Momentum and Impulse - B McMULLEN

- momentum is in loose terms the ability of an object to inflict damage.
- Momentum is the product of the mass of an object and the velocity that it is travelling at.
- momentum is a vector quantity → M + D's.

$$p = mv$$

- p — momentum ($kgms^{-1}$)
- m — mass (kg)
- v — Velocity (ms^{-1})

Ex 1

A person of mass 75kg runs with a speed of $8ms^{-1}$.

Calculate the momentum of the person.

$p = ?$
$m = 75kg$
$v = 8ms^{-1}$

$p = mv = 75 \times 8 = 735 kgms^{-1}$

* You need to mention a direction in the answer if it is given in the question *

Conservation of momentum

Total momentum before a collision = Total momentum after a collision

(This is provided that no external forces are acting)

ie T.M.B.C = T.M.A.C (Providing no external forces are acting)

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

m_1 → mass of object 1 (kg)
m_2 → mass of object 2 (kg)
u_1 → velocity of object 1 before the collision
u_2 → velocity of object 2 before the collision
v_1 → velocity of object 1 after the collision
v_2 → velocity of object 2 after the collision.

(ALL of the velocities in ms^{-1})

Ex2

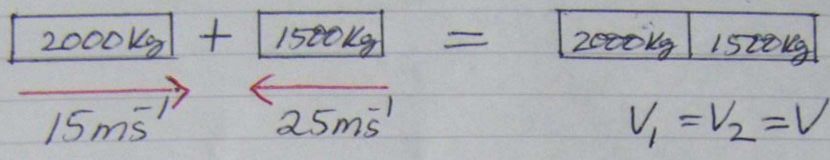
Q A car travelling along a road at $15ms^{-1}$ due East with a mass of 2000kg, collides head on with a car of mass 1500kg travelling at $25ms^{-1}$.

The cars join together after the collision.

Calculate the velocity of the cars. Combined mass immediately after the collision.

Remember direction with velocities

A T.M.B.C = T.M.A.C (Providing no external forces are acting)



$$m_1u_1 + m_2u_2 = m_1V_1 + m_2V_2$$

$$\Rightarrow (2000 \times 15) + (1500 \times -25) = 2000V + 1500V$$

$$\Rightarrow 30{,}000 - 37{,}500 = 3500V$$

$$\Rightarrow -7{,}500 = 3500V \Rightarrow v = \frac{-7500}{3500} = -2.14ms^{-1}$$

ie Combined mass moves with a speed of $2.14ms^{-1}$ due West. (ie -ve direction!!)

Ex3

Q A 700kg car moving at $20ms^{-1}$ due East runs into the back of a 1000kg car moving at $11.5ms^{-1}$.

If the cars lock together after impact, then calculate the cars common velocity after impact.

A T.M.B.C = T.M.A.C (Providing no external forces are acting)

[700Kg] + [1000Kg] = [700Kg | 1000Kg]

$20ms^{-1}$ → $11{\cdot}5ms^{-1}$ → $V_1 = V_2 = V$

$$m_1u_1 + m_2u_2 = m_1V_1 + m_2V_2$$

$$\Rightarrow (700 \times 20) + (1000 \times 11{\cdot}5) = 700V + 1000V$$

$$\Rightarrow 14{,}000 + 11{,}500 = 1700V$$

$$\Rightarrow 1700V = 25{,}500 \Rightarrow V = \frac{25{,}500}{1700} = 15ms^{-1}$$

ie combined mass moves at $15ms^{-1}$ due East after the collision.

Ex4

Q A white snooker ball collides with a black snooker ball as shown in the diagram below:

Before : ○ + ● — white $2ms^{-1}$ →, black Rest

After : ○ + ● — white $V_1 = ?$, black $3ms^{-1}$ →

If both balls have a mass of 160g, calculate the velocity of the white ball after the collision.

$(160g = 0.16kg)$

A T.M.B.C = T.M.A.C (Providing no external forces are acting)

$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

$\Rightarrow (0.16 \times 2) + (0.16 \times 0) = (0.16v_1) + (0.16 \times 3)$

$\Rightarrow 0.32 + 0 = 0.16v_1 + 0.48$

$\Rightarrow 0.32 - 0.48 = 0.16v_1$

$\Rightarrow 0.16v_1 = -0.16 \Rightarrow v_1 = \frac{-0.16}{0.16} = -1ms^{-1}$

ie The white ball backspins at (-)1ms^{-1} (backwards movement).

COLLISIONS

There are three types of collision:

Elastic, Inelastic and Explosion.

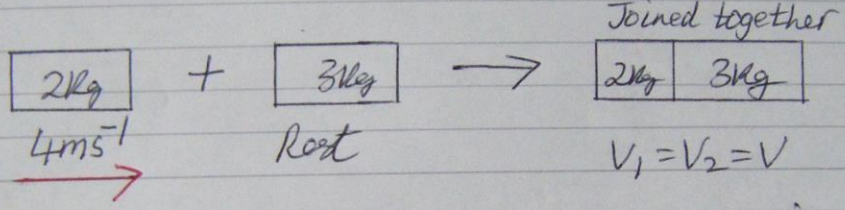
TYPE OF COLLISION	TOTAL MOMENTUM	TOTAL ENERGY	KINETIC ENERGY.
Elastic	Conserved	Conserved	Conserved
Inelastic	Conserved	Conserved	Not Conserved
Explosion	Conserved	Conserved	Gained.

