# EQUATIONS OF MOTION - B McMULLEN

1) $v = u + at$
2) $v^2 = u^2 + 2as$
3) $s = ut + \frac{1}{2}at^2$

uvast

u = Initial speed/Velocity ($ms^{-1}$)
v = Final speed/velocity ($ms^{-1}$)
a = acceleration ($ms^{-2}$)
s = distance/displacement (m)
t = time taken (s)

## Acceleration unit

g = acceleration due to gravity

$g = 9.8\,ms^{-2}$. What does this mean?

This means that an objects vertical speed or vertical velocity will increase by $9.8\,ms^{-1}$ every second.

eg An object is dropped from rest at the edge of a cliff.

At 0s ⇒ vertical speed/velocity = $0\,ms^{-1}$
1s ⇒ vertical speed/velocity = $9.8\,ms^{-1}$
2s ⇒ vertical speed/velocity = $19.6\,ms^{-1}$
3s ⇒ vertical speed/velocity = $29.4\,ms^{-1}$
etc

∴ after 5s ⇒ Vertical speed/Velocity = $49\,ms^{-1}$

## Ex1

**Q** A vehicle travelling at $15ms^{-1}$ then accelerates at $1.4ms^{-2}$ for 7 seconds.

Calculate the final speed of the vehicle.

**A**

$u = 15ms^{-1}$
$v = ?$
$a = 1.4ms^{-2}$
$s = /$
$t = 7s.$

Equation ① ⇒ $V = u + at$

$V = u + at$
$\Rightarrow V = 15 + 1.4 \times 7$
$\Rightarrow V = 15 + 9.8 = 24.8ms^{-1}$

## Ex2

**Q** A speed boat is initially travelling at $18ms^{-1}$. It then accelerates at $0.8ms^{-2}$ until it reaches a final speed of $22ms^{-1}$.

Calculate how far the speed boat travelled while it was accelerating.

**A**

$u = 18ms^{-1}$
$v = 22ms^{-1}$
$a = 0.8ms^{-2}$
$s = ?$
$t = /$

Equation ② ⇒ $v^2 = u^2 + 2as$

$v^2 = u^2 + 2as$
$\Rightarrow 22^2 = 18^2 + 2 \times 0.8 \times s$
$\Rightarrow 484 = 324 + 1.6s$
$\Rightarrow 1.6s = 484 - 324$
$\Rightarrow 1.6s = 160$
$\Rightarrow s = \frac{160}{1.6} = 100m$

## Ex 3

Q/ A tractor has an initial speed of $5ms^{-1}$ and accelerates at $0{\cdot}7ms^{-2}$ for 6·5 seconds.

Calculate how far the tractor has travelled in the 6·5 seconds.

A/

$u = 5ms^{-1}$
$v = /$
$a = 0{\cdot}7ms^{-2}$
$s = ?$
$t = 6{\cdot}5s$

Equation (3) $\Rightarrow s = ut + \frac{1}{2}at^2$

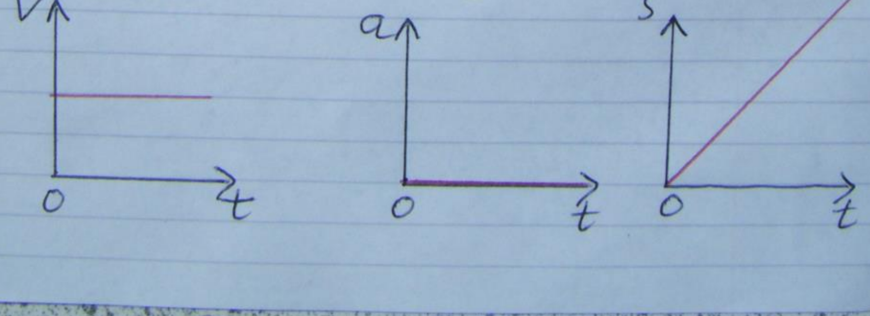
$$s = ut + \tfrac{1}{2}at^2$$
$$\Rightarrow s = 5 \times 6{\cdot}5 + \tfrac{1}{2} \times 0{\cdot}7 \times 6{\cdot}5^2$$
$$\Rightarrow s = 32{\cdot}5 + 14{\cdot}8$$
$$\Rightarrow s = 47{\cdot}3m$$

## Motion Graphs.

In Higher Physics deceleration is treated as a negative acceleration.

