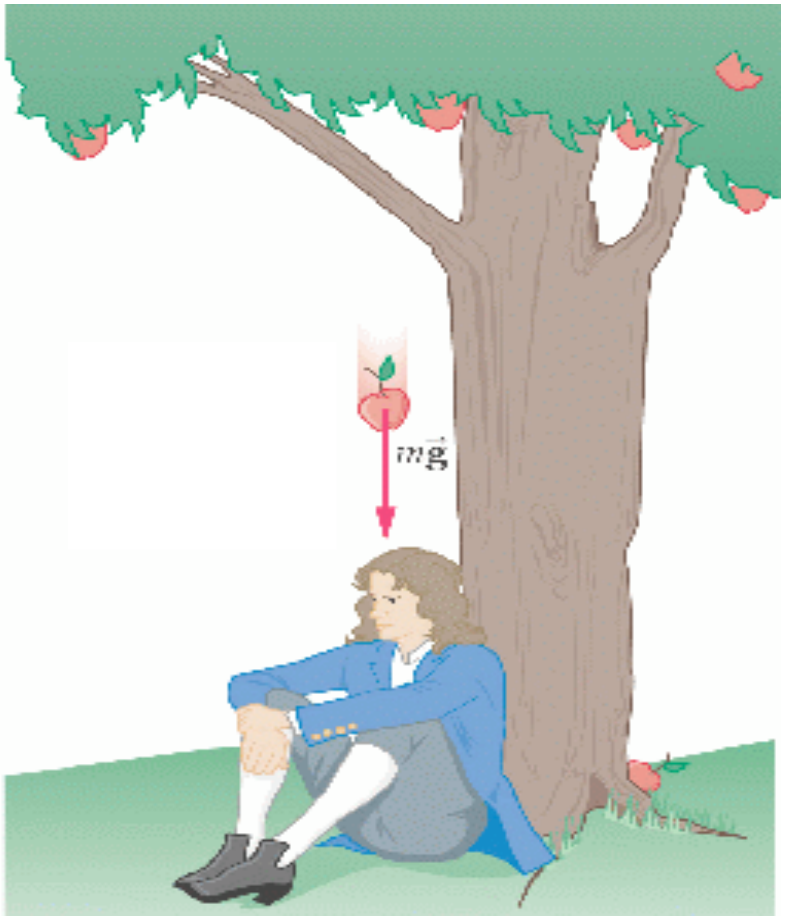
GENERAL RELATIVITY - Einstein 1915



Starting problem:
Special Relativity only applies to inertial reference frames, where no forces are involved.

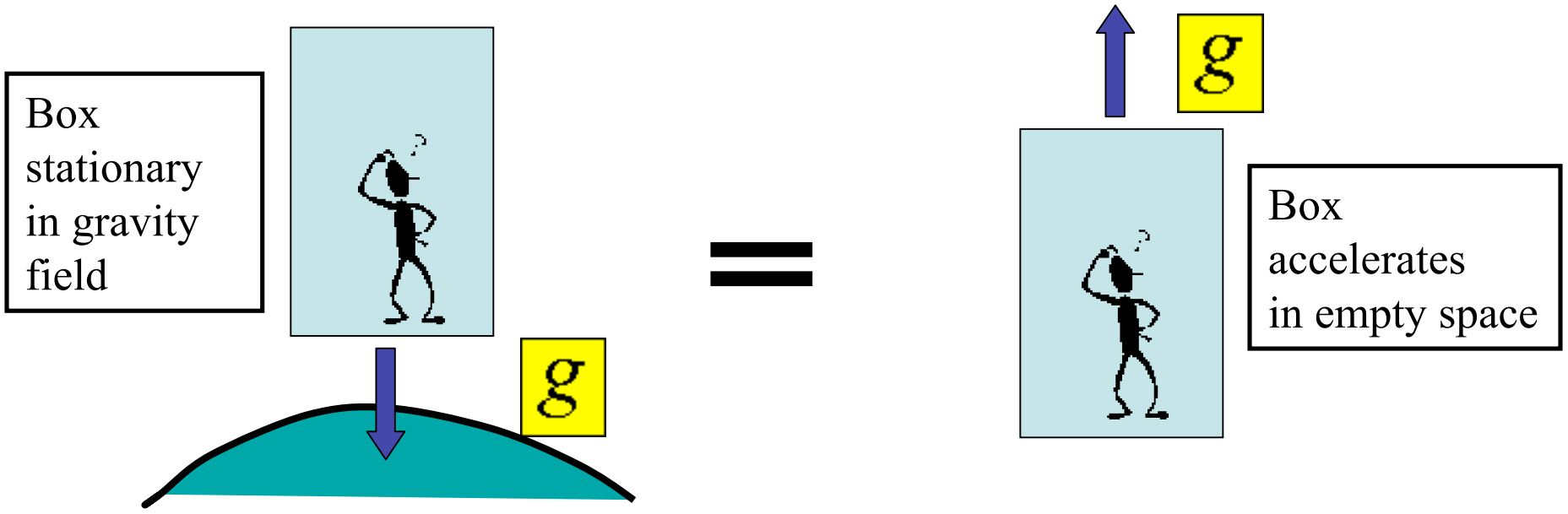
How to describe gravity then?

[Note: some material in these notes courtesy of D. Watson and M. Begelman]

Principle of Equivalence: Einstein 1907

(in Einstein's own words: "The happiest thought of my life")

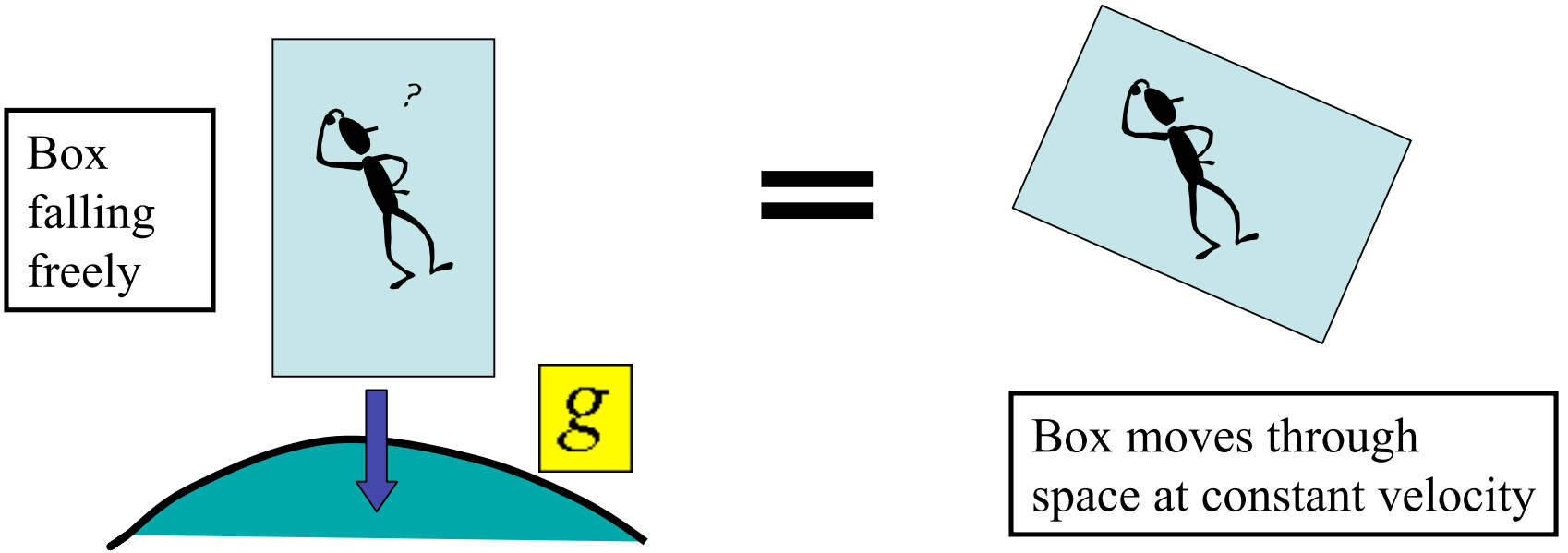
The effects of gravity are exactly equivalent to the effects of acceleration.



