is represented on a spacetime diagram as a vertical line.



Objects moving at a constant velocity

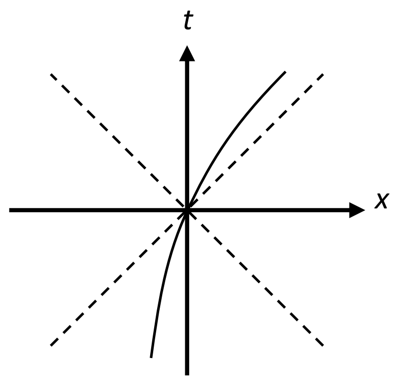
These are shown as straight world lines. All objects will have speed between zero (a vertical line) and the speed of light (a line at 45°).

Line B has a higher velocity than line A.

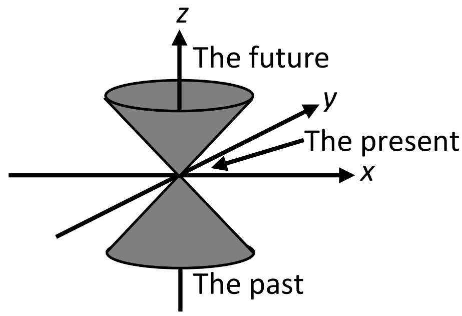


Accelerating objects

These are shown as curved world lines.

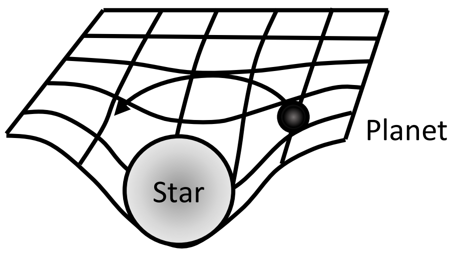


When extended to three dimensions (two of space and one of time) the spacetime diagram representing light rays becomes a light cone.



5.5 I know that general relativity leads to the interpretation that mass curves spacetime, and that gravity arises from the curvature of spacetime.

In general relativity, the concept of a gravitational force does not occur. Gravity is interpreted as a mass altering the shape of spacetime. Objects in within the gravitational field are influenced by the shape of the spacetime. The diagram shows a star curving the spacetime around it. The planet in orbit is following a geodesic path as it follows the curved spacetime around the star.



RMA Question Book pages 23 and 24 questions 8 to 17

5.6 I understand what is meant by "a black hole"

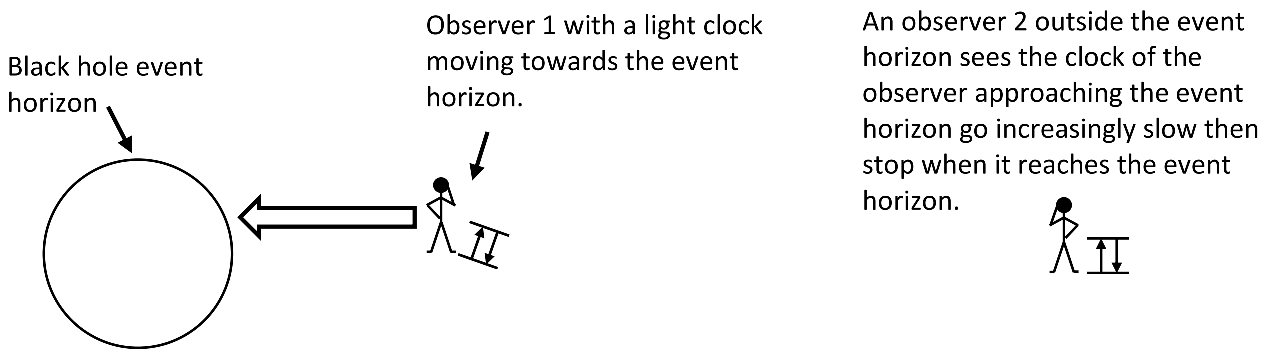
A black hole is a region of space where the gravitational field is so high that radiation and matter cannot escape. They are areas of space where the density of matter is extremely high. Black holes are formed by the collapse of massive stars, where the inward spacetime curvature produced by gravity overcomes the outward pressure of nuclear processes within the star. As the star collapses its diameter decreases which produces a higher gravitational field which again caused the star to reduce in diameter. This continues until all the matter of the star is compressed to a single point called a singularity.

5.7 I understand what is meant by the terms "event horizon" and "Schwarzschild radius".

Near a black hole spacetime is curved to such an extent that light cannot escape. Moving away from the black hole the curvature of spacetime will decrease to a point where light can escape. This radius is call the **event horizon** or the **Schwarzschild radius**. This radius is the boundary between areas of space where radiation and matter can escape from a black hole and where they cannot.

5.8 I know that time appears to be “frozen” at the event horizon of a black hole.

Observer 1 moving towards the event horizon of a black hole will experience an increasing gravitational field strength. Observer 2 outside the event horizon will see the clock of observer 1 run increasingly slow. At the event horizon, it will appear that the clock of observer 1 will have stopped i.e. frozen.
According to Observer 1 they will pass through the event horizon without any noticeable change.



5.9 I can use the relationship $r = \frac{2GM}{c^2}$ to solve problems relating to the Schwarzschild radius of a black hole.

The Schwarzschild radius can be calculated using the relationship

Universal constant of gravitation ($\mathrm{m^3kg^{-1}s^{-2}}$)

Mass of the black hole (kg)

$$r = \frac{2GM}{c^2}$$

Schwarzschild Radius (m)

Speed of light ($\mathrm{ms^{-1}}$)

Example

A star of mass ten times that of the Sun collapses to form a black hole. Calculate the Schwarzschild radius of the black hole.

Solution

$G = 6.67 \times 10^{-11}\text{m}^3\text{kg}^{-1}\text{s}^{-2}$

$M = 10 \times 2.0 \times 10^{30}\text{kg} = 2.0 \times 10^{31}\text{kg}$

$c = 3.0 \times 10^{8}\text{ms}^{-1}$

$$r = \frac{2GM}{c^2}$$

$$r = \frac{2 \times 6.67 \times 10^{-11} \times 2.0 \times 10^{31}}{3.0 \times 10^{8^2}}$$

$$r = 3.0 \times 10^{4}\text{m} = 30\text{km}$$

RMA Question Book pages 24 and 25 questions 18 to 22

Key Area: Stellar Physics

Previous Knowledge

Inverse square law.
Peak wavelength and temperature.
Convert between light years and metres.

Success Criteria

6.1 I can describe the properties of stars in terms of their radius, surface temperature, luminosity and apparent brightness.

6.2 I can solve problems involving surface temperature, power per unit area, luminosity, apparent brightness and stellar radius.