(Mathematically speaking acceleration in the reverse direction is the same as deceleration in the forward direction!!)

✳ 1) Constant Velocity



Constant velocity ⇒ zero acceleration

This means that the displacement is increasing at a constant rate.

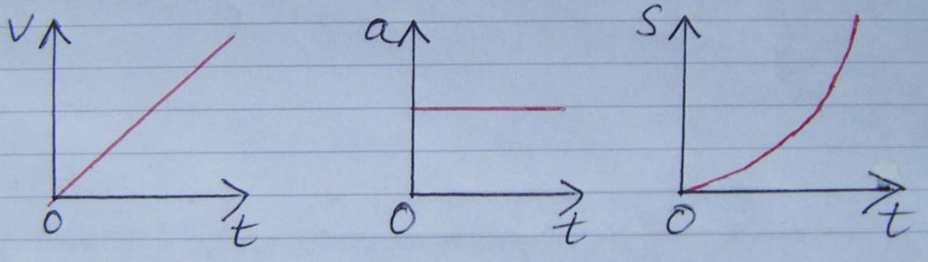
Calculus ⇒ $v = \frac{ds}{dt}$

⇓

differentiation.

$$a = \frac{dv}{dt} = \frac{d^2s}{dt^2}$$

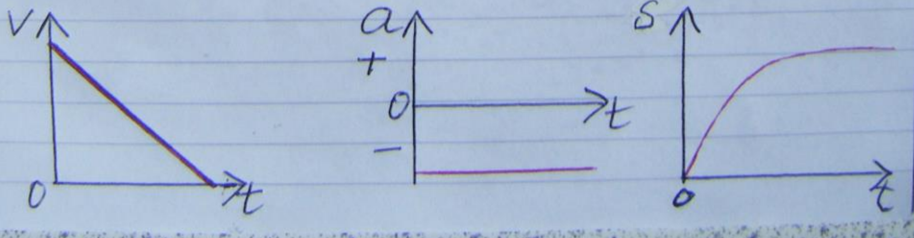
2) <u>Constant acceleration</u>




The velocity is increasing at a constant rate.

The displacement will increase at a non-constant rate ie the gradient continually increases.

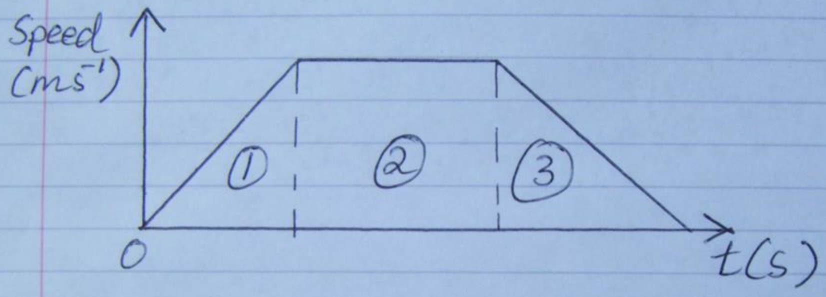
3) <u>Constant deceleration</u>



The velocity is decreasing at a constant rate.

The displacement will increase at a non-constant rate ie the gradient continually decreases.

## Speed-time graphs




The distance travelled can be found from the area under the speed-time graph.