In other words:

A freely-falling reference frame is under the influence of gravity but is equivalent to an inertial frame.

→ The laws of Special Relativity apply.



Consequences of the Equivalence Principle:

Gravitational Time Dilation

To a distant observer, time appears to pass more slowly in places where gravity is strong.

To an observer in a place where gravity is strong, time appears to pass more quickly in places where gravity is weak.

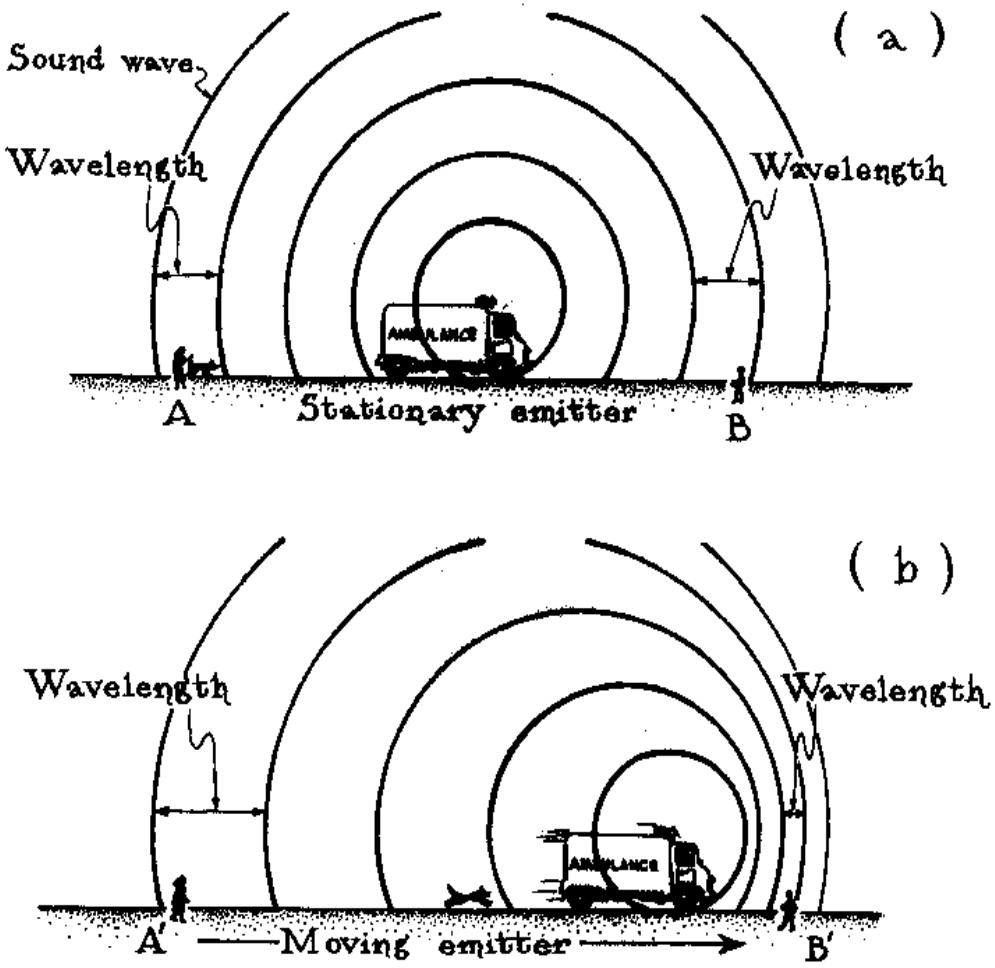
Both statements embody the idea that **gravity warps time**.

This sort of time dilation is importantly different from the special-relativistic version of time dilation!

- In special relativity, two observers in inertial frames moving with respect to each other each see the other's clock as moving slowly.

Einstein thought of gravitational time dilation as an example of the **Doppler effect**.

Doppler effect (example of sound waves)



The pitch one perceives for a sound changes depending upon the motion of the source: the pitch is higher (**wavelength shorter**) for approaching sources, and lower (**wavelength longer**) for receding sources.

All waves, including light, exhibit the Doppler effect.

Figure from Thorne, *Black holes and time warps*

Doppler effect for light: approaching objects look bluer, receding objects redder, than their natural colors

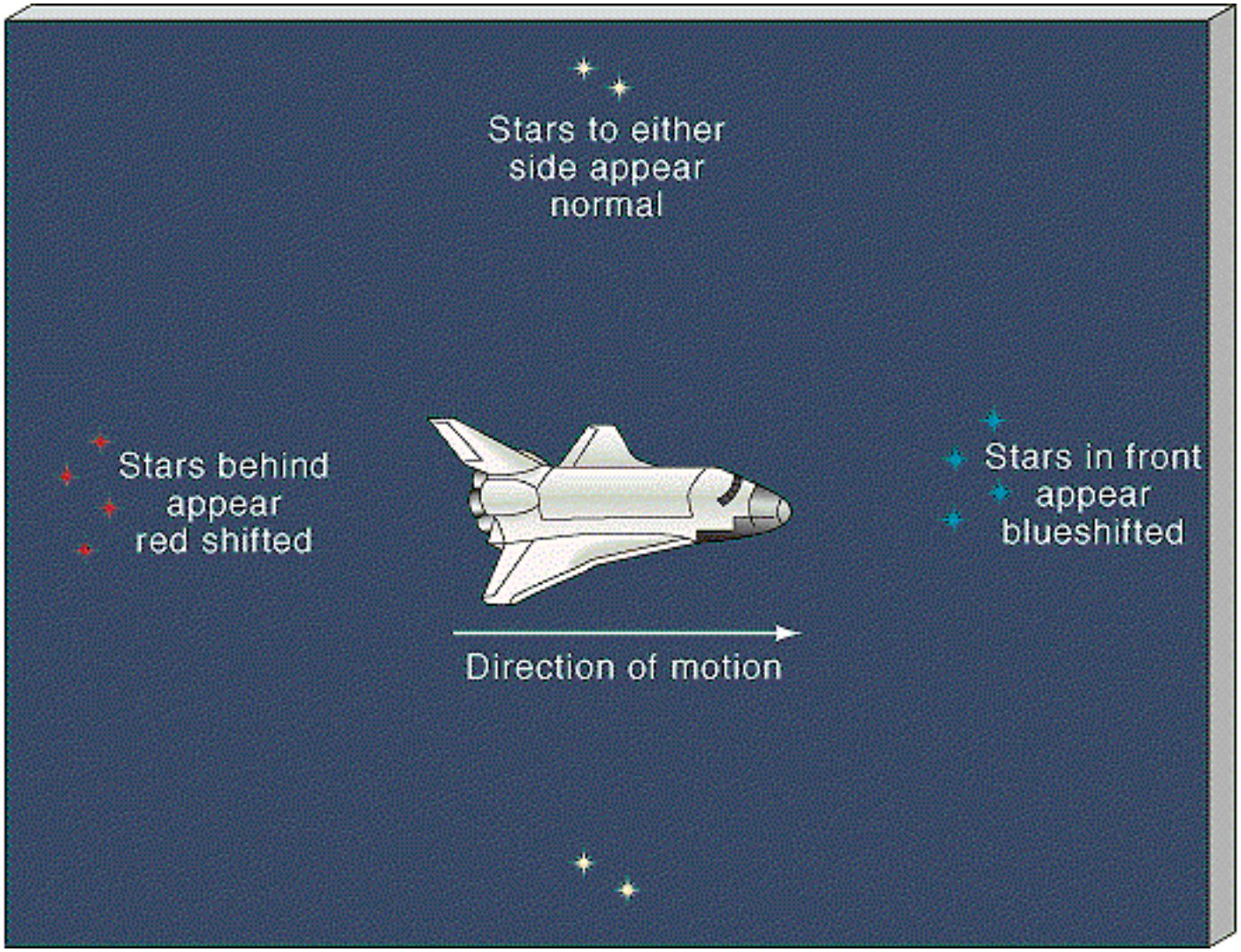
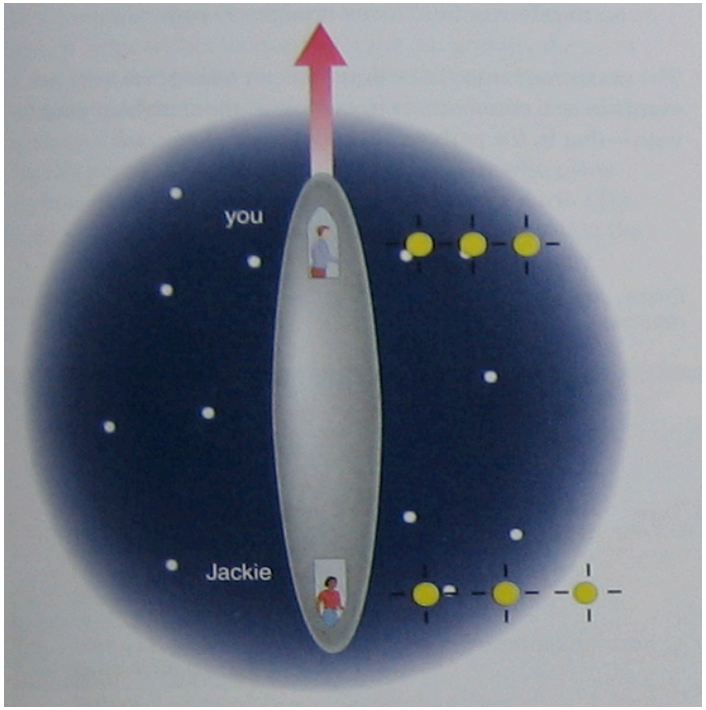