6.3 I know the stages of the proton-proton chain in stellar fusion reactions which convert hydrogen to helium.

6.4 I know how stars are formed from interstellar dust

6.5 I know and understand the stages in stellar evolution.

6.6 I understand the Hertzsprung-Russell (H-R) diagram and where main sequence stars, giant stars, super giant stars and white dwarves occur on the diagram.

6.7 I can predict the colour of a star from its position on the Hertzsprung-Russell diagram.

6.1 I can describe the properties of stars in terms of their radius, surface temperature, luminosity and apparent brightness

Radius

The Sun, the star in our solar system, has a radius of approximately 695,500km. This is about 109 times the size of the Earth.

When examining the size of stars, their radius is usually expressed as a multiple or a fraction of the size of the Sun, $R_{\odot}$.

There are a large range of sizes of star a few examples are given in the table 1.

Table 1

Star	Type	Radius$(\boldsymbol{R_{\odot}})$	Surface Temperature (K)	Luminosity $(\boldsymbol{L_{\odot}})$
Sirius B	White Dwarf	0.0084	25200	25.4
Procyon B	White Dwarf	0.01234	7740	0.00049
Sun	Main Sequence	1	5778	1
Beta Cassiopeiae	Main Sequence	3.5	7079	27.3
Arcturus	Giant	25.4	4286	170
Hamal	Giant	14.9	4480	91
Alpha Persei	Supergiant	68	6350	5400
Betelgeuse	Supergiant	887	3590	140,000

Surface Temperature

The surface temperature of a stars varies with their radius and the energy produced by nuclear fusion within the stars. Examples of some surface temperatures are given in table 1.

The surface temperature the star determines the power radiated. This is given by the Stefan-Boltzmann relationship.

Surface Temperature (K)

$$\text{Power per unit area} = \sigma T^4$$

Power per unit area (Wm^{-2})

Stefan-Boltzmann constant ($Wm^{-2}K^{-4}$)
See the data sheet

Luminosity

When the Stefan-Boltzmann relationship is multiplied by the surface area of a star it gives the total power emitted which is called the star's luminosity, *L*.
The luminosity depends on the surface area of the star as well as its surface temperature. Large stars with a low temperature can be equally as luminous as small star with a high surface temperature. Table 1 gives some typical values of luminosity.

The area of a sphere is given by $A = 4\pi r^2$. When multiplied by the Stefan-Boltzmann relationship gives the relationship for luminosity.

Radius of the star (m)

Surface Temperature (K)

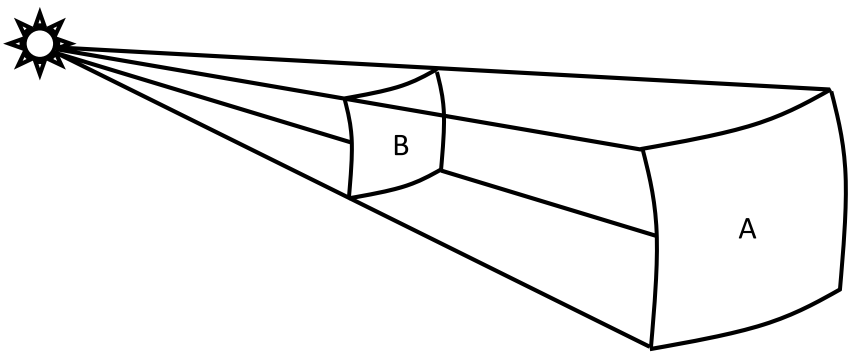
$$L = 4\pi \mathrm{r}^2 \sigma T^4$$

Luminosity (W)

Stefan-Boltzmann constant ($Wm^{-2}K^{-4}$)

Apparent Brightness

As light radiates away from a star it is spread out over a larger area. Area A in the diagram is bigger than area B. The power per unit area on area A is less than that on area B. So, at the distance of area A the star will appear dimmer than at the distance of area B.



The apparent brightness of a star can be calculated from

Luminosity (W)

$$b = \frac{L}{4\pi \mathrm{r}^2}$$

Apparent brightness (Wm^{-2})

Distance from the star (m)

> **Note**
> The *r* term in the apparent brightness relationship is a different quantity from the *r* term in the luminosity relationship. These *r* terms cannot be cancelled.

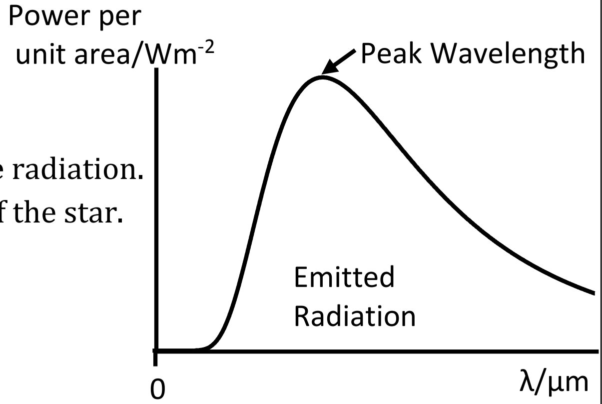
When the distance to a star is known, the apparent brightness allows the luminosity of a star to be calculated.

6.2 I can solve problems involving surface temperature, power per unit area, luminosity, apparent brightness and stellar radius.

Note Wien's Displacement Law may be required to solve some of the problems in the problem book. This is a relationship between the surface temperature of a star and the peak wavelength of the radiation emitted. Knowledge of Wien's Displacement Law is not part of the Advanced Higher Physics course.

$$\lambda_{peak} = \frac{2.9 \times 10^{-3}}{T}$$

$$\text{where} \begin{cases} \lambda_{peak} = \text{Peak wavelength of the radiation.} \\ T = \text{Surface temperature of the star.} \end{cases}$$



Example 1

The surface temperature of the star Hamal is 4480K, see Table 1 in section 6.1. Find

a. The power emitted by each square metre of the Hamal.

b. The luminosity of the Hamal.

c. The apparent brightness of Hamal (Earth Hamal distance is 65.8 light years)

Solution 1

a. $\text{Power per unit area} = \sigma T^4$

$\text{Power per unit area} = 5.67 \times 10^{-8} \times 4480^4$

$\text{Power per unit area} = 2.3 \times 10^7\text{W} \quad (2.28 \times 10^7\text{W})$

from the data sheet $\sigma = 5.67 \times 10^{-8}\text{Wm}^{-2}\text{K}^{-4}$

b. To find the luminosity first calculate the radius of Hamal.

From table 1, $r = 14.9R_{\odot}$ for the star Hamal.