distance travelled = ① + ② + ③

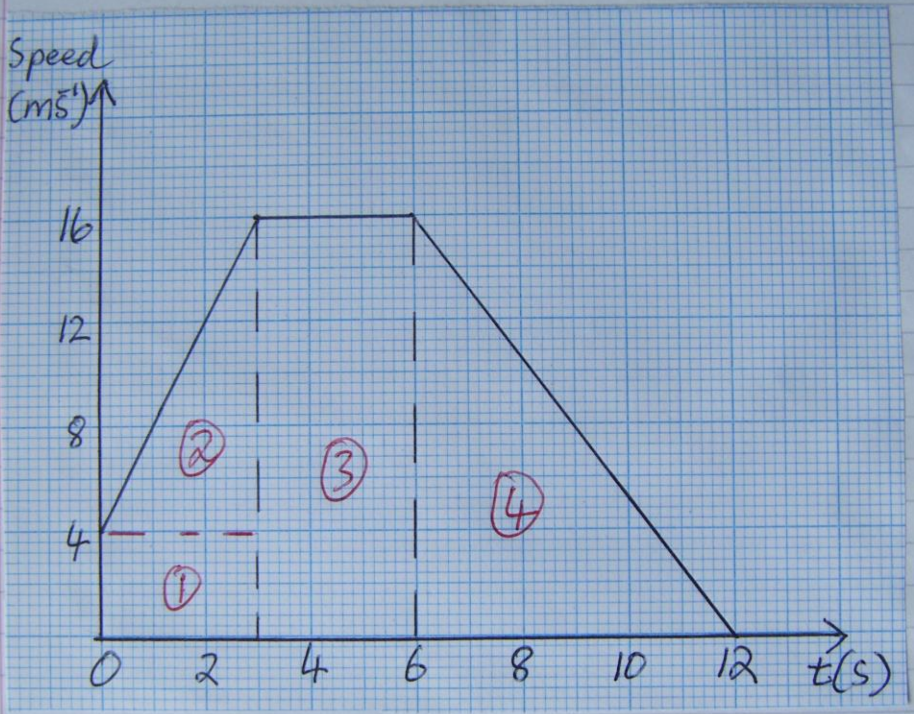
= △ + □ + ◺

Then the average speed can be found from

$$\text{average speed} = \frac{\text{distance travelled}}{\text{time taken}}$$

## Ex4

A speed-time graph was drawn which describes the motion of a motorcycle over a period of 12 seconds, as shown below.




Q. Calculate or find:

a) acceleration of the motorcycle
   i) 0 → 3s   ii) 3 → 6s   iii) 6 → 12s

b) i) The total distance travelled by the motorcycle over the 12s.

   ii) Average speed over the 12s.

A) a) i) $a = \frac{v-u}{t} = \frac{16-4}{3} = \frac{12}{3} = \underline{4ms^{-2}}$

ii) $a = \underline{0}$

iii) $a = \frac{v-u}{t} = \frac{0-16}{6} = \frac{-16}{6} = \underline{-2.67ms^{-2}}$

b) i) Total distance travelled = Area under the speed-time graph

$= ① + ② + ③ + ④$

$= (3 \times 4) + (\frac{1}{2} \times 3 \times 12) + (3 \times 16) + (\frac{1}{2} \times 6 \times 16)$

$= 12 + 18 + 48 + 48$

Total distance travelled $= \underline{126m}$

ii) Average speed $= \frac{\text{distance travelled}}{\text{time taken}}$

Average speed $= \frac{126}{12} = \underline{10.5ms^{-1}}$

Graphs involving velocity against time are used to find the displacement of the object in question.

As velocity and displacement are vector quantities then the direction of the moving object is extremely important.

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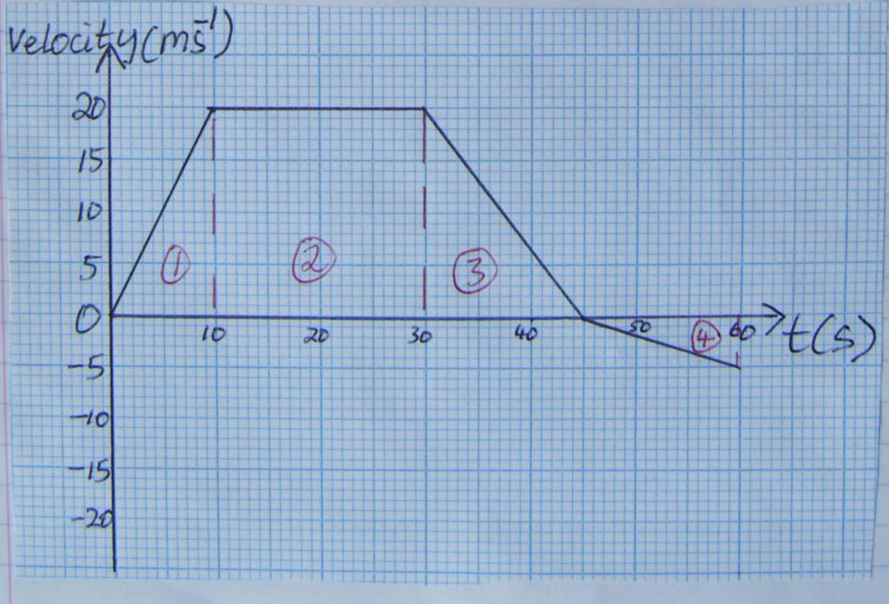
| Area under a velocity-time graph | = displacement |
| --- | --- |