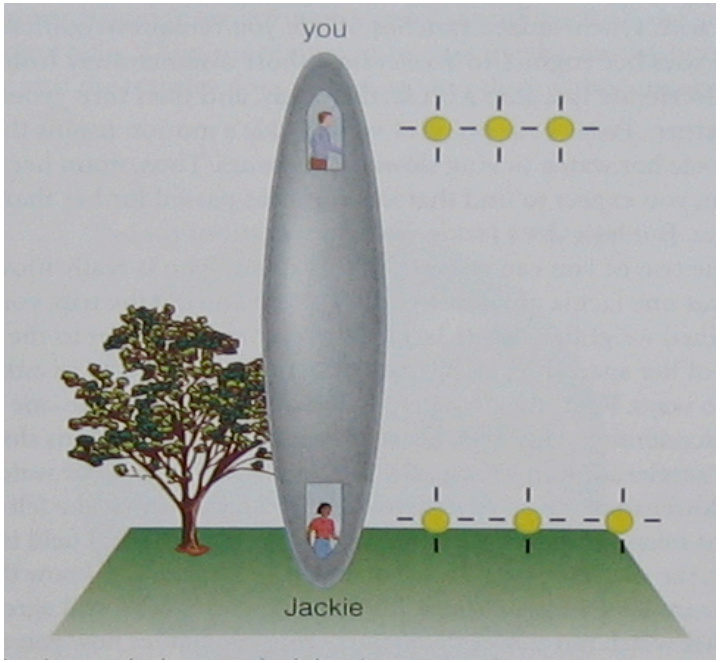
Figure: Chaisson and McMillan, *Astronomy today*

Time Dilation in a Gravitational Field



A thought experiment with flashing watches shows that time runs slower at the back of an accelerating space-ship. The yellow dots represent the flashes from the watches, and the spacing between the dots represent the time between the flashes.

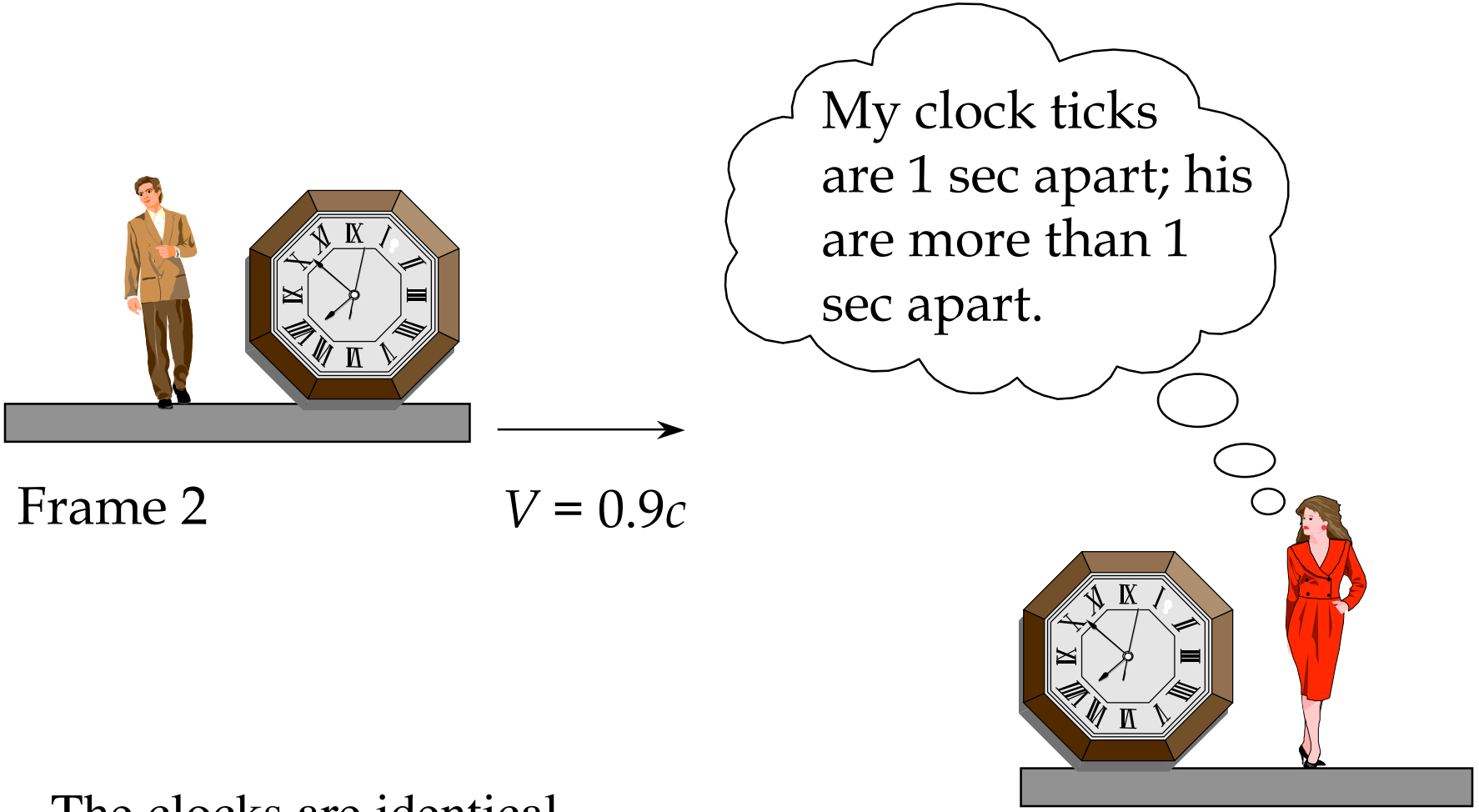
By the equivalence principle, time must also run slower at lower altitudes in a gravitational field.