By calculating the kinetic energy before and after a collision then you can deduce what type of collision is taking place.

(In reality Elastic collisions do not occur in practice, they are just a theoretical ideal!!!)

Ex5

Q



a) Calculate the velocity of the combined mass after the collision.

b) Show by calculation which type of collision is taking place.

A a) T.M.B.C = T.M.A.C (Providing no external forces are acting)

$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$

$\Rightarrow (2 \times 4) + (3 \times 0) = 2V + 3V$

$\Rightarrow 8 = 5V$

$\Rightarrow V = \frac{8}{5} = 1.6ms^{-1} \rightarrow$

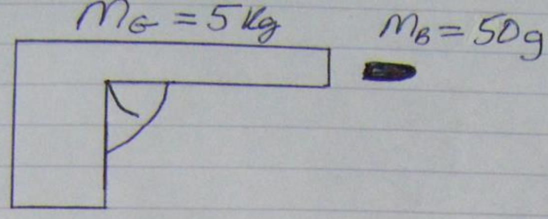
b) $E_{K\,before} = \frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2 = \frac{1}{2} \times 2 \times 4^2 + \frac{1}{2} \times 3 \times 0^2 = 16J$

$E_{K\,after} = \frac{1}{2} m_1 V_1^2 + \frac{1}{2} m_2 V_2^2 = \frac{1}{2} \times 2 \times 1.6^2 + \frac{1}{2} \times 3 \times 1.6^2 = 6.4J$

$\therefore E_{K\,before} > E_{K\,after}$ $\therefore$ E_K is not conserved

$\therefore$ Inelastic Collision.

Ex6

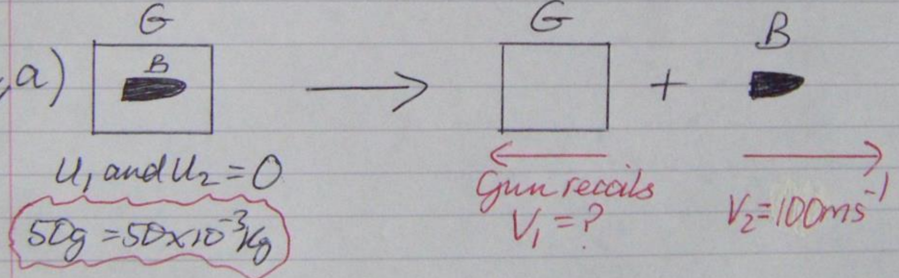


Q A bullet of mass 50g is fired from a gun of mass 5kg. If the bullet leaves the gun at $100ms^{-1}$, then calculate or find:

a) The recoil velocity of the gun.

b) The type of collision taking place.

A a)



T.M.B.C = T.M.A.C (Providing no external forces are acting)

$$\overset{G}{m_1u_1} + \overset{B}{m_2u_2} = \overset{G}{m_1V_1} + \overset{B}{m_2V_2}$$

$\Rightarrow (5\times0) + (50\times10^{-3}\times0) = 5V_1 + 50\times10^{-3}\times100$

$\Rightarrow \quad 0 = 5V_1 + 5$

$\Rightarrow \quad 5V_1 = -5 \quad \Rightarrow \quad \underline{V_1 = -1\ ms^{-1}}$

ie The gun recoils back at $1ms^{-1}$

b) $E_{k\,BEFORE} = \frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2$

$E_{k\,BEFORE} = \frac{1}{2}\times5\times0^2 + \frac{1}{2}\times50\times10^{-3}\times0^2 = \underline{0J}$

$E_{k\,AFTER} = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$

$= \frac{1}{2}\times5\times(-1)^2 + \frac{1}{2}\times50\times10^{-3}\times100^2$

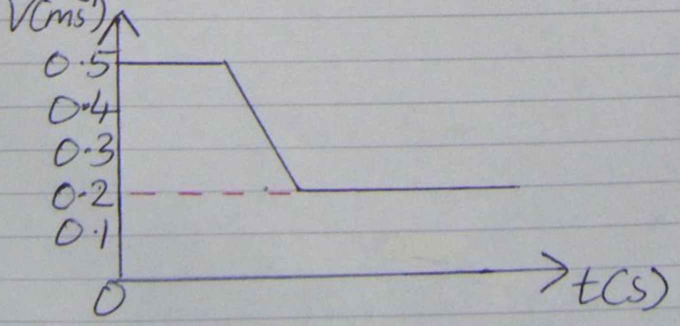
$E_{k\,AFTER} = 2.5J + 250J = \underline{252.5J}$

$E_{k\,AFTER} > E_{k\,BEFORE}