From the Data Sheet $R_{\odot} = 6.9550 \times 10^8\text{m}$

$$r = 14.9 \times 6.9550 \times 10^8 = 1.04 \times 10^{10}\text{m}$$

$$L = 4\pi r^2 \sigma T^4$$

$$L = 4\pi \times (1.04 \times 10^{10})^2 \times 2.28 \times 10^7$$

From part a.

$$L = 3.1 \times 10^{28}\text{W} \quad (3.08 \times 10^{28}\text{W})$$

c. Convert 65.8 light years to metres

$$1 \text{ light year} = 60 \times 60 \times 24 \times 365 \times 3.0 \times 10^8 = 9.46 \times 10^{15}\text{m}$$

$$r = 65.8 \text{ light years} = 65.8 \times 9.46 \times 10^{15} = 6.23 \times 10^{17}\text{m}$$

$$b = \frac{L}{4\pi r^2}$$

$$b = \frac{3.08 \times 10^{28}}{4\pi(6.23 \times 10^{17})^2}$$

$$b = 6.3 \times 10^{-9}\text{Wm}^{-2}$$

<u>Example 2</u>

The star Arcturus has a luminosity of $6.54 \times 10^{28}\text{W}$ and an apparent brightness of $4.3 \times 10^{-8}\text{Wm}^{-2}$. Calculate its distance from the Earth in light years.

<u>Solution 2</u>

The relationship for apparent brightness

$$b = \frac{L}{4\pi r^2}$$

rearranges to

$$r = \sqrt{\frac{L}{4\pi b}}$$

$$r = \sqrt{\frac{6.54 \times 10^{28}}{4\pi \times 4.3 \times 10^{-8}}}$$

$$r = 3.49 \times 10^{17}\text{m}$$

$$1 \text{ light year} = 60 \times 60 \times 24 \times 365 \times 3.0 \times 10^8 = 9.46 \times 10^{15}\text{m}$$

$$r = \frac{3.49 \times 10^{17}}{9.46 \times 10^{15}} = 36.8 \text{ light years}$$

RMA Question Book pages 26 and 27 questions 1 to 13

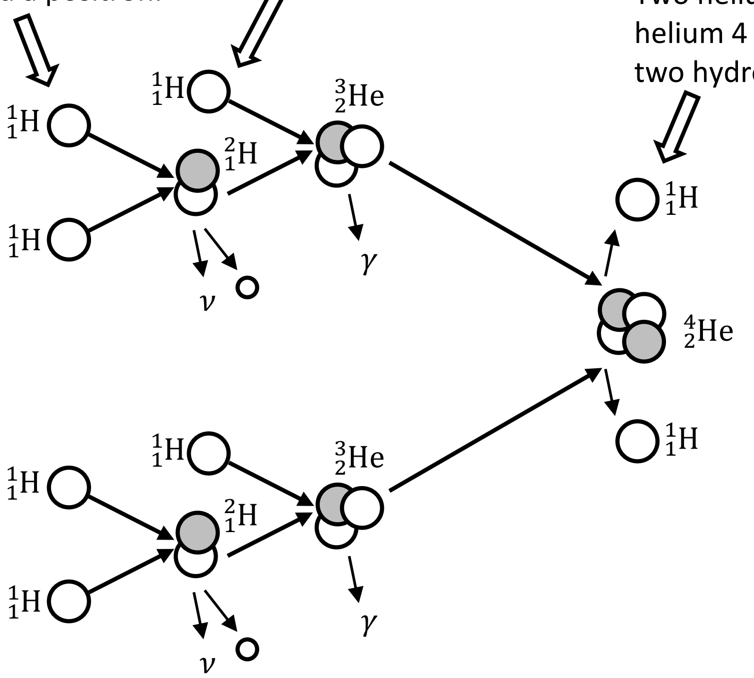
6.3 I know the stages of the proton-proton chain in stellar fusion reactions which convert hydrogen to helium.

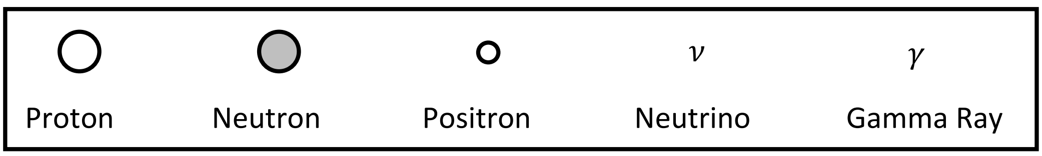
In main sequence stars (see section 6.4) hydrogen nuclei are fused to form helium nuclei releasing energy which powers the star. High temperatures and pressures are required to give the positive nuclei sufficient energy to overcome the electrostatic repulsion between them. This means that fusion reactions only occur in the core of a star. The main fusion reaction pathway is called the proton-proton chain.
Overall the fusion reaction converts six hydrogen nuclei to one helium nucleus, two hydrogen nuclei, two positrons, two neutrinos and energy. The energy of the reaction is released as two gamma rays and as the kinetic energy of the particles. The proton-proton chain occurs in three stages.

Stage 2
The deuterium nucleus fuses with a hydrogen nucleus to form a helium 3 nucleus. This also releases a gamma ray.

Stage 1
Two hydrogen nuclei fuse to form a deuterium nucleus. This also releases a neutrino and a positron.

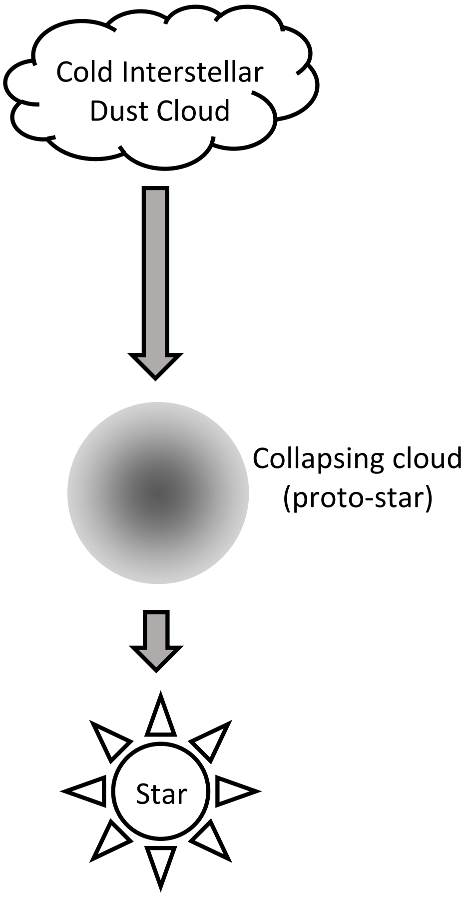
Stage 3
Two helium 3 nuclei fuse to form a helium 4 nucleus. This also releases two hydrogen nuclei.