[Example and figures from "The Cosmic Perspective", by Bennett et al.]

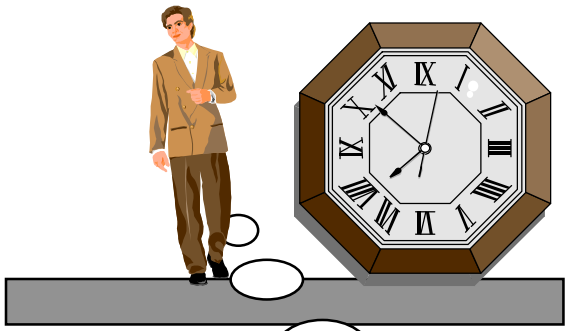
Special-relativistic time dilation is symmetrical

Here is *Beverly's* point of view:



The clocks are identical.

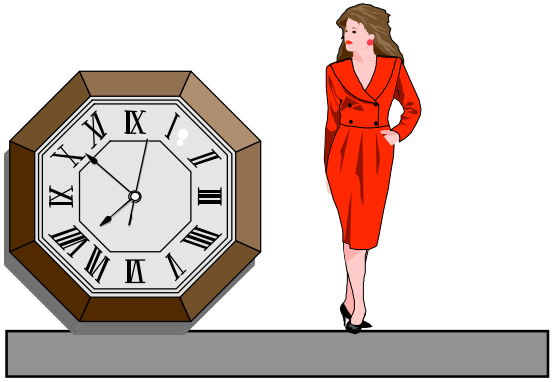
….and here is *Chuck's* point of view for the same identical clocks:



Frame 1

My clock ticks are 1 sec apart; hers are more than 1 sec apart.

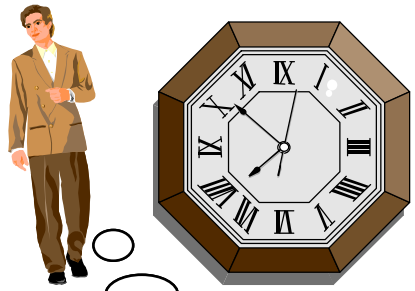
$V = 0.9c$



But: gravitational time dilation is not symmetrical.

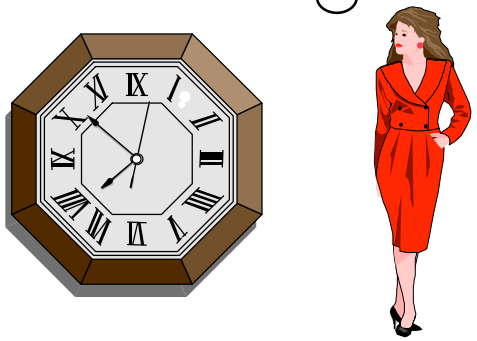
The clocks are still identical.

Weak gravity



My clock ticks are 1 sec apart; his are less than 1 sec apart.

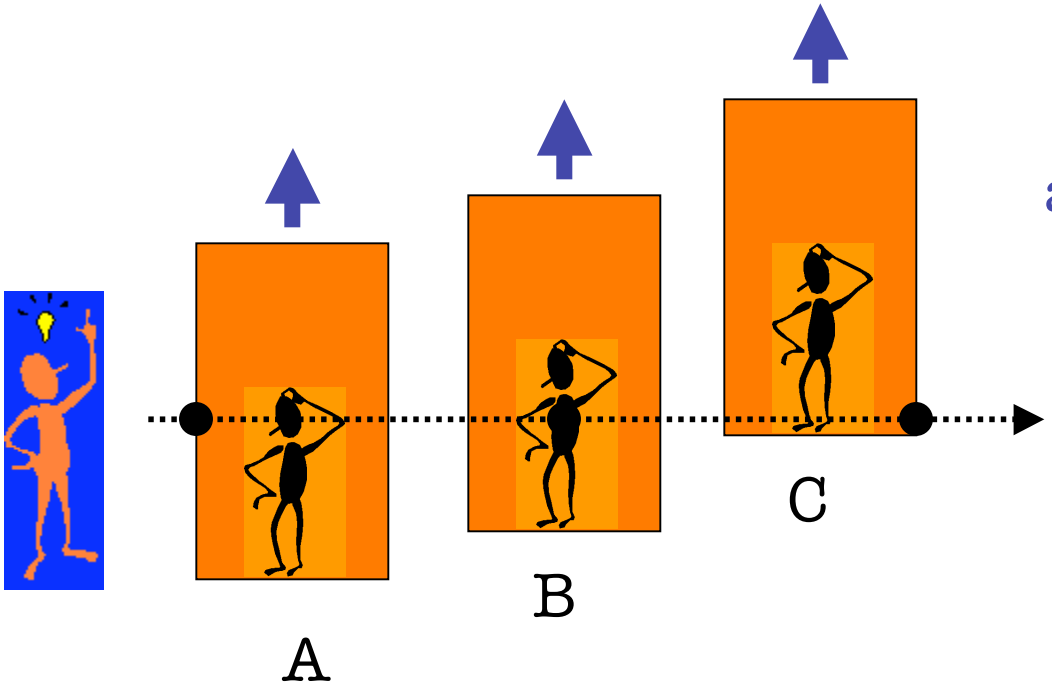
My clock ticks are 1 sec apart; hers are more 1 sec apart.



Extremely strong gravity

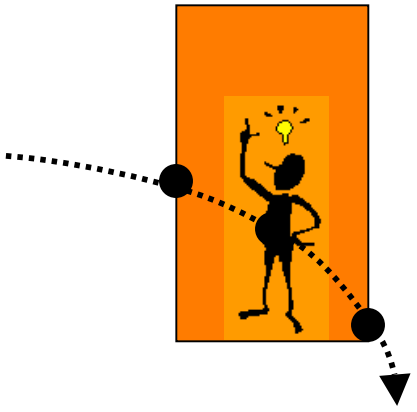
Another consequence of the principle of equivalence:

Gravitational deflection of light



accelerating observer
=
gravity

If the elevator is accelerating, the distance traveled between points B and C is larger than the distance traveled between A and B (assuming identical time intervals between B and C, and between A and B). To the observer inside the elevator, **the trajectory of the light ray will appear bent.**



Light Rays and Gravity

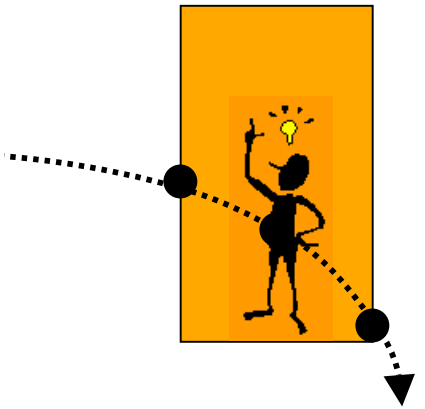
- In SR: light rays travel on straight lines

