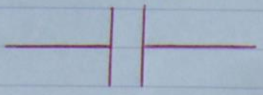
Capacitors — B McMULLEN

Circuit symbol



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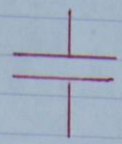


A capacitor stores electrical charge and electrical energy.

Think Abstract !!

Capacitor ⇒ Bucket
Charge stored in capacitor ⇒ water in bucket
Resistance ⇒ hole in the bucket

Capacitors are used in time-delay circuits.

$$\boxed{t = RC}$$

Capacitance (F) ⇓ Farads

Time delay (s)

Resistance (Ω)

- For a capacitor to charge up quickly ie a small time-delay
  ⇒ $t \downarrow \therefore R$ and/or $C \downarrow$

- For a capacitor to charge up slowly ie a large time-delay
  ⇒ $t \uparrow \therefore R$ and/or $C \uparrow$

Applying to the Bucket theory

- To fill a small bucket with a small hole ⇒ small time

ie small capacitance with a small resistance ⇒ small time-delay.

- To fill a large bucket with a large hole ⇒ large time

ie large capacitance with a large resistance ⇒ large time-delay.

Frequency and time-delay

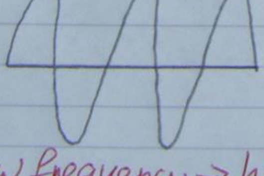
$$f = \frac{1}{T} \Rightarrow f \propto \frac{1}{T}$$

Conclusion

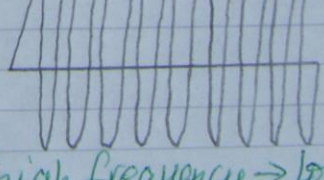
Time-delay ↓ ∴ frequency ↑

and

Time-delay ↑ ∴ frequency ↓



Low frequency → high period



high frequency → low period.

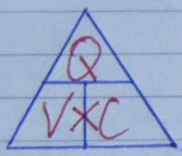
Capacitance Equation

In DB!! $\Rightarrow$ $\boxed{C = \frac{Q}{V}}$

Charge
(Coulombs)
$\Downarrow$
(C)

Capacitance
(Farads)
$\Downarrow$
(F)

Voltage/Potential difference
(Volts)
$\Downarrow$
(V)



QVC
$\Downarrow$
Shopping channel!!

1. $Q = VC$ 2. $C = Q/V$ 3. $V = Q/C$

Alternative units for Capacitance

From $C = \frac{Q}{V}$ $\Rightarrow$ $1F = \frac{1C}{1V}$

$\Rightarrow$ xx $1F = 1CV^{-1}$ xx

$\Rightarrow$ 1 Farad = 1 Coulomb per Volt.

Magnitude of Capacitance

DO NOT use the words size or value when you talk about capacitance or Resistance.

ie Increase or decrease the Capacitance or Resistance.

- pF = picofarads $= \times 10^{-12}$ F ⇒ Low Capacitance
- nF = nanofarads $= \times 10^{-9}$ F
- μF = microfarads $= \times 10^{-6}$ F
- mF = millifarads $= \times 10^{-3}$ F ⇒ high Capacitance.

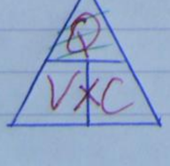
* I have only ever seen one capacitor which has a capacitance of greater than one Farad. *

Ex 1

Q Calculate the charge stored in a 400μF capacitor when a voltage of 7V is dropped across it.

A
$Q = ?$
$V = 7V$
$C = 400\mu F = 400 \times 10^{-6} F$

$Q = VC$
$\Rightarrow Q = 7 \times 400 \times 10^{-6}$
$\Rightarrow Q = 2.8 \times 10^{-3} C$



Ex2

Q: A capacitor is marked 2500nF on its side. How long will it take for an average current of 12A to charge it up to 150V

A: $C = 2500nF$
$I = 12A$
$V = 150V$
$t = ?$
$Q = ?$

STEP 1 $\Rightarrow C = \frac{Q}{V}$

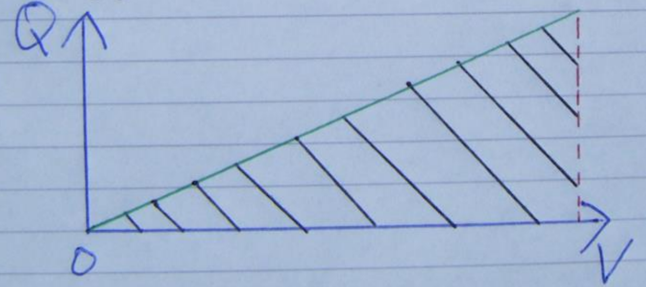
$\Rightarrow Q = VC = 150 \times 2500 \times 10^{-9}$

$\Rightarrow Q = 3.75 \times 10^{-4} C$

STEP 2 $\Rightarrow Q = It$

$\Rightarrow t = \frac{Q}{I} = \frac{3.75 \times 10^{-4}}{12} = 3.125 \times 10^{-5} s$

Energy stored in a capacitor



Energy stored in a capacitor = Area under a Q–V graph

$$\boxed{E_{CAP} = \frac{1}{2}QV}$$

• From $C = \frac{Q}{V} \Rightarrow Q = VC$

Sub $Q = VC$ into $E_{CAP} = \frac{1}{2}QV$

$\Rightarrow E_{CAP} = \frac{1}{2}(VC)V \Rightarrow \boxed{E_{CAP} = \frac{1}{2}CV^2}$

• From $C = \frac{Q}{V} \Rightarrow V = \frac{Q}{C}$

Sub $V = \frac{Q}{C}$ into $E_{CAP} = \frac{1}{2}QV$

$\Rightarrow E_{CAP} = \frac{1}{2}Q\left(\frac{Q}{C}\right) \Rightarrow \boxed{E_{CAP} = \frac{1}{2}\frac{Q^2}{C}}$

Summary

* • $E_{CAP} = \frac{1}{2}QV$

* • $E_{CAP} = \frac{1}{2}CV^2$

most common in use !!

• $E_{CAP} = \frac{1}{2} \cdot \frac{Q^2}{C}$

Ex3

Q. Calculate the energy stored in a 650μF capacitor when fully charged by a 8V dc supply.

A. $E_{CAP} = ?$

$C = 650\mu F = 650 \times 10^{-6} F$

$V = 8V$

$E_{CAP} = \frac{1}{2} C V^2$

$\Rightarrow E_{CAP} = \frac{1}{2} \times 650 \times 10^{-6} \times 8^2$

$\Rightarrow \underline{\underline{E_{CAP} = 2.08 \times 10^{-2} J}}$

Ex4

Q. A 2500μF capacitor has 7.2×10^{-5}J of electrical energy stored in it. Calculate the charge stored in the capacitor.

A. $E_{CAP} = 7.2 \times 10^{-5} J$

$C = 2500\mu F = 2500 \times 10^{-6} F$

$Q = ?$

METHOD 1

$E_{CAP} = \frac{1}{2} \cdot \frac{Q^2}{C} \Rightarrow 7.2 \times 10^{-5} = \frac{1}{2} \cdot \frac{Q^2}{2500 \times 10^{-6}}$

$\Rightarrow$ multiply each side by 2 to get rid of the $\frac{1}{2}$.

$\Rightarrow 2 \times 7.2 \times 10^{-5} = 2 \times \frac{1}{2} \cdot \frac{Q^2}{2500 \times 10^{-6}}$

$$\Rightarrow 14{\cdot}4 \times 10^{-5} = \frac{Q^2}{2500 \times 10^{-6}}$$

$$\Rightarrow Q^2 = 14{\cdot}4 \times 10^{-5} \times 2500 \times 10^{-6} = 3{\cdot}6 \times 10^{-7}$$

$$\Rightarrow Q = \sqrt{3{\cdot}6 \times 10^{-7}} = \underline{\underline{6 \times 10^{-4}\,C}}$$

METHOD 2

$\boxed{\text{STEP 1}}$ $E_{CAP} = \frac{1}{2}CV^2$ $\boxed{\text{To find } V}$

$$\Rightarrow 7{\cdot}2 \times 10^{-5} = \tfrac{1}{2} \times 2500 \times 10^{-6} \times V^2$$

MULTIPLY EACH SIDE BY 2

$$\Rightarrow 14{\cdot}4 \times 10^{-5} = 2500 \times 10^{-6} \times V^2$$

$$\Rightarrow V^2 = \frac{14{\cdot}4 \times 10^{-5}}{2500 \times 10^{-6}} = 0{\cdot}0576 \Rightarrow \underline{\underline{V = 0{\cdot}24V}}$$

$\boxed{\text{STEP 2}}$ $E_{CAP} = \frac{1}{2}QV$ $\boxed{\text{To find } Q}$

$$\Rightarrow 7{\cdot}2 \times 10^{-5} = \tfrac{1}{2} \times Q \times 0{\cdot}24$$

MULTIPLY EACH SIDE BY 2.

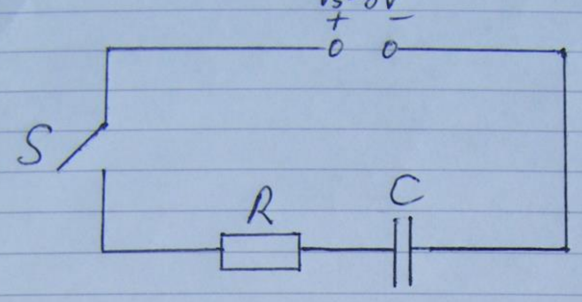
$$\Rightarrow 14{\cdot}4 \times 10^{-5} = Q \times 0{\cdot}24$$

$$\Rightarrow Q = \frac{14{\cdot}4 \times 10^{-5}}{0{\cdot}24}$$

$$\Rightarrow \underline{\underline{Q = 6 \times 10^{-4}\,C}}$$

* I would advise that you should use METHOD 1 to do the calculation in one stage. *

Ex 5 (NB!! Explains the difference between $E_W = QV$ and $E_{CAP} = \frac{1}{2}QV$)



Q// When the switch is closed the capacitor charges up constantly at 6×10^{-4}C.

a) Calculate the electrical work done required to fully charge up the capacitor from being uncharged.

b) Calculate the energy stored in the capacitor when it is fully charged.

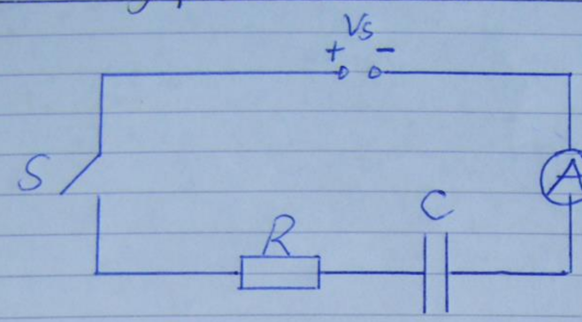
c) Explain the difference in the answers found in a) and b).

A// a) $E_W = QV = 6 \times 10^{-4} \times 8 = \underline{4.8 \times 10^{-3} J}$

b) $E_{CAP} = \frac{1}{2}QV = \frac{1}{2} \times 6 \times 10^{-4} \times 8 = \underline{2.4 \times 10^{-3} J}$

c) Half of the electrical work done is stored as electrical energy in the capacitor with the other half given off as heat energy through the resistor.

Voltage/Potential Dividers with R and C



The capacitor C is initially uncharged.

$$\boxed{V_S = V_C + V_R}$$

1) At the instant when S is closed, a current will flow.

$I = \frac{V_R}{R}$ As C is uncharged then $V_C = 0 \Rightarrow V_R = V_S$.

As V_R is at a maximum at this point then the current is at a maximum

ie $I = \frac{V_R \leftarrow \text{max}}{R \leftarrow \text{constant}}$ $\therefore I \propto V_R$.

2) As the capacitor charges up V_C increases from 0V and V_R decreases from its original Vs.

$$\begin{matrix} \leftrightarrow & = & \uparrow & + & \downarrow \\ V_S & = & V_C & + & V_R \end{matrix}$$ (charging!!)

As $I = \frac{V_R}{R}$ as V_R decreases then the current I decreases.

ie $I \propto V_R$.

3) When the capacitor fully charges up, V_C is at a maximum.

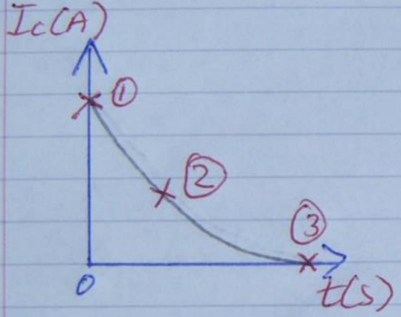
ie ($V_C = V_S$).

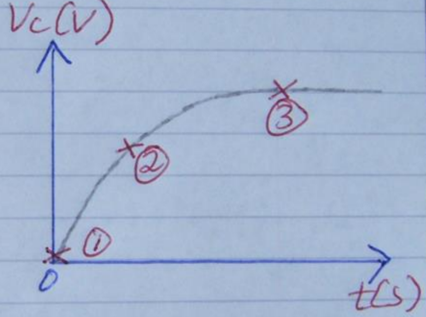
$$\leftrightarrow = \uparrow + \downarrow$$

$$V_S = V_C + V_R$$

As $V_R = 0$ then $I = \frac{V_R}{R} \Rightarrow I = 0$.

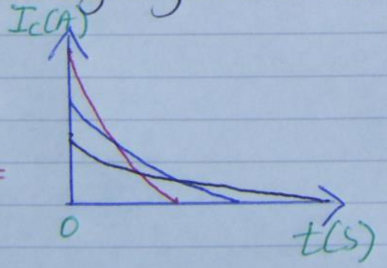
Graphically





Varying R and C in charging graphs.

Varying R



- Small R $\Rightarrow I_{max}$ increases $\left(I = \frac{V_R}{R}\right)$

Also from $t = RC$ $(t \propto R)$.

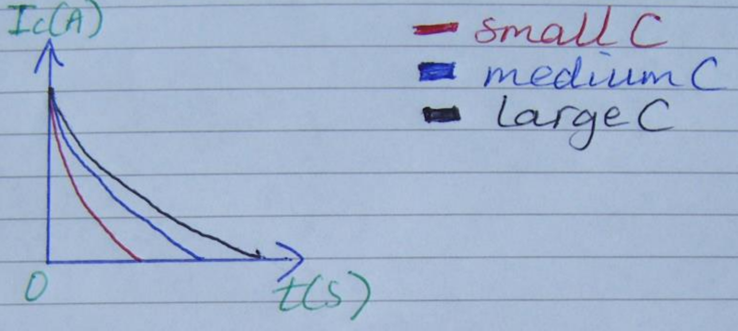
$\Rightarrow$ As R is small then the time delay is small.

• Large R $\Rightarrow$ I_{MAX} decreases $\left(I = \frac{V_R}{R}\right)$

Also from $t = RC$ $(t \propto R)$

$\Rightarrow$ As R is large then the time delay is large.

Varying C



Varying the capacitance C will only vary the time-delay.

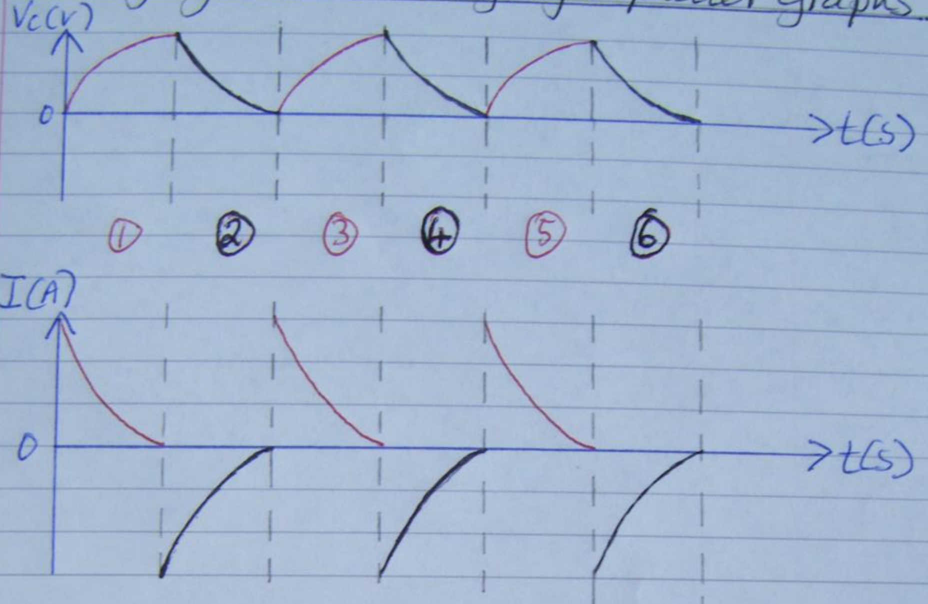
ie It does not have any baring on the current.

(Only the Resistance can vary the current in a dc circuit)

From $t = RC$ $\therefore$ $t \propto C$

ie The greater the capacitance the greater the time-delay.

Charging and discharging capacitor graphs



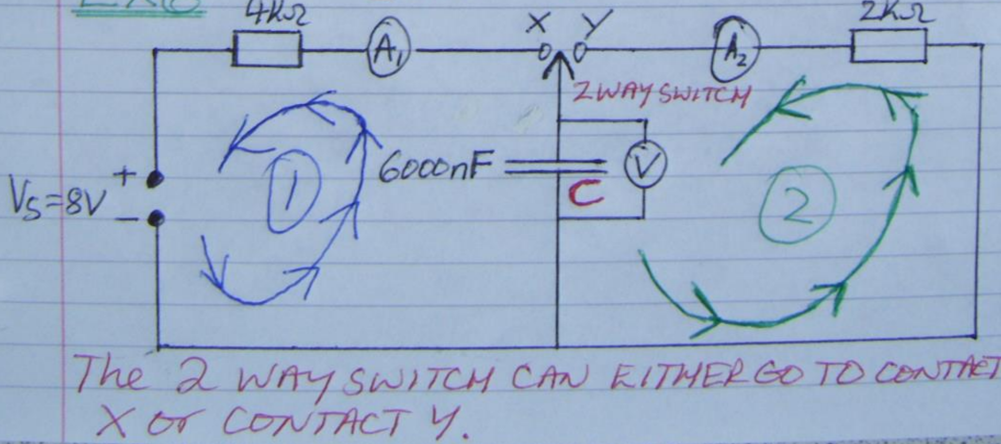
KEY

1, 3 and 5 ⇒ Charging cycles

2, 4 and 6 ⇒ discharging cycles.

Why does the current start at a negative maximum in the discharge cycle?

Ex6 (A/B STANDARD IN EXAM!!).



The 2 WAY SWITCH CAN EITHER GO TO CONTACT X or CONTACT Y.

When connected to X ⇒ Charging cycle

When connected to Y ⇒ discharging cycle

Q+A Calculate or find:

a) i) max reading on $-(A_1)-$ during the charging cycle.

$$I = \frac{V_R}{R} = \frac{8}{4 \times 10^3} = 2 \times 10^{-3} A$$

ii) Max reading on $-(A_2)-$ during the discharging cycle.

$$I = \frac{V_R}{R} = \frac{8}{2 \times 10^3} = 4 \times 10^{-3} A$$

b) Charge stored on the capacitor when fully charged.

$Q = ?$

$V = 8V$

$C = 6000nF = 6000 \times 10^{-9} F$

$Q = VC$

$\Rightarrow Q = 8 \times 6000 \times 10^{-9}$

$\Rightarrow Q = 4 \cdot 8 \times 10^{-5} C$

c) Energy stored on the capacitor when fully charged.

① $E_{CAP} = \frac{1}{2} QV$

$= \frac{1}{2} \times 4 \cdot 8 \times 10^{-5} \times 8$

$E_{CAP} = 1 \cdot 92 \times 10^{-4} J$

OR

$E_{CAP} = \frac{1}{2} CV^2$

$\Rightarrow E_{CAP} = \frac{1}{2} \times 6000 \times 10^{-9} \times 8^2$

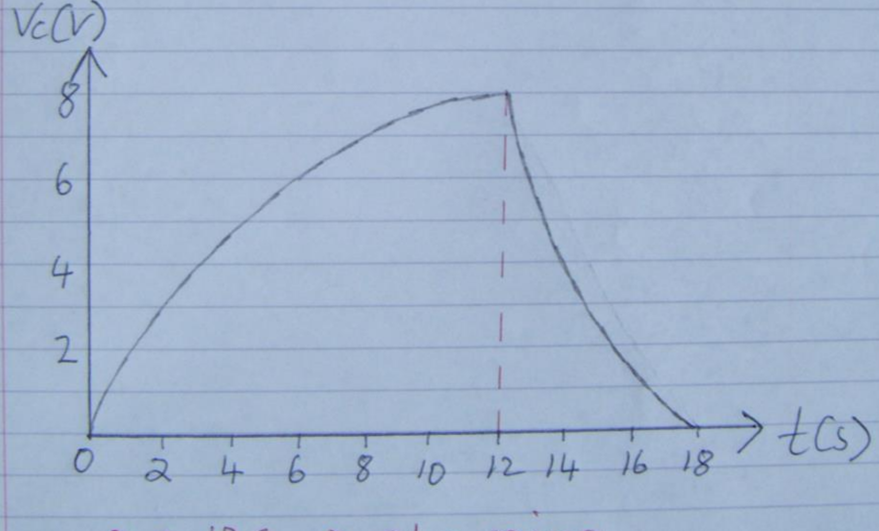
$\Rightarrow E_{CAP} = 1 \cdot 92 \times 10^{-4} J$

d) The capacitor takes 12 seconds to fully charge up. The capacitor then takes a further period of time to then discharge.
Draw a graph with readings to show V_c against time for the 12s. charging period and the discharging period thereafter.

Charging cycle $\Rightarrow R = 4k\Omega$ and time $= 12s$

Discharging cycle $\Rightarrow R = 2k\Omega \therefore$ time $= 6s$.

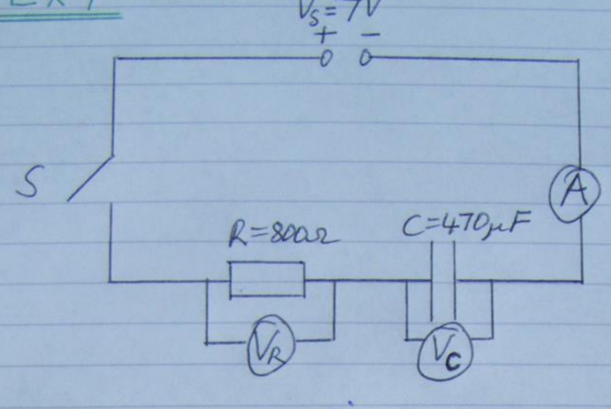
Why 6s? $\boxed{t = RC} \Rightarrow t \propto R$.



$0 \rightarrow 12s \Rightarrow$ charging

$12 \rightarrow 18s \Rightarrow$ discharging.

Ex 7



C is initially uncharged and starts to charge when S is closed.

Calculate or find:

a) The max reading on (A) when S is closed. (at that instant)

$$I = \frac{V_R}{R} = \frac{7}{800} = 8.75 \times 10^{-3} A$$

b) Suggest a suitable range for the ammeter.

$I_{MAX} = 8.75 \times 10^{-3} A = 8.75 mA$ ∴ Range $0 \rightarrow 10 mA$.

c) When $V_C = 2.5V$ during the charging process then what is the reading on the ammeter at this instant?

$$I = \frac{V_R}{R} = \frac{7 - 2.5}{800} = \frac{4.5}{800} = 5.63 \times 10^{-3} A.$$

d) Calculate the energy stored in the capacitor when it is fully charged.

$$E_{CAP} = \tfrac{1}{2} C V^2 = \tfrac{1}{2} \times 470 \times 10^{-6} \times 7^2$$

$$\Rightarrow E_{CAP} = 1.15 \times 10^{-2} J$$

e) Calculate the charge stored on the capacitor when it is fully charged.

$Q = VC = 7 \times 470 \times 10^{-6} = \underline{\underline{3.29 \times 10^{-3} C}}$

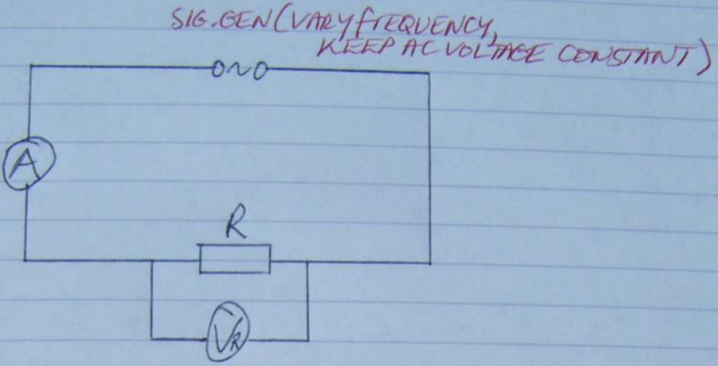
f) What effect will increasing the resistance from 800Ω to 1kΩ have in terms of:

i) I_{max}, the maximum current
ii) Time to charge up the capacitor.

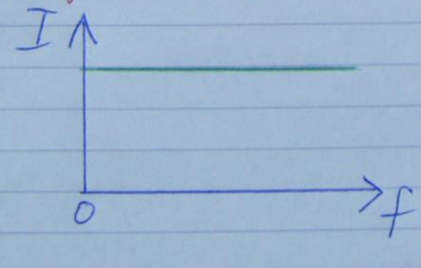
i) As $R \uparrow \therefore I_{MAX} \downarrow$

ii) From $t = RC$ as $R \uparrow \therefore t \uparrow$ ($t = RC$)
$t \propto R$.

A Resistive ac Circuit

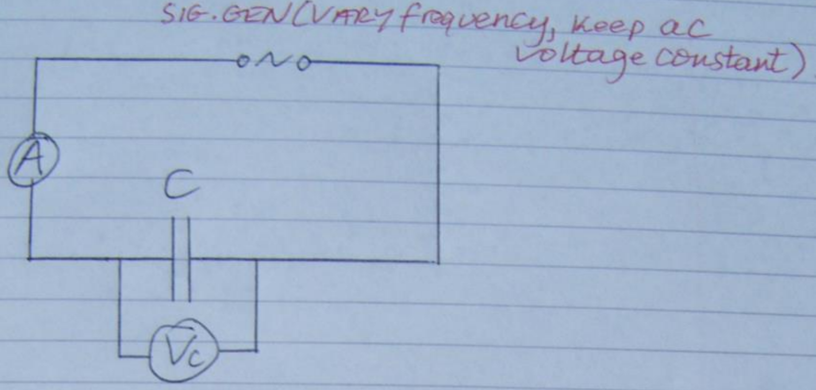


- The frequency from the sig.gen is varied from 200Hz to 600Hz.
- The corresponding ac currents are taken from the -(A)- for each of the frequencies.
- The voltage across the resistor must be kept constant (≈2V) throughout the experiment.
- Graph of I v f



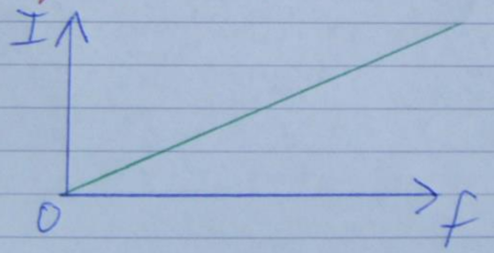
- Conclusion
  The current stays constant when the frequency is varied in an ac Resistive Circuit.

A capacitive ac circuit



- The same three steps as used in the ac resistive circuit.

- Graph of I v f



$I \propto f$

SLTO

- Conclusion

As the frequency increases the ac current will also increase in direct proportion.
This can be shown in the I v f graph which shows an SLTO.

NB One or both of these graphs will come up in the exam - 100% certainty!!