6.4 I know how stars are formed from interstellar dust

Formation of Stars



Stars are formed from cold relatively dense interstellar dust clouds. These clouds can be stable with the motion of the particles in the clouds producing an outward pressure which balances the inward gravitational force.

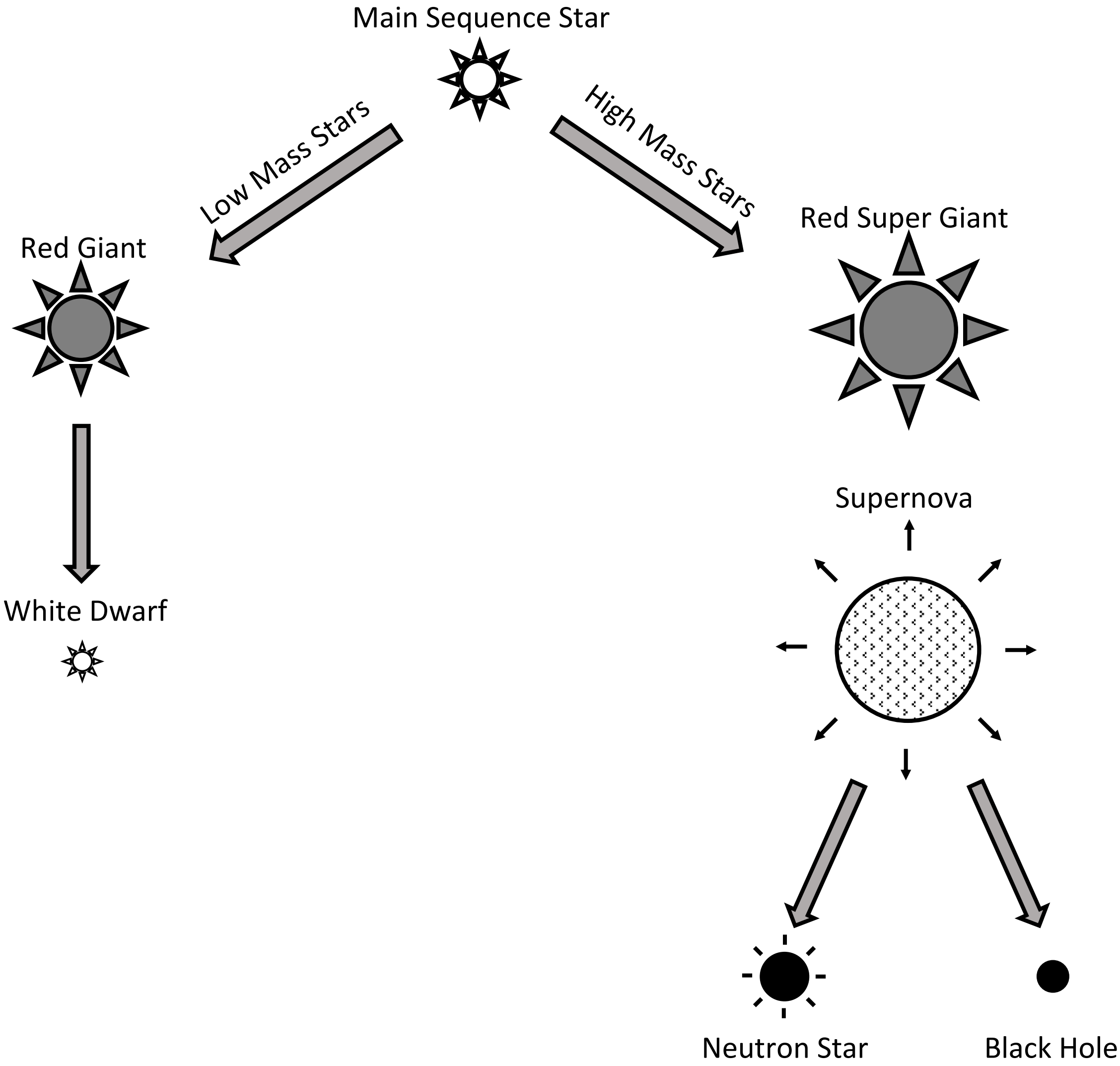
A trigger (e.g. a nearby supernova explosion) which causes and increase in density in some parts of the clouds without an increase in temperature can cause gravitational attraction to overcome the outward pressure. This leads to the inward collapse of the cloud.

As the cloud contracts the gravitational potential energy of the particles in the cloud are converted kinetic energy. This causes and increase in temperature.

When the temperature and pressure are sufficiently high, fusion reactions start within the core. This produces energy which further heats the star. When the outward thermal and radiation pressure produced by the hot gas balances the inward force of gravity the star reaches equilibrium.
The star at this stage will be a main sequence star which is converting hydrogen to helium. This can continue for millions to billions of years. How long the star remains as a main sequence star and its future evolution is determined by its initial mass.
High mass stars have lifetimes as short as a few tens of millions of years. Low mass stars can have life time of thousands of billions of years.

6.5 I know and understand the stages in stellar evolution.

The diagram below show the evolution of main sequence stars. The follow one of two paths which depends on their mass.



Main Sequence – Most stars are in this group as most of the lifetime of a star is spent as a main sequence star converting hydrogen to helium.

Giants and Super Giants – After the hydrogen is converted to helium in a main sequence star the outward thermal pressure decreases. This allows gravity to compress the core increasing its temperature. A new equilibrium is reached when this increased temperature allows the further fusion reactions converting helium to carbon. As helium to carbon fusion occurs at a higher temperature this increases the thermal pressure making the star expand. Low mass stars expand to form giant. High mass stars go through further stages of fusion at higher temperatures and expand to form super giants.

White Dwarves – These occur at the end of the life of low mass stars after they have run out of helium to convert to carbon. The expanded outer layers of a giant star is dispersed into the space surrounding the star. This leaves the hot core of the star where no further nuclear fusion occurs. This is a white dwarf which continues to cool.

Neutron Stars and Black Holes

These are formed from super giant stars. These stars continue to fuse helium to carbon then go through a series of fusion processes until iron is reached. After iron, fusion does not produce any energy. Once fusion in the star stops its core will suddenly collapse releasing large amounts of energy blowing away the outer layers of the star. This is called a supernova and can for a short period make the star brighter than the whole galaxy.
What remains after a supernova depends of the mass of the star. It can leave a highly compressed core consisting of neutrons, a neutron star. If the mass of the star is large enough the gravitational contraction of the core can continue until a single point in space called a singularity is reached. The resulting object is a black hole.

6.6 I understand the Hertzsprung-Russell (H-R) diagram and where main sequence stars, giant stars, super giant stars and white dwarves occur on the diagram.

When the luminosity of stars are plotted against their temperature a Hertzsprung-Russell (H-R) is obtained.