  => in freely falling fame, light travels on ***straight*** lines

- BUT: to stationary observer light travels on curved paths

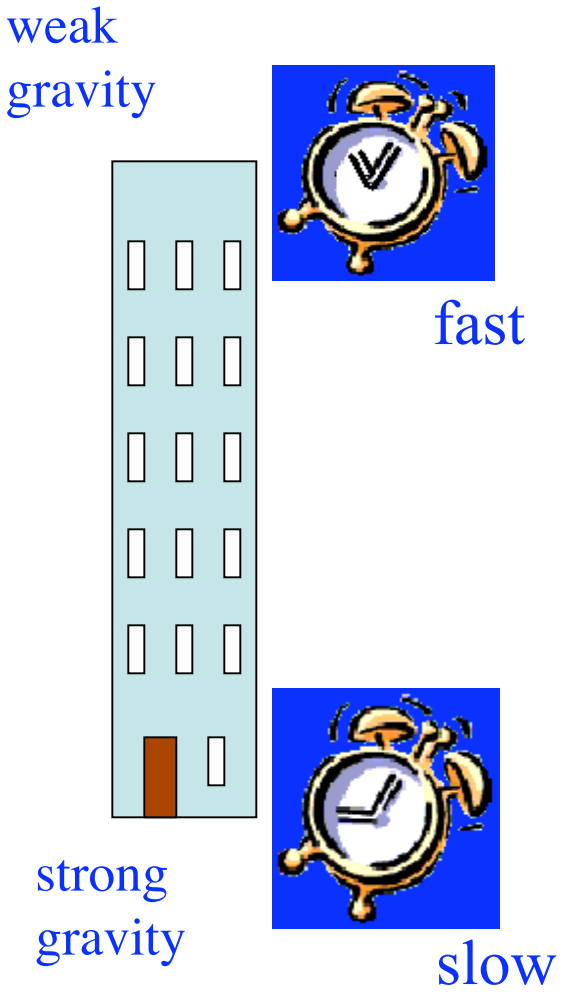
  => Maybe gravity has something to do with…

  curvature of space ?



Curved Spacetime

- Remember: Gravity warps time



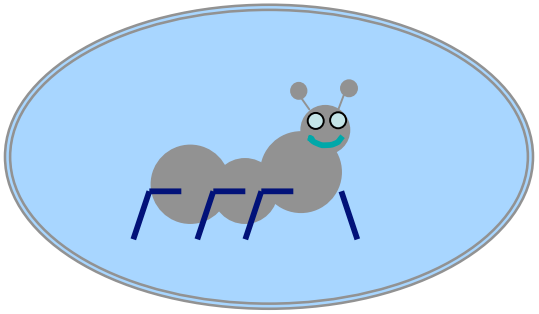
BUT: in spacetime, time and space are not separable

=> Both space and time are curved (warped)

This is a bit hard to visualize (spacetime already 4D…)

CURVATURE IN 2D

- Imagine being an ant… living in 2D



- You would understand:

  left, right, forward, backward,

  but ***NOT*** up/down…

- How do you know your world is curved?

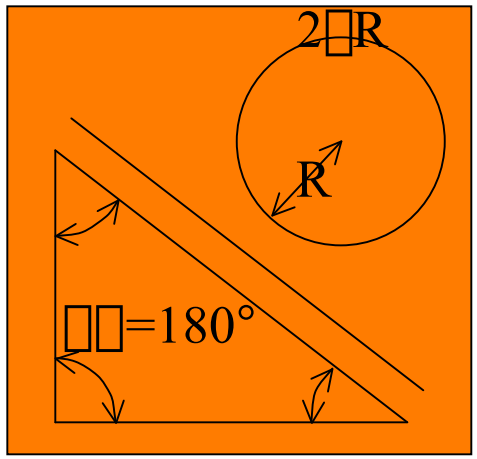
Curvature in 2D (continued)

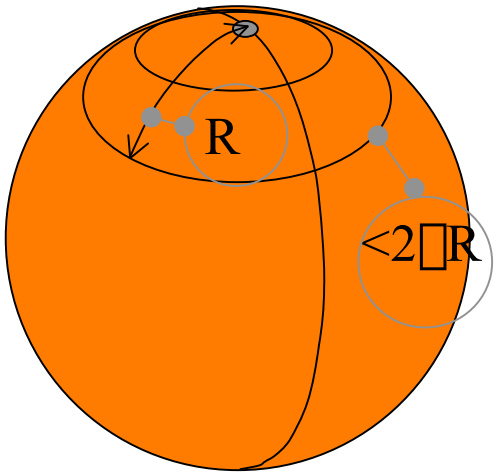
- In a curved space, Euclidean geometry does not apply:

