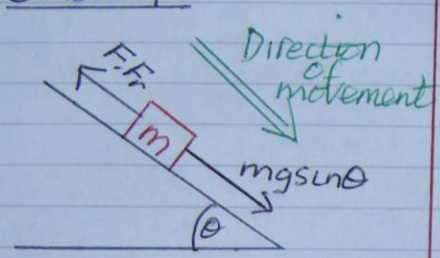
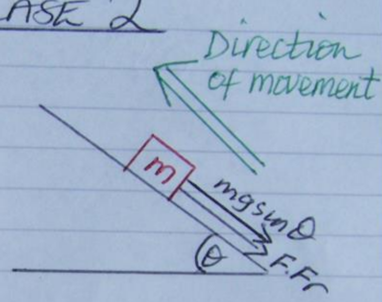
Forces acting at angles - B McMullen

We will look at the cases of an object of mass m moving up or down an inclined plane, at an angle θ to the horizontal.

✱ CASE 1



✱ CASE 2



- mgsinθ is the component of force or weight, which always acts down the slope.
- The force of friction F.Fr always acts in the opposite direction to the direction of movement.

Component of force/weight parallel to the slope ⇒ ✱ $\boxed{W_{\parallel} = mg\sin\theta}$ ✱

* CASE 1

• If $mg\sin\theta = F.Fr$, then the forces acting on the slope are balanced.

The object moving down the slope must then either be stationary or moving with a constant speed down the slope.

∴ F, the unbalanced force $= 0$.

• If $mg\sin\theta > F.Fr$, then the object will be accelerating down the slope.

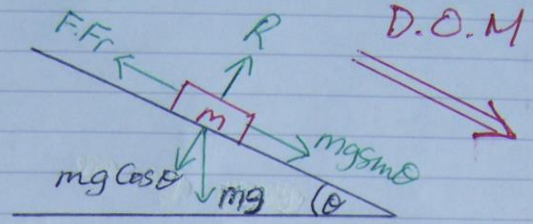
• If $mg\sin\theta < F.Fr$, then the object will be decelerating down the slope.

* CASE 2

• If the object is moving up the slope, then it will be decelerating until it comes to rest.

Why? As $mg\sin\theta$ and the force of friction F.Fr will be acting against the objects movement, as they are directed down the slope.

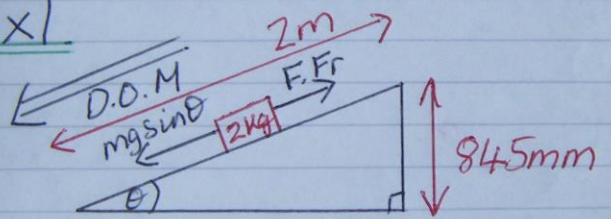
A more in-depth look at the forces.



R = Reaction force of the slope on m.

Component of force perpendicular to the slope = mgcosθ.
(normal to the slope)   ie * $W_{\perp r}$ = mgcosθ *

Ex1



The 2kg mass runs down a 2m slope which is inclined by 845mm.

Q a) Label all the forces acting parallel to the slope.

b) Calculate the angle of the slope.

c) Calculate the unbalanced force acting on the 2kg mass if the force of friction on the slope is 4·28N.

d) Calculate the acceleration of the 2kg mass.

e) Calculate what angle θ would be needed for the 2kg mass to go down the slope with a constant speed.

A. a) Listed on diagram.

b) Trigonometry (2m = 2000mm)

$$\sin\theta = \frac{O}{H} = \frac{845}{2000} = 0.4225$$

$$\therefore \theta = \sin^{-1}(0.4225) = \underline{25^\circ}$$

c) Component of force/weight, $W_{\parallel} = mg\sin\theta$
$\parallel$ to the slope

$$W_{\parallel} = 2 \times 9.8 \times \sin 25^\circ = \underline{8.28N}$$

Unbalanced Force, $F = mg\sin\theta - F.Fr = 8.28 - 4.28 = \underline{4N}$

d) $a = \frac{F}{m} = \frac{4}{2} = \underline{2ms^{-2}}$

e) Constant speed $\therefore a = 0 \therefore mg\sin\theta = F.Fr$

$$\therefore 4.28 = mg\sin\theta \Rightarrow 4.28 = 2 \times 9.8 \times \sin\theta$$

$$\Rightarrow \sin\theta = \frac{4.28}{2 \times 9.8} = \frac{4.28}{19.6} = 0.218$$

$$\therefore \theta = \sin^{-1}(0.218) = \underline{12.6^\circ}$$

CONSERVATION OF ENERGY

Ex2 (C STANDARD)

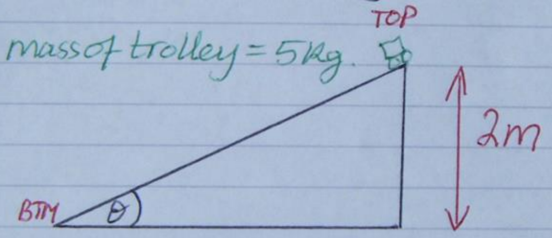
This involves a friction compensated slope where F.Fr = 0.

The energy conversion is from gravitational potential energy (E_p) into kinetic energy (E_K).

$$\boxed{E_p = mgh \Longrightarrow E_K = \tfrac{1}{2}mv^2}$$

$\Rightarrow E_p$ at top of slope $= E_K$ at btm of slope.

ie Loss in E_p = gain in E_K.



Calculate the speed of the trolley at the bottom of the slope.

E_p at top $= E_K$ at btm

$\Rightarrow mgh = \frac{1}{2}mv^2$

$\Rightarrow 5 \times 9{\cdot}8 \times 2 = \frac{1}{2} \times 5 \times v^2$

$\Rightarrow 98 = 2{\cdot}5v^2$

$\Rightarrow v^2 = \frac{98}{2{\cdot}5} = 39{\cdot}2 \quad \therefore v = \sqrt{39{\cdot}2} = \underline{6{\cdot}26\,ms^{-1}}$

Alternatively

$\cancel{m}gh = \frac{1}{2}\cancel{m}v^2$

$\Rightarrow v^2 = 2gh$

$\Rightarrow v^2 = 2 \times 9{\cdot}8 \times 2$

$\Rightarrow v^2 = 39{\cdot}2$

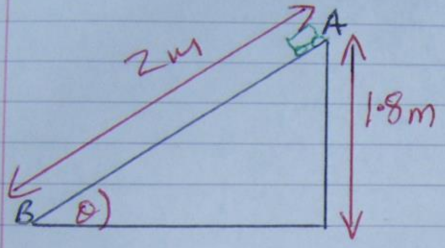
$\Rightarrow \underline{v = 6{\cdot}26\,ms^{-1}}$

Ex3 (A/B Standard)

Here we need to take into account the force of friction on the slope.

This relates to $E_W = F_{FR} \times d$

E_W: work done against friction (J)
F_{FR}: force of friction (N)
d: distance (m).



A 1kg trolley is released from rest at point A down a slope which is not friction compensated.
If the trolley reaches point B with a speed of $4{\cdot}39\,ms^{-1}$, then calculate or find:

Q a) The work done against friction as the trolley moves from A to B.

b) The force of friction, F_{Fr} acting on the 1kg trolley.

A a) $\boxed{E_P = E_K + E_W}$

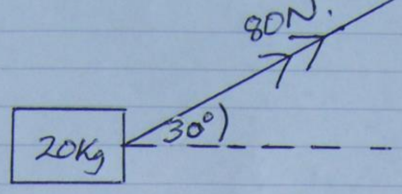
$E_P = mgh = 1 \times 9{\cdot}8 \times 1{\cdot}8 = \underline{17{\cdot}64J}$

$E_K = \frac{1}{2}mv^2 = \frac{1}{2} \times 1 \times 4{\cdot}39^2 = \underline{9{\cdot}64J}$

$E_W = 17{\cdot}64 - 9{\cdot}64 = \underline{8J}$

b) $E_W = F_{FR} \times d \Rightarrow F_{FR} = \frac{E_W}{d} = \frac{8}{2} = \underline{4N}$

Ex4



A 20Kg object is pulled by a force of 80N at an angle of 30° to the horizontal.

Q Calculate or find:

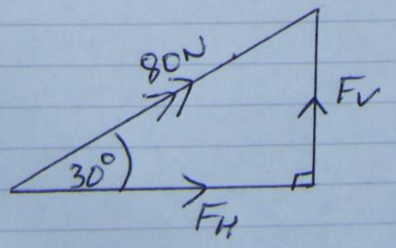
a) i) Horizontal component of force acting on the object.

ii) acceleration of the object

iii) Work done in pulling the object over a distance of 6m.

b) If the angle is reduced from 30° to 25° then how would this affect the work done calculated over the 6m.

A



A a) i) $\cos 30^\circ = \frac{A}{H} = \frac{F_H}{80}$

$\Rightarrow F_H = 80\cos 30^\circ = \underline{69.3N}$

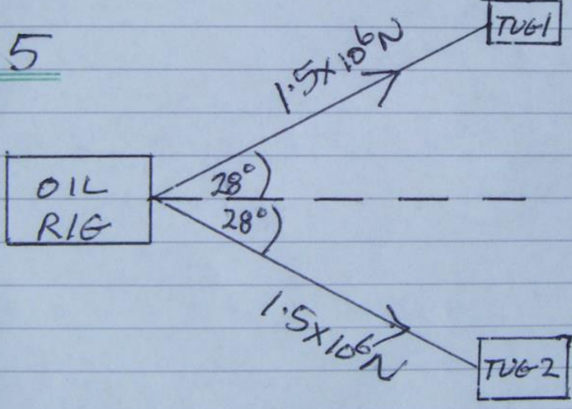
ii) $a = \frac{F_H}{m} = \frac{69.3}{20} = \underline{3.47ms^{-2}}$

iii) $E_W = F_H \times d = 69.3 \times 6 = \underline{416J}$

b) $\theta \Rightarrow 30^\circ \rightarrow 25^\circ \Rightarrow F_H \uparrow$

$\therefore$ As $F_H \uparrow \therefore E_W \uparrow$
Work done over the 6m increases.

Ex 5



Q a) Calculate the resultant force pulling the oil rig

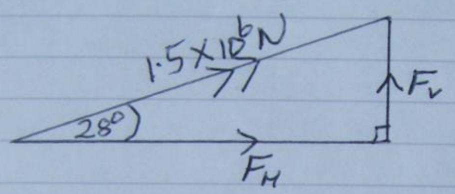
b) i) What would the force of friction of the water against the oil rig be if it was moving with a constant speed?

ii) Explain your answer in b) i).

A • The two triangles here have the same force acting at the same angle, above and below the horizontal.

• Find the horizontal force acting in the top triangle and then double it to find the resultant force.

a)



$$\cos 28^\circ = \frac{A}{H} = \frac{F_H}{1.5\times10^6} \Rightarrow F_H = 1.5\times10^6 \times \cos 28^\circ$$

$$\Rightarrow \underline{F_H = 1.32\times10^6 N}$$

M + D's

$\therefore F_{H\,TOTAL} = 2\times1.32\times10^6 = \underline{2.64\times10^6 N \text{ to the right}}$

b) • Constant speed $\therefore$ acceleration = 0

• acceleration = 0 $\therefore$ forces are balanced

• Forces are equal in magnitude and opposite in direction

M + D's

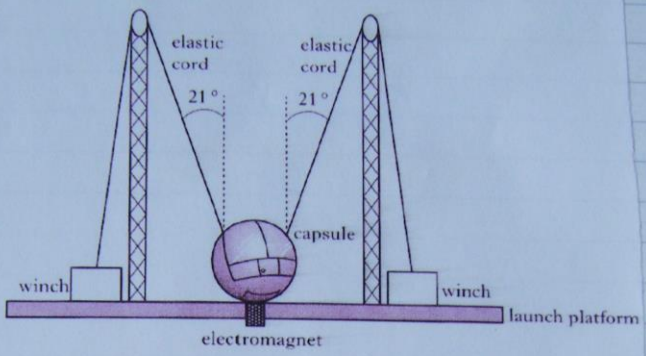
• Force of friction = $\underline{2.64\times10^6 N \text{ to the left}}$

EX6 – 2005 HIGHER PAPER Q22

(10)

Q//

22. A "giant catapult" is part of a fairground ride.



Two people are strapped into a capsule. The capsule and the occupants have a combined mass of 236 kg.

The capsule is held stationary by an electromagnet while the tension in the elastic cords is increased using the winches.

The mass of the elastic cords and the effects of air resistance can be ignored.

(*a*) When the tension in each cord reaches $4\cdot5 \times 10^3$ N the electromagnet is switched off and the capsule and occupants are propelled vertically upwards.

(i) Calculate the vertical component of the force exerted by **each** cord just before the capsule is released. **1**

(ii) Calculate the initial acceleration of the capsule. **3**

(iii) Explain why the acceleration of the capsule decreases as it rises. **1**

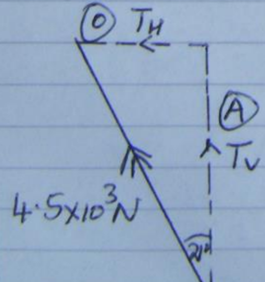
(*b*) Throughout the ride the occupants remain upright in the capsule.

A short time after release the occupants feel no force between themselves and the seats.

Explain why this happens. **1**

(6)

A//



a) i) $\cos 21^\circ = \frac{A}{H} = \frac{T_V}{4{\cdot}5\times10^3}$

$\Rightarrow T_V = 4{\cdot}5\times10^3 \times \cos 21^\circ$

$\Rightarrow \underline{T_V = 4{\cdot}2\times10^3 \text{N}}$

Vertical component of Tension. 11

a) ii) $\uparrow 2 \times T_v$ (Left and right chord) $\downarrow W$

- Total upward force $= 2T_v = 2 \times 4.2 \times 10^3 = \underline{8.4 \times 10^3 N} \uparrow$

- Total downward force $= W = mg = 236 \times 9.8 = \underline{2313N} \downarrow$

- Unbalanced force $= 8.4 \times 10^3 N \uparrow - 2.313 \times 10^3 \downarrow = \underline{6087N} \uparrow$

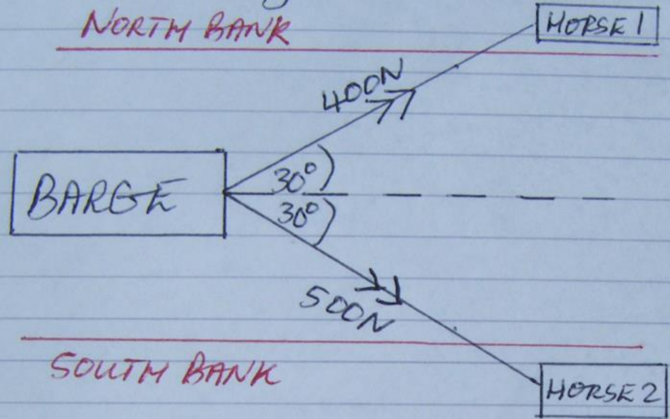
- acceleration, $a = \frac{F}{m} = \frac{6087}{236} = \underline{25.8 ms^{-2}}$

iii) The acceleration decreases due to the tension in the elastic chords decreasing. (angle $\theta \uparrow \therefore \cos\theta \downarrow$)

b) i) The seat and your body are in freefall towards the ground. They are both under the force of gravity, where the acceleration due to gravity is $9.8 ms^{-2}$ downwards.

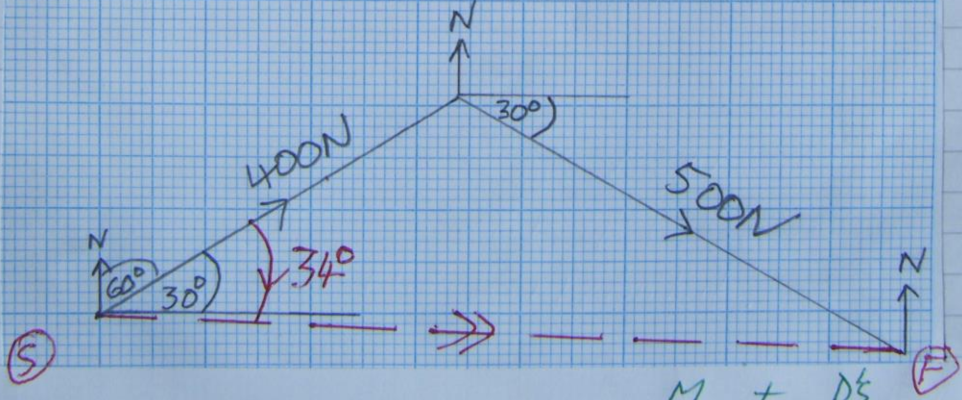
Ex 7 (Two non identical forces acting at the same angle)

A scale diagram is required here as the two forces do not have the same magnitude.



Calculate the resultant force pulling the barge.

Scale 1cm:50N



M + D's

Resultant Force = 780N at 4° below the horizontal.

* NO BEARINGS WITH FORCES!! *