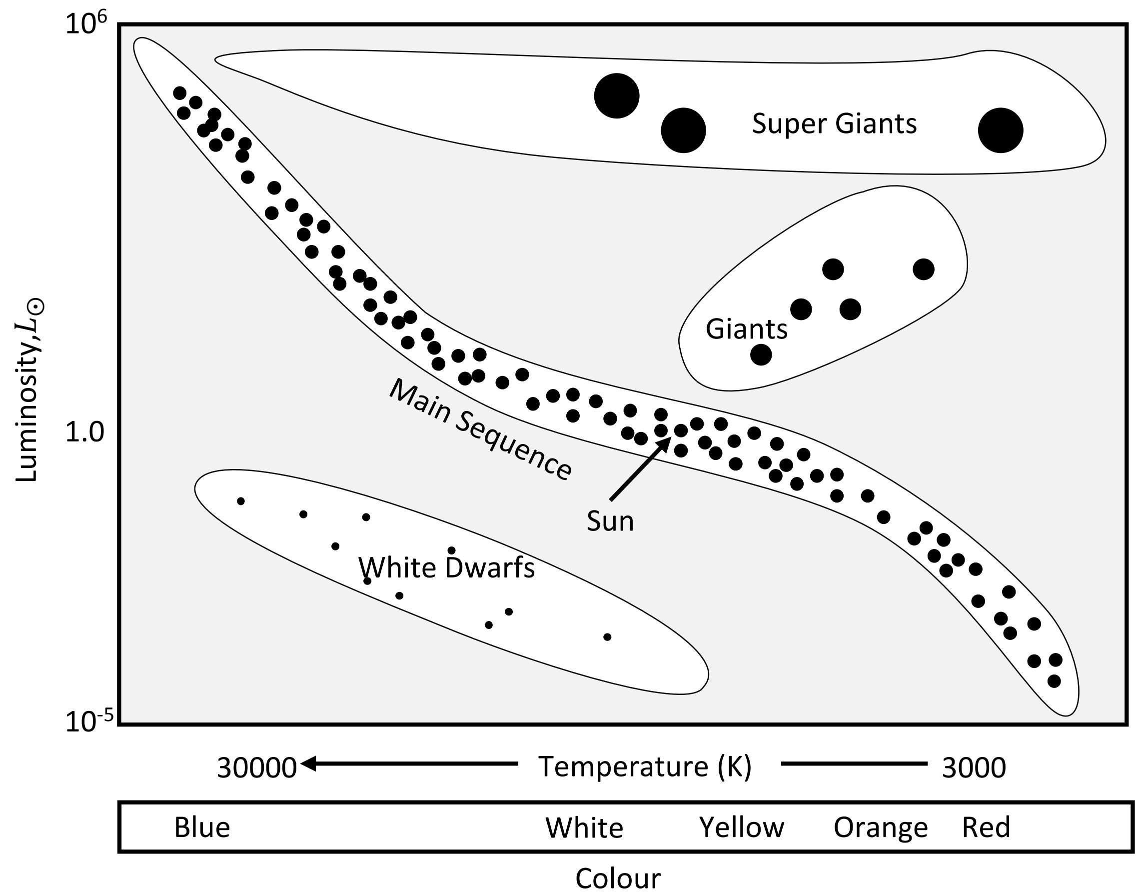
Main sequence stars occur in a broad diagonal band. High mass stars occur in the top left of the diagram and low mass stars towards the bottom right.

Giants and super giants occur in the top right of the diagram as they are high luminosity, low temperature stars. Main sequence stars will change their temperature an luminosity as they run out of hydrogen and move across the diagram into the giant or super giant region depending on their mass.

White dwarves occur in the bottom left of the diagram as they are low luminosity stars. They are formed from giant stars which have lost their outer layers when fusion reactions end in their cores.

Neutron stars and black holes cannot be plotted on the H-R diagram as their temperatures are beyond the scale of the diagram and their luminosity is very low or zero.

Throughout their lives stars will move across the H-R diagram from the main sequence to giant or supergiant then to white dwarves.



6.7 I can predict the colour of a star from its position on the Hertzsprung-Russell diagram.

The position of a star on the H-R diagram is determined by its temperature and luminosity. As the temperature increases from right to left the colour changes from red to blue. By examining the position of a star on the diagram its colour can be found by reading the colour scale at the bottom of the diagram.

RMA Question Book pages 28 and 29 question 14

Quantities, Units and Multiplication Factors

Quantity	Quantity Symbol	Unit	Unit Abbreviation
acceleration	a	Metre per second squared	ms^{-2}
Angle	θ	Degree/Radians	° /Rad
Angular acceleration	α	Radians per second squared	rad s^{-2}
angular displacement	θ	Radians	rad
Angular momentum	L	Kilogram metres squared per second	$\mathrm{kgm^2\ s^{-1}}$
angular velocity	ω, ω_0	Radians per second	rad s^{-1}
Apparent Brightness	b	Watt per metre squared	$\mathrm{Wm^{-2}}$
Displacement	s	Metre	m
Force	F	Newton	N
Gravitational Field Strength	g	Newtons per kilogram	$\mathrm{Nkg^{-1}}$
Gravitational Potential	V_p	Joules per kilogram	$\mathrm{Jkg^{-1}}$
Gravitational Potential Energy	E_p	Joules	J
Height	h	Metre	m
Kinetic Energy	E_k	Joules	J
Luminosity	L	Watt	W
Luminosity	L	Watt	W
Mass	m	Kilogram	Kg
Moment of Inertia	I	Kilogram metre squared	$\mathrm{kgm^2}$
Period	T	Second	s
Power per unit Area	I	Watt per metre squared	Wm^{-2}
Power per unit area	I	Watt per metre squared	$\mathrm{Wm^{-2}}$
Radial acceleration	a_r	Metre per second squared	ms^{-2}
Radius	r	Metre	m
Schwarzschild radius	r	Metre	m
Tangential acceleration	a_t	Radians per second squared	rad s^{-2}
Temperature	T	Kelvin	K
Time	t	Second	s
Torque	T	Newton metre	Nm
velocity	v	Metre per second	ms^{-1}

Prefix Name	Prefix Symbol	Multiplication Factor
Pico	p	$\times 10^{-12}$
Nano	n	$\times 10^{-9}$
Micro	µ	$\times 10^{-6}$
Milli	m	$\times 10^{-3}$
Kilo	k	$\times 10^{3}$
Mega	M	$\times 10^{6}$
Giga	G	$\times 10^{9}$
Tera	T	$\times 10^{12}$