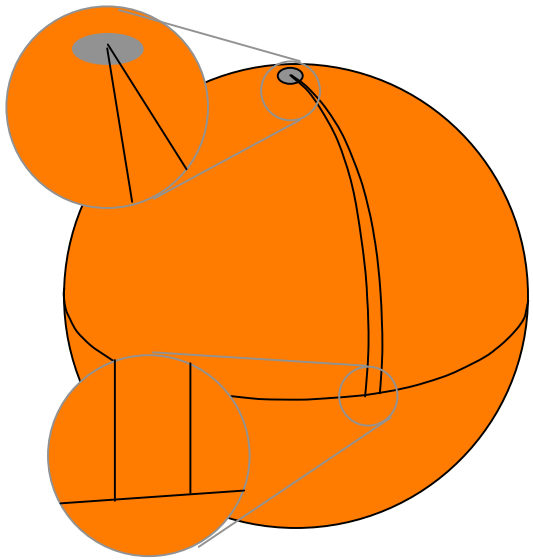
  - circumference ≠ 2□ R

  - triangles ≠ 180°

  - parallel lines don't stay parallel

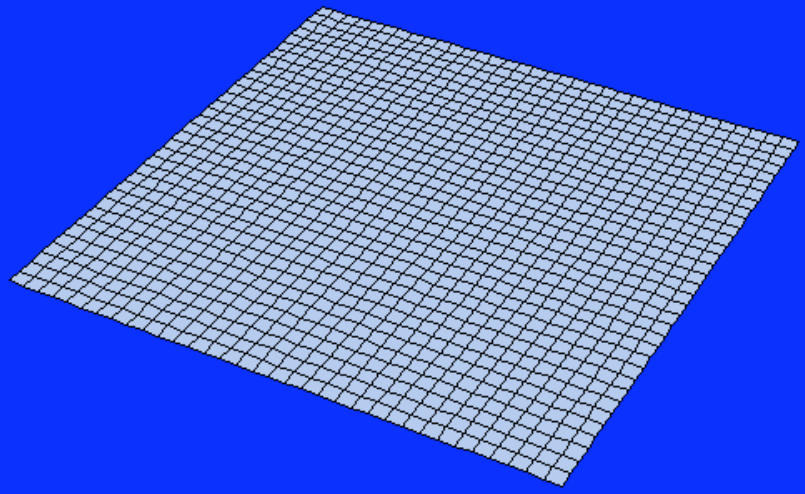


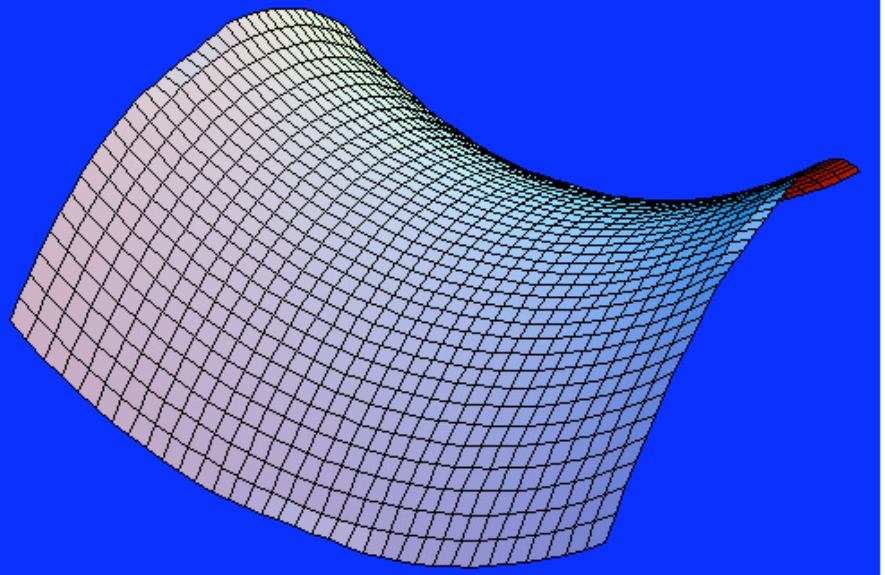




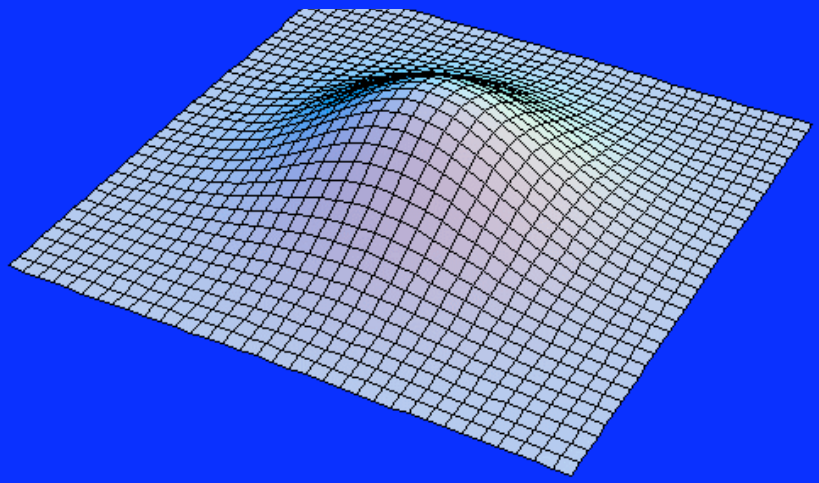
Curvature in 2D (continued)

Zero Curvature



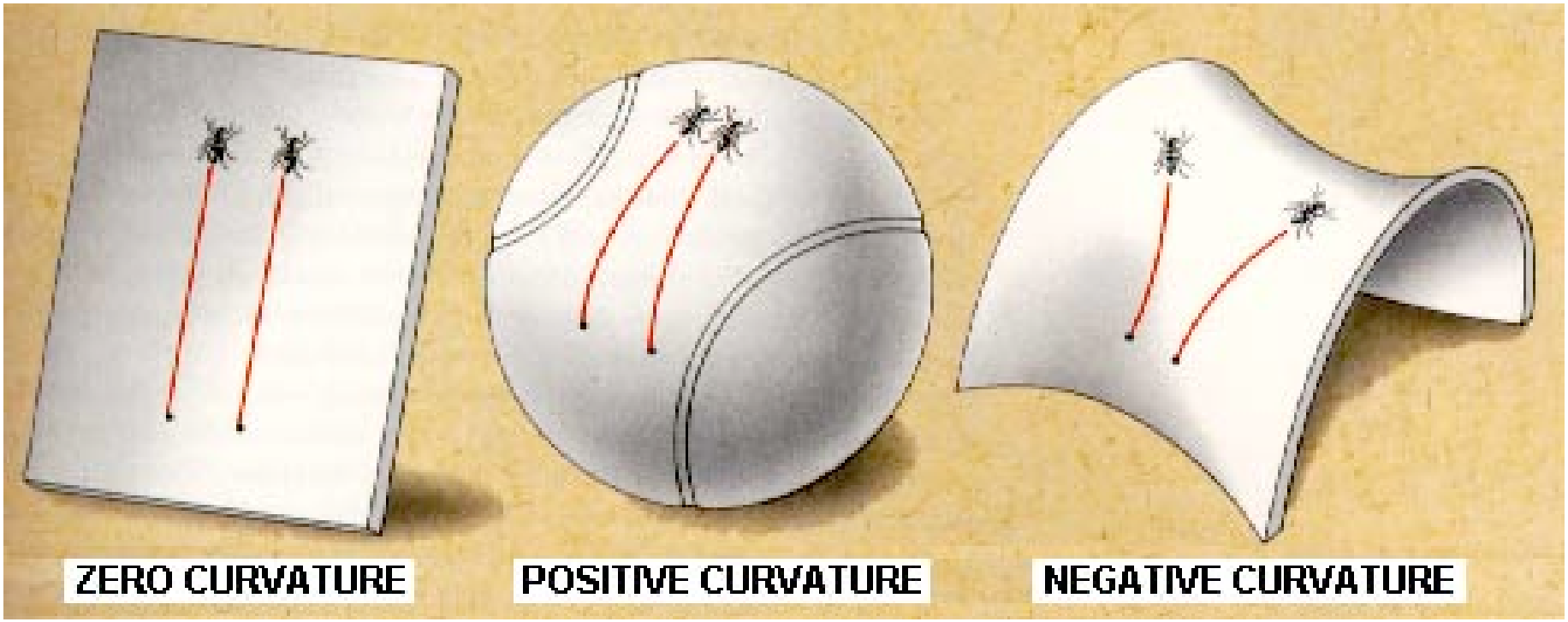


Negative Curvature



Positive Curvature

Curvature in 2D (continued)



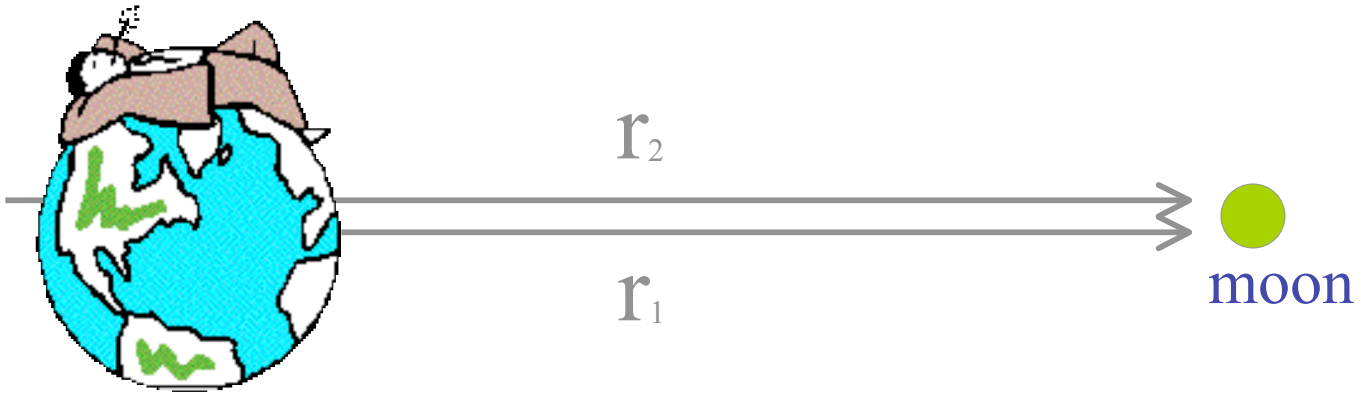
Parallel lines remain parallel

Parallel lines eventually converge

Parallel lines eventually diverge

Curvature of Space: Tides

- The Earth and the moon:



- Gravity decreases with distance => stretch…

Tides (continued)

One can think of tides as
an effect of gravitational force
(as Newton did), or as the effect
of a curvature, warping or
stretching of space created by
the presence of the gravitating mass
("space is warped by gravity").

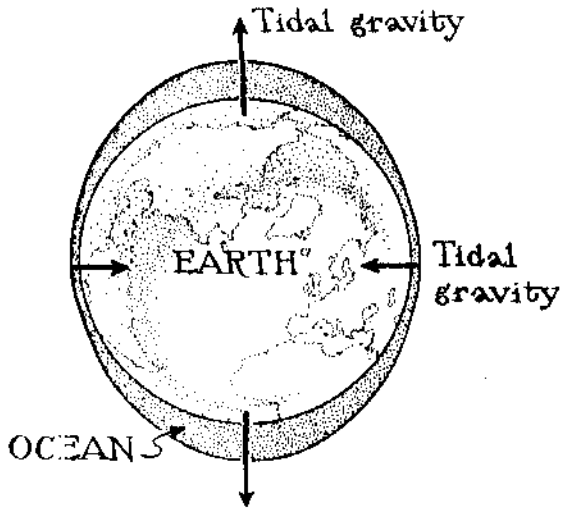


Figure from Thorne,
Black holes and time warps

Tidal gravity

Earth pulls all things toward its center, and pulls harder on nearer things.

To you, it seems as if you are stretched in the direction of the earth's center, and squeezed in the other direction.

Figure from Thorne, *Black holes and time warps*

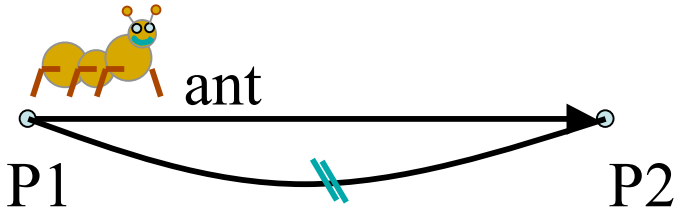
Tidal forces are a characteristic of gravity but not of accelerating frames.

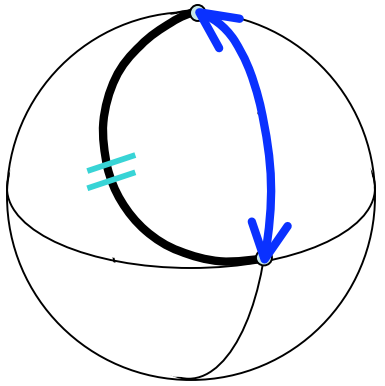
Travel in a curved space: **Geodesics**

- To do geometry, we need a way to measure distances

=> use *ant* (let's call the ant "***metric***"), count steps it has to take on its way from P1 to P2 (in spacetime, the ant-walk is a bit funny looking, but never mind that)

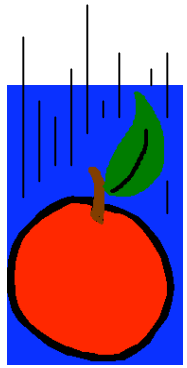
- *Geodesic*: ***shortest line*** between P1 and P2 (the fewest possible ant steps)





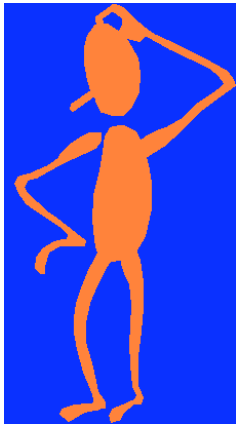
Geodesics (continued)

- To the ant, the geodesic is a ***straight line***, i.e., the ant ***never*** has to ***turn***

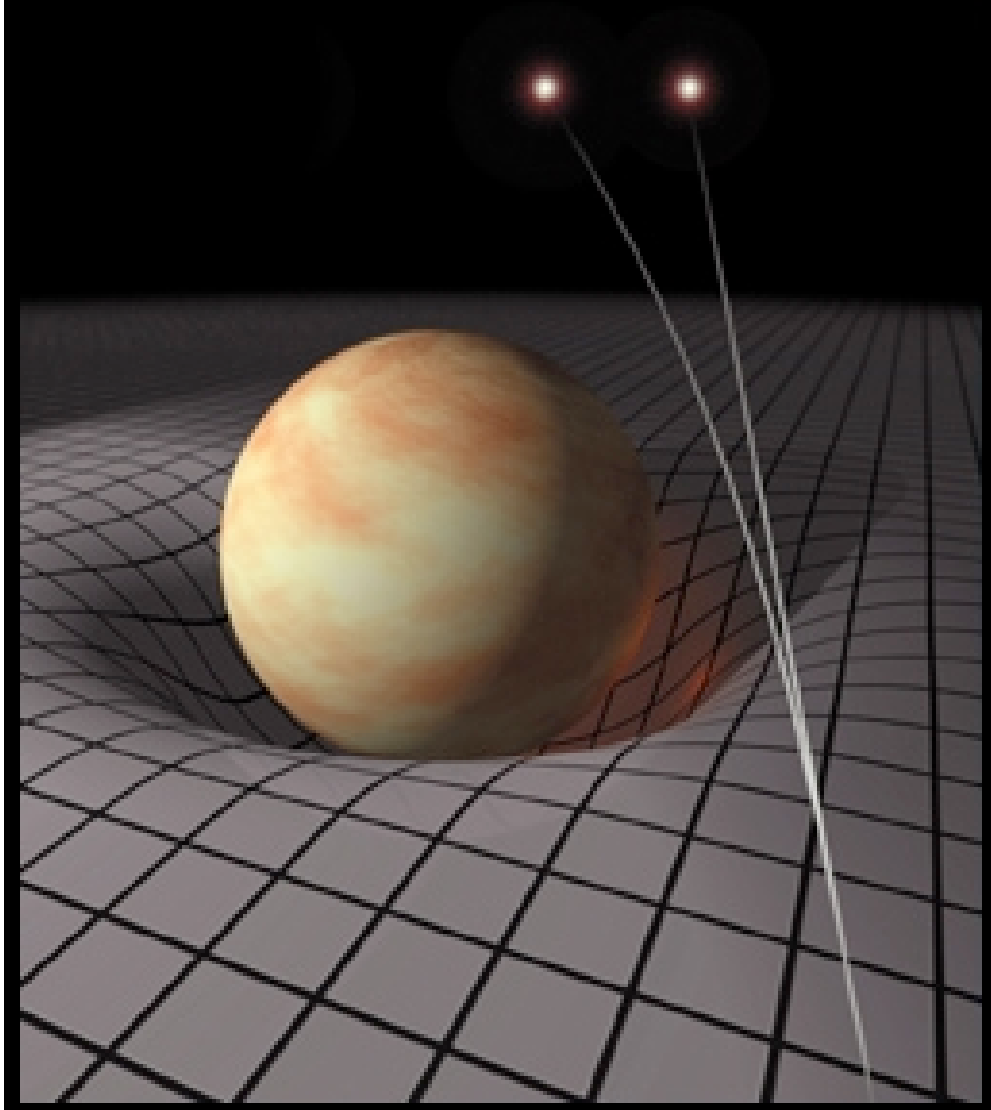


- In SR and in freely falling frames, objects move in straight lines (uniform motion)

- In GR, ***freely falling objects*** (*freely falling*: under the influence of gravity only, no rocket engines and such; *objects*: apples, photons, etc.) ***move on geodesics*** in spacetime.