## Ex5

The velocity-time graph shown below is of the motion of an object over a 60second time interval.



Q a) Describe the motion of the object from:

i) 0 → 10s

ii) 10 → 30s

iii) 30 → 45s

iv) 45 → 60s.

b) Calculate the total displacement of the object.

c) Calculate the magnitude of the average velocity over the 60s.

d) Draw the corresponding graph of acceleration versus time over the 60s using graph paper.

A a) i) Constant acceleration

ii) Constant Speed (NOT steady speed or steady pace)

iii) Constant negative acceleration (deceleration)

iv) Constant acceleration in the opposite direction.

b) Total displacement = Area under the velocity-time graph

= ① + ② + ③ + ④

$= (\frac{1}{2} \times 10 \times 20) + (20 \times 20) + (\frac{1}{2} \times 15 \times 20) + (\frac{1}{2} \times 15 \times -5)$

$= 100m + 400m + 150m - 37.5m$

$= \underline{612.5m}$

c) Average velocity $= \frac{\text{displacement}}{\text{time}} = \frac{612.5}{60} = \underline{10.2ms^{-1}}$

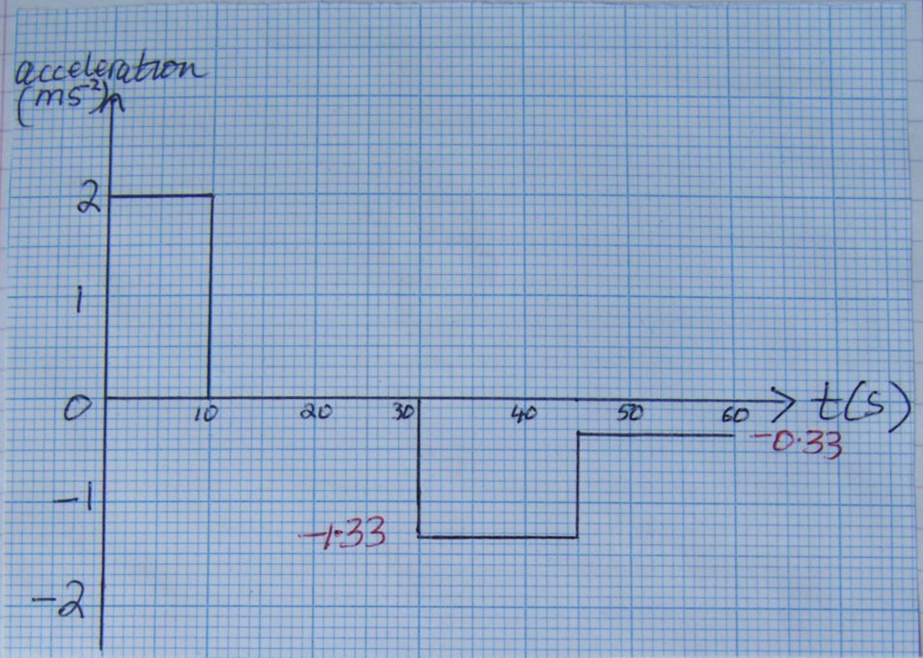
d) $0 \rightarrow 10s$, $a = \frac{v-u}{t} = \frac{20-0}{10} = \underline{2ms^{-2}}$

$10 \rightarrow 30s$, $\underline{a = 0}$

$30 \rightarrow 45s$, $a = \frac{v-u}{t} = \frac{0-20}{15} = \underline{-1.33ms^{-2}}$

$45 \rightarrow 60s$, $a = \frac{v-u}{t} = \frac{-5-0}{15} = \underline{-0.33ms^{-2}}$

* Remember that decelerating in the forward direction is mathematically equivalent to accelerating in the reverse direction. *




## Ex6

An object is initially travelling at $5ms^{-1}$, then for the next four seconds it accelerates at $1.5ms^{-2}$, then for the next eight seconds it accelerates at $2.5ms^{-2}$.