Relationships required for Physics Advanced Higher

$$v = \frac{ds}{dt}$$

$$a = \frac{dv}{dt} = \frac{d^2s}{dt^2}$$

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$v^2 = u^2 + 2as$$

$$\omega = \frac{d\theta}{dt}$$

$$\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$$

$$\omega = \omega_o + \alpha t$$

$$\theta = \omega_o t + \frac{1}{2}\alpha t^2$$

$$\omega^2 = \omega_o{}^2 + 2\alpha\theta$$

$$s = r\theta$$

$$v = r\omega$$

$$a_t = r\alpha$$

$$a_r = \frac{v^2}{r} = r\omega^2$$

$$F = \frac{mv^2}{r} = mr\omega^2$$

$$T = Fr$$

$$T = I\alpha$$

$$L = mvr = mr^2\omega$$

$$L = I\omega$$

$$E_K = \frac{1}{2}I\omega^2$$

$$F = G\frac{Mm}{r^2}$$

$$V = -\frac{GM}{r}$$

$$v = \sqrt{\frac{2GM}{r}}$$

$$\text{apparent brightness, } b = \frac{L}{4\pi r^2}$$

$$\text{Power per unit area} = \sigma T^4$$

$$L = 4\pi r^2 \sigma T^4$$

$$r_{Schwarzschild} = \frac{2GM}{c^2}$$

$$E = hf$$

$$\lambda = \frac{h}{p}$$

$$mvr = \frac{nh}{2\pi}$$

$$\Delta x \, \Delta p_x \geq \frac{h}{4\pi}$$

$$\Delta E \, \Delta t \geq \frac{h}{4\pi}$$

$$F = qvB$$

$$\omega = 2\pi f$$

$$a = \frac{d^2y}{dt^2} = -\omega^2 y$$

$$y = A\cos\omega t \quad \text{or} \quad y = A\sin\omega t$$

$$v = \pm\omega\sqrt{(A^2 - y^2)}$$

$$E_K = \frac{1}{2}m\omega^2(A^2 - y^2)$$

$$E_P = \frac{1}{2}m\omega^2 y^2$$

$$y = A\sin 2\pi(ft - \frac{x}{\lambda})$$

$$E = kA^2$$

$$\phi = \frac{2\pi x}{\lambda}$$

$$\text{optical path difference} = m\lambda \quad \text{or} \quad \left(m + \frac{1}{2}\right)\lambda$$

where $m = 0, 1, 2....$

$$\Delta x = \frac{\lambda l}{2d}$$

$$d = \frac{\lambda}{4n}$$

$$\Delta x = \frac{\lambda D}{d}$$

$$n = \tan i_P$$

$$F = \frac{Q_1 Q_2}{4\pi\varepsilon_o r^2}$$

$$E = \frac{Q}{4\pi\varepsilon_o r^2}$$

$$V = \frac{Q}{4\pi\varepsilon_o r}$$

$$F = QE$$

$$V = Ed$$

$$F = IlB\sin\theta$$

$$B = \frac{\mu_o I}{2\pi r}$$

$$c = \frac{1}{\sqrt{\varepsilon_o \mu_o}}$$

$$t = RC$$

$$X_C = \frac{V}{I}$$

$$X_C = \frac{1}{2\pi fC}$$

$$\mathcal{E} = -L\frac{dI}{dt}$$

$$E = \frac{1}{2}LI^2$$

$$X_L = \frac{V}{I}$$

$$X_L = 2\pi fL$$

$$\frac{\Delta W}{W} = \sqrt{\left(\frac{\Delta X}{X}\right)^2 + \left(\frac{\Delta Y}{Y}\right)^2 + \left(\frac{\Delta Z}{Z}\right)^2}$$

$$\Delta W = \sqrt{\Delta X^2 + \Delta Y^2 + \Delta Z^2}$$

$d = \bar{v}t$

$s = \bar{v}t$

$v = u + at$

$s = ut + \frac{1}{2}at^2$

$v^2 = u^2 + 2as$

$s = \frac{1}{2}(u+v)t$

$W = mg$

$F = ma$

$E_W = Fd$

$E_P = mgh$

$E_K = \frac{1}{2}mv^2$

$P = \frac{E}{t}$

$p = mv$

$Ft = mv - mu$

$F = G\frac{Mm}{r^2}$

$t' = \frac{t}{\sqrt{1-\left(\frac{v}{c}\right)^2}}$

$l' = l\sqrt{1-\left(\frac{v}{c}\right)^2}$

$f_o = f_s\left(\frac{v}{v \pm v_s}\right)$

$z = \frac{\lambda_{observed} - \lambda_{rest}}{\lambda_{rest}}$

$z = \frac{v}{c}$

$v = H_0 d$

$E_W = QV$

$E = mc^2$

$E = hf$

$E_K = hf - hf_0$

$E_2 - E_1 = hf$

$T = \frac{1}{f}$

$v = f\lambda$

$d\sin\theta = m\lambda$

$n = \frac{\sin\theta_1}{\sin\theta_2}$

$\frac{\sin\theta_1}{\sin\theta_2} = \frac{\lambda_1}{\lambda_2} = \frac{v_1}{v_2}$

$\sin\theta_c = \frac{1}{n}$

$I = \frac{k}{d^2}$

$I = \frac{P}{A}$

path difference $= m\lambda$ or $\left(m + \frac{1}{2}\right)\lambda$ where $m = 0,\ 1,\ 2...$

random uncertainty $= \frac{\text{max. value - min. value}}{\text{number of values}}$

$V_{peak} = \sqrt{2}V_{rms}$

$I_{peak} = \sqrt{2}I_{rms}$

$Q = It$

$V = IR$

$P = IV = I^2R = \frac{V^2}{R}$

$R_T = R_1 + R_2 + ...$

$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + ...$

$E = V + Ir$

$V_1 = \left(\frac{R_1}{R_1 + R_2}\right)V_S$

$\frac{V_1}{V_2} = \frac{R_1}{R_2}$

$C = \frac{Q}{V}$

$E = \frac{1}{2}QV = \frac{1}{2}CV^2 = \frac{1}{2}\frac{Q^2}{C}$

Additional Relationships

Circle

$circumference = 2\pi r$

$area = \pi r^2$

Sphere

$area = 4\pi r^2$

$volume = \frac{4}{3}\pi r^3$

Trigonometry

$$\sin\theta = \frac{opposite}{hypotenuse}$$

$$\cos\theta = \frac{adjacent}{hypotenuse}$$

$$\tan\theta = \frac{opposite}{adjacent}$$

$$\sin^2\theta + \cos^2\theta = 1$$

Moment of inertia

point mass
$I = mr^2$

rod about centre
$I = \frac{1}{12}ml^2$

rod about end
$I = \frac{1}{3}ml^2$

disc about centre
$I = \frac{1}{2}mr^2$

sphere about centre
$I = \frac{2}{5}mr^2$

Table of standard derivatives

$f(x)$	$f'(x)$
$\sin ax$	$a\cos ax$
$\cos ax$	$-a\sin ax$

Table of standard integrals

$f(x)$	$\int f(x)dx$
$\sin ax$	$-\frac{1}{a}\cos ax + C$
$\cos ax$	$\frac{1}{a}\sin ax + C$

Electron Arrangements of Elements

Key

Atomic number
Symbol
Electron arrangement
Name

Transition Elements

Group 1 (1)	Group 2 (2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	Group 3 (13)	Group 4 (14)	Group 5 (15)	Group 6 (16)	Group 7 (17)	Group 0 (18)
1 H 1 Hydrogen																	2 He 2 Helium
3 Li 2, 1 Lithium	4 Be 2, 2 Beryllium											5 B 2, 3 Boron	6 C 2, 4 Carbon	7 N 2, 5 Nitrogen	8 O 2, 6 Oxygen	9 F 2, 7 Fluorine	10 Ne 2, 8 Neon
11 Na 2, 8, 1 Sodium	12 Mg 2, 8, 2 Magnesium											13 Al 2, 8, 3 Aluminium	14 Si 2, 8, 4 Silicon	15 P 2, 8, 5 Phosphorus	16 S 2, 8, 6 Sulphur	17 Cl 2, 8, 7 Chlorine	18 Ar 2, 8, 8 Argon
19 K 2, 8, 8, 1 Potassium	20 Ca 2, 8, 8, 2 Calcium	21 Sc 2, 8, 9, 2 Scandium	22 Ti 2, 8, 10, 2 Titanium	23 V 2, 8, 11, 2 Vanadium	24 Cr 2, 8, 13, 1 Chromium	25 Mn 2, 8, 13, 2 Manganese	26 Fe 2, 8, 14, 2 Iron	27 Co 2, 8, 15, 2 Cobalt	28 Ni 2, 8, 16, 2 Nickel	29 Cu 2, 8, 18, 1 Copper	30 Zn 2, 8, 18, 2 Zinc	31 Ga 2, 8, 18, 3 Gallium	32 Ge 2, 8, 18, 4 Germanium	33 As 2, 8, 18, 5 Arsenic	34 Se 2, 8, 18, 6 Selenium	35 Br 2, 8, 18, 7 Bromine	36 Kr 2, 8, 18, 8 Krypton
37 Rb 2, 8, 18, 8, 1 Rubidium	38 Sr 2, 8, 18, 8, 2 Strontium	39 Y 2, 8, 18, 9, 2 Yttrium	40 Zr 2, 8, 18, 10, 2 Zirconium	41 Nb 2, 8, 18, 12, 1 Niobium	42 Mo 2, 8, 18, 13, 1 Molybdenum	43 Tc 2, 8, 18, 13, 2 Technetium	44 Ru 2, 8, 18, 15, 1 Ruthenium	45 Rh 2, 8, 18, 16, 1 Rhodium	46 Pd 2, 8, 18, 18, 0 Palladium	47 Ag 2, 8, 18, 18, 1 Silver	48 Cd 2, 8, 18, 18, 2 Cadmium	49 In 2, 8, 18, 18, 3 Indium	50 Sn 2, 8, 18, 18, 4 Tin	51 Sb 2, 8, 18, 18, 5 Antimony	52 Te 2, 8, 18, 18, 6 Tellurium	53 I 2, 8, 18, 18, 7 Iodine	54 Xe 2, 8, 18, 18, 8 Xenon
55 Cs 2, 8, 18, 18, 8, 1 Caesium	56 Ba 2, 8, 18, 18, 8, 2 Barium	57 La 2, 8, 18, 18, 9, 2 Lanthanum	72 Hf 2, 8, 18, 32, 10, 2 Hafnium	73 Ta 2, 8, 18, 32, 11, 2 Tantalum	74 W 2, 8, 18, 32, 12, 2 Tungsten	75 Re 2, 8, 18, 32, 13, 2 Rhenium	76 Os 2, 8, 18, 32, 14, 2 Osmium	77 Ir 2, 8, 18, 32, 15, 2 Iridium	78 Pt 2, 8, 18, 32, 17, 1 Platinum	79 Au 2, 8, 18, 32, 18, 1 Gold	80 Hg 2, 8, 18, 32, 18, 2 Mercury	81 Tl 2, 8, 18, 32, 18, 3 Thallium	82 Pb 2, 8, 18, 32, 18, 4 Lead	83 Bi 2, 8, 18, 32, 18, 5 Bismuth	84 Po 2, 8, 18, 32, 18, 6 Polonium	85 At 2, 8, 18, 32, 18, 7 Astatine	86 Rn 2, 8, 18, 32, 18, 8 Radon
87 Fr 2, 8, 18, 32, 18, 8, 1 Francium	88 Ra 2, 8, 18, 32, 18, 8, 2 Radium	89 Ac 2, 8, 18, 32, 18, 9, 2 Actinium	104 Rf 2, 8, 18, 32, 32, 10, 2 Rutherfordium	105 Db 2, 8, 18, 32, 32, 11, 2 Dubnium	106 Sg 2, 8, 18, 32, 32, 12, 2 Seaborgium	107 Bh 2, 8, 18, 32, 32, 13, 2 Bohrium	108 Hs 2, 8, 18, 32, 32, 14, 2 Hassium	109 Mt 2, 8, 18, 32, 32, 15, 2 Meitnerium									

Lanthanides	57 La 2, 8, 18, 18, 9, 2 Lanthanum	58 Ce 2, 8, 18, 20, 8, 2 Cerium	59 Pr 2, 8, 18, 21, 8, 2 Praseodymium	60 Nd 2, 8, 18, 22, 8, 2 Neodymium	61 Pm 2, 8, 18, 23, 8, 2 Promethium	62 Sm 2, 8, 18, 24, 8, 2 Samarium	63 Eu 2, 8, 18, 25, 8, 2 Europium	64 Gd 2, 8, 18, 25, 9, 2 Gadolinium	65 Tb 2, 8, 18, 27, 8, 2 Terbium	66 Dy 2, 8, 18, 28, 8, 2 Dysprosium	67 Ho 2, 8, 18, 29, 8, 2 Holmium	68 Er 2, 8, 18, 30, 8, 2 Erbium	69 Tm 2, 8, 18, 31, 8, 2 Thulium	70 Yb 2, 8, 18, 32, 8, 2 Ytterbium	71 Lu 2, 8, 18, 32, 9, 2 Lutetium
Actinides	89 Ac 2, 8, 18, 32, 18, 9, 2 Actinium	90 Th 2, 8, 18, 32, 18, 10, 2 Thorium	91 Pa 2, 8, 18, 32, 20, 9, 2 Protactinium	92 U 2, 8, 18, 32, 21, 9, 2 Uranium	93 Np 2, 8, 18, 32, 22, 9, 2 Neptunium	94 Pu 2, 8, 18, 32, 24, 8, 2 Plutonium	95 Am 2, 8, 18, 32, 25, 8, 2 Americium	96 Cm 2, 8, 18, 32, 25, 9, 2 Curium	97 Bk 2, 8, 18, 32, 27, 8, 2 Berkelium	98 Cf 2, 8, 18, 32, 28, 8, 2 Californium	99 Es 2, 8, 18, 32, 29, 8, 2 Einsteinium	100 Fm 2, 8, 18, 32, 30, 8, 2 Fermium	101 Md 2, 8, 18, 32, 31, 8, 2 Mendelevium	102 No 2, 8, 18, 32, 32, 8, 2 Nobelium	103 Lr 2, 8, 18, 32, 32, 9, 2 Lawrencium

DATA SHEET

COMMON PHYSICAL QUANTITIES

Quantity	Symbol	Value	Quantity	Symbol	Value
Gravitational acceleration on Earth	g	$9{\cdot}8\ \mathrm{m\,s^{-2}}$	Mass of electron	m_e	$9{\cdot}11 \times 10^{-31}$ kg
Radius of Earth	R_E	$6{\cdot}4 \times 10^{6}$ m	Charge on electron	e	$-1{\cdot}60 \times 10^{-19}$ C
Mass of Earth	M_E	$6{\cdot}0 \times 10^{24}$ kg	Mass of neutron	m_n	$1{\cdot}675 \times 10^{-27}$ kg
Mass of Moon	M_M	$7{\cdot}3 \times 10^{22}$ kg	Mass of proton	m_p	$1{\cdot}673 \times 10^{-27}$ kg
Radius of Moon	R_M	$1{\cdot}7 \times 10^{6}$ m	Mass of alpha particle	m_α	$6{\cdot}645 \times 10^{-27}$ kg
Mean Radius of Moon Orbit		$3{\cdot}84 \times 10^{8}$ m	Charge on alpha particle		$3{\cdot}20 \times 10^{-19}$ C
Solar radius		$6{\cdot}955 \times 10^{8}$ m	Planck's constant	h	$6{\cdot}63 \times 10^{-34}$ J s
Mass of Sun		$2{\cdot}0 \times 10^{30}$ kg	Permittivity of free space	ε_0	$8{\cdot}85 \times 10^{-12}\ \mathrm{F\,m^{-1}}$
1 AU		$1{\cdot}5 \times 10^{11}$ m	Permeability of free space	μ_0	$4\pi \times 10^{-7}\ \mathrm{H\,m^{-1}}$
Stefan-Boltzmann constant	σ	$5{\cdot}67 \times 10^{-8}\ \mathrm{W\,m^{-2}\,K^{-4}}$	Speed of light in vacuum	c	$3{\cdot}0 \times 10^{8}\ \mathrm{m\,s^{-1}}$
Universal constant of gravitation	G	$6{\cdot}67 \times 10^{-11}\ \mathrm{m^3\,kg^{-1}\,s^{-2}}$	Speed of sound in air	v	$3{\cdot}4 \times 10^{2}\ \mathrm{m\,s^{-1}}$

REFRACTIVE INDICES

The refractive indices refer to sodium light of wavelength 589 nm and to substances at a temperature of 273 K.

Substance	Refractive index	Substance	Refractive index
Diamond	2·42	Glycerol	1·47
Glass	1·51	Water	1·33
Ice	1·31	Air	1·00
Perspex	1·49	Magnesium Fluoride	1·38

SPECTRAL LINES

Element	Wavelength/nm	Colour
Hydrogen	656	Red
	486	Blue-green
	434	Blue-violet
	410	Violet
	397	Ultraviolet
	389	Ultraviolet
Sodium	589	Yellow

Element	Wavelength/nm	Colour
Cadmium	644	Red
	509	Green
	480	Blue

Lasers

Element	Wavelength/nm	Colour
Carbon dioxide	9550 10590	Infrared
Helium-neon	633	Red

PROPERTIES OF SELECTED MATERIALS

Substance	Density/ $\mathrm{kg\,m^{-3}}$	Melting Point/ K	Boiling Point/K	Specific Heat Capacity/ $\mathrm{J\,kg^{-1}\,K^{-1}}$	Specific Latent Heat of Fusion/ $\mathrm{J\,kg^{-1}}$	Specific Latent Heat of Vaporisation/ $\mathrm{J\,kg^{-1}}$
Aluminium	$2{\cdot}70 \times 10^{3}$	933	2623	$9{\cdot}02 \times 10^{2}$	$3{\cdot}95 \times 10^{5}$	....
Copper	$8{\cdot}96 \times 10^{3}$	1357	2853	$3{\cdot}86 \times 10^{2}$	$2{\cdot}05 \times 10^{5}$	....
Glass	$2{\cdot}60 \times 10^{3}$	1400	....	$6{\cdot}70 \times 10^{2}$	....	....
Ice	$9{\cdot}20 \times 10^{2}$	273	....	$2{\cdot}10 \times 10^{3}$	$3{\cdot}34 \times 10^{5}$	....
Glycerol	$1{\cdot}26 \times 10^{3}$	291	563	$2{\cdot}43 \times 10^{3}$	$1{\cdot}81 \times 10^{5}$	$8{\cdot}30 \times 10^{5}$
Methanol	$7{\cdot}91 \times 10^{2}$	175	338	$2{\cdot}52 \times 10^{3}$	$9{\cdot}9 \times 10^{4}$	$1{\cdot}12 \times 10^{6}$
Sea Water	$1{\cdot}02 \times 10^{3}$	264	377	$3{\cdot}93 \times 10^{3}$	....	....
Water	$1{\cdot}00 \times 10^{3}$	273	373	$4{\cdot}19 \times 10^{3}$	$3{\cdot}34 \times 10^{5}$	$2{\cdot}26 \times 10^{6}$
Air	1·29	....	....	....	....	....
Hydrogen	$9{\cdot}0 \times 10^{-2}$	14	20	$1{\cdot}43 \times 10^{4}$	....	$4{\cdot}50 \times 10^{5}$
Nitrogen	1·25	63	77	$1{\cdot}04 \times 10^{3}$	....	$2{\cdot}00 \times 10^{5}$
Oxygen	1·43	55	90	$9{\cdot}18 \times 10^{2}$	....	$2{\cdot}40 \times 10^{4}$

The gas densities refer to a temperature of 273 K and a pressure of $1{\cdot}01 \times 10^{5}$ Pa.