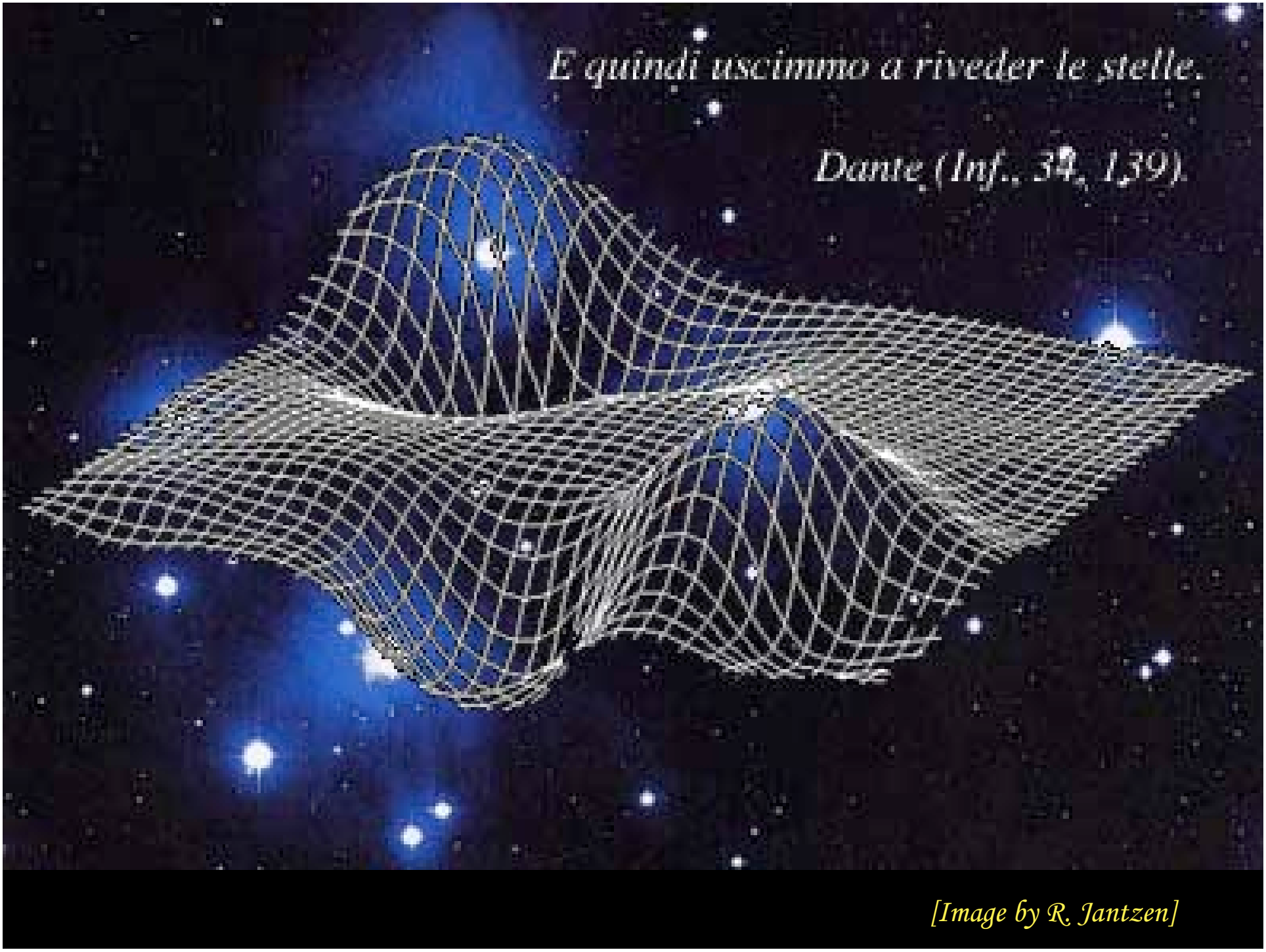
GENERAL RELATIVITY: (in a nutshell)



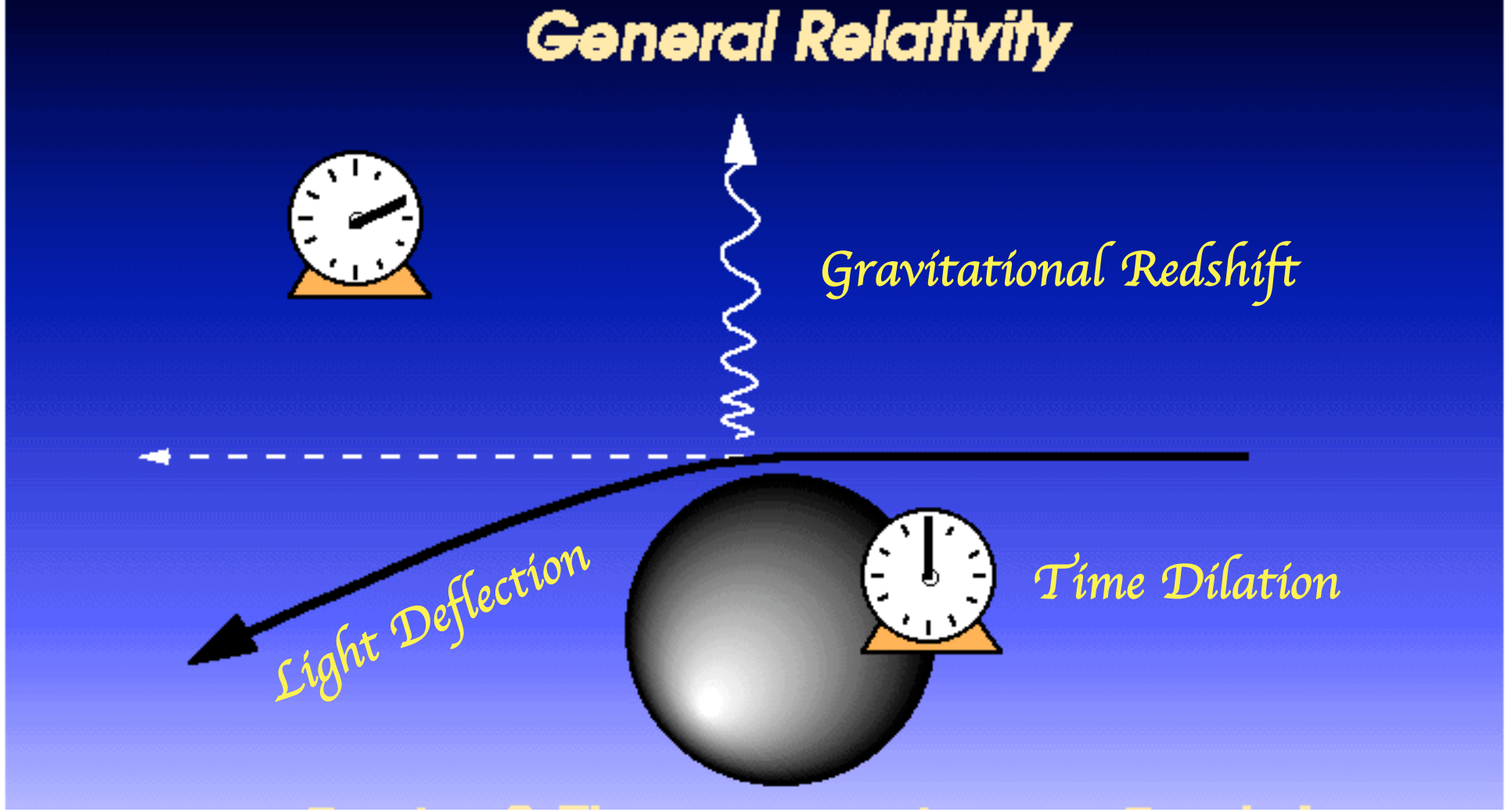
Masses tell spacetime how to curve.

Spacetime, with its curvature, tells masses how to move.

[Image from "Spacetime and Gravity" by S. Carroll]



[Image by R. Jantzen]

[Image credit: NASA/

SUMMARY OF THE PREDICTIONS OF