Q/ Calculate or find:

a) The velocity of the object after 4s.

b) The velocity of the object after 12s.

c) Draw a velocity-time graph over the 12s.

d) i) The displacement over the 12s.
ii) Average velocity over the 12s.

A a) $0 \rightarrow 4s$

$u = 5ms^{-1}$
$v = ?$
$a = 1{\cdot}5ms^{-2}$
$s = /$
$t = 4s$

$v = u + at$
$\Rightarrow v = 5 + 1{\cdot}5 \times 4$
$\Rightarrow v = 5 + 6$
$\underline{v = 11ms^{-1}}$

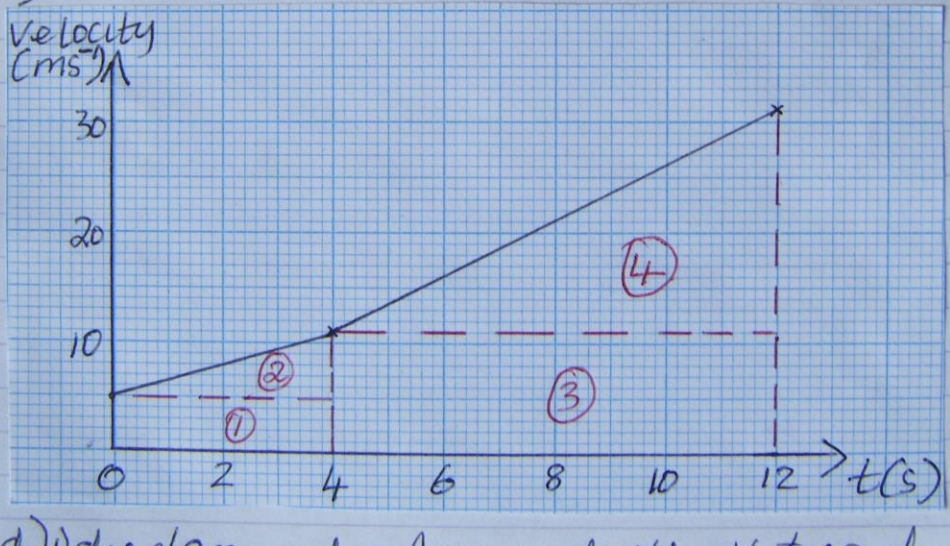
b) $4 \rightarrow 12s$.

$u = 11ms^{-1}$
$v = ?$
$a = 2{\cdot}5ms^{-2}$
$s = /$
$t = 8s$

$v = u + at$
$\Rightarrow v = 11 + 2{\cdot}5 \times 8$
$\Rightarrow v = 11 + 20$
$\underline{v = 31ms^{-1}}$

* Final velocity $0 \rightarrow 4s$ is the initial velocity between $4 \rightarrow 12s$. *

c).




d) i) displacement = Area under the v-t graph

= ① + ② + ③ + ④
$= (4 \times 5) + (\tfrac{1}{2} \times 4 \times 6) + (8 \times 11) + (\tfrac{1}{2} \times 8 \times 20)$
$= 20m + 12m + 88m + 80m = 200m$.

ii) Average velocity $= \dfrac{\text{displacement}}{\text{time}} = \dfrac{200}{12} = \underline{16{\cdot}7ms^{-1}}$

## The Bouncing Ball

A ball is dropped from rest at a height of 1 metre.

While at rest the ball possesses gravitational potential energy ($E_p$)

$$E_p = mgh$$

- $E_p$ → gravitational potential energy (Joules) → (J)
- $m$ → mass (Kg)
- $g$ → gravitational field strength ($NKg^{-1}$ or $ms^{-2}$)
- $h$ → height (m)

When the ball is released it will accelerate at $9.8ms^{-2}$, which is the gravitational field strength on Earth.

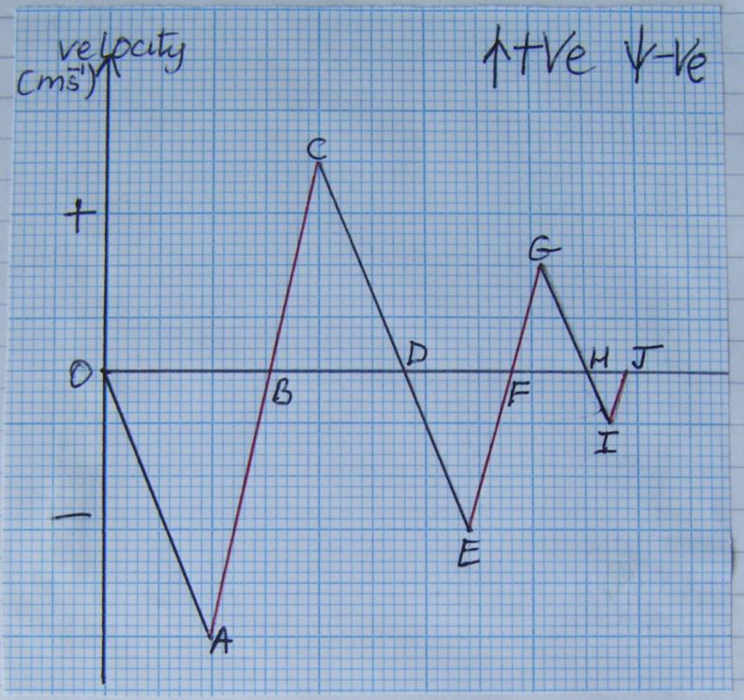
Just before hitting the ground the gravitational potential energy of the ball when released will be converted into Kinetic Energy ($E_K$).

$$E_K = \frac{1}{2}mv^2$$

- $E_K$ → Kinetic Energy (J)
- $m$ → mass (Kg)
- $v$ → Speed/Velocity ($ms^{-1}$)

* Remember to square the reading for speed or velocity *.

The graph below shows the velocity-time graph for a ball dropped from rest at a height.




* This graph could also be written with the opposite sign convention, with ↑-ve and ↓+ve. *

O → A ⇒ Ball is dropped from rest and accelerates until it hits the ground at A.

A → B ⇒ Ball decelerates to rest and changes shape while in contact with the ground.

B→C ⇒ Ball starts to regain its shape and start to accelerate from rest

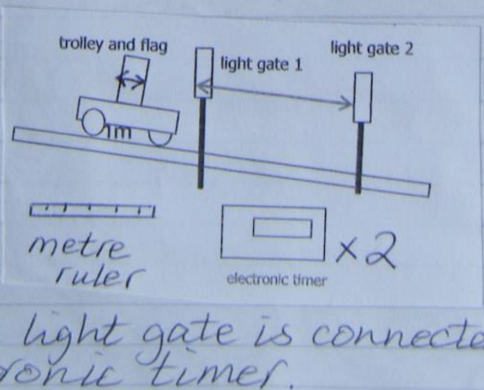
C→D ⇒ Ball leaves the ground and decelerates upwards until it reaches its max height at D.

This motion then repeats itself until the ball comes to rest.

## Measuring Acceleration.

Aim – To measure the acceleration of a trolley down a ramp.

Apparatus. – Ramp
Trolley and flag
2 light gates
2 Electronic Timers
A metre ruler




Each light gate is connected to an electronic timer.

## Method

- Set up the apparatus as shown above.

- Release the trolley from rest at the top of the ramp.

- $u = \frac{\text{length of flag}}{\text{time that light beam 1 is broken}}$

- $v = \frac{\text{length of flag}}{\text{time that light beam 2 is broken}}$

- measure the distance with a metre ruler from light beam (gate) 1 to light beam (gate) 2, $s$.

- The acceleration between light gate 1 and light gate 2 can be found using

$$v^2 = u^2 + 2as$$

$$\Rightarrow \frac{v^2 - u^2}{2s} = a$$

$$\Rightarrow \boxed{a = \frac{v^2 - u^2}{2s}}$$

(17)

# Ex 7 – 1999 HIGHER PAPER Q1

Q

**1.** (*a*) A sports car is being tested along a straight track.

(i) In the first test, the car starts from rest and has a constant acceleration of $4{\cdot}0\ \text{m s}^{-2}$ in a straight line for 7·0 seconds.

Calculate the distance the car travels in the 7·0 seconds.

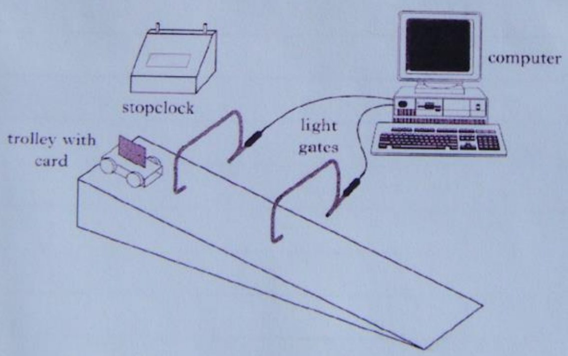
(ii) In a second test, the car again starts from rest and accelerates at $4{\cdot}0\ \text{m s}^{-2}$ over twice the distance covered in the first test.

What is the **increase** in the final speed of the car at the end of the second test compared with the final speed at the end of the first test?

(iii) In a third test, the car reaches a speed of $40\ \text{m s}^{-1}$. It then decelerates at $2{\cdot}5\ \text{m s}^{-2}$ until it comes to rest.

Calculate the distance travelled by the car while it decelerates to rest. **7**

(*b*) A student measures the acceleration of a trolley as it moves freely down a sloping track.




The trolley has a card mounted on it. As it moves down the track the card cuts off the light at each of the light gates in turn. Both the light gates are connected to the computer which is used for timing.

The student uses a stopclock to measure the time it takes the trolley to move from the first light gate to the second light gate.

(i) List all the **measurements** that have to be made by the student and the computer to allow the acceleration of the trolley to be calculated.

(ii) Explain fully how each of these measurements is used in calculating the acceleration of the trolley as it moves down the slope.

**3**

**(10)**

4. a) i) $u = 0$ $\quad s = ut + \frac{1}{2}at^2$
$v = ?$
$a = 4ms^{-2}$ $\quad \Rightarrow s = 0 + \frac{1}{2} \times 4 \times 7^2$
$s = ?$
$t = 7s$ $\quad \Rightarrow \underline{s = 98m}$

ii) First Test $\Rightarrow v = u + at$

$\Rightarrow v = 0 + 4 \times 7 = \underline{28ms^{-1}}$

Second Test $\Rightarrow s = 2 \times 98 = \underline{196m}$.

$u = 0$ $\quad v^2 = u^2 + 2as$
$v = ?$
$a = 4ms^{-2}$ $\quad \Rightarrow v^2 = 0^2 + 2 \times 4 \times 196 = 1568$
$s = 196m$
$t = ?$ $\quad \Rightarrow \underline{v = 39.6ms^{-1}}$

$\therefore$ Increase in speed $= 39.6 - 28 = \underline{11.6ms^{-1}}$

iii). $u = 40ms^{-1}$ $\quad v^2 = u^2 + 2as$
$v = 0$
$a = -2.5ms^{-2}$ $\quad \Rightarrow 0^2 = 40^2 + 2 \times -2.5 \times s$
$s = ?$ $\quad \Rightarrow 0 = 1600 - 5s$
$t = ?$ $\quad \Rightarrow 5s = 1600 \quad \Rightarrow s = \frac{1600}{5}$

$\Rightarrow \underline{s = 320m}$

b) i). 
- length of card
- time taken for the trolley to go from light gate 1 to light gate 2.
- The computer measures the times for the card to cut light beams 1+2.

11). $u = \dfrac{\text{length of Card}}{\text{time that light beam 1 is broken for}}$

$v = \dfrac{\text{length of Card}}{\text{time that light beam 2 is broken for}}$

$t$ = time for the trolley to go from light gate 1 to light gate 2 with the stopwatch.

Then $a = \dfrac{v-u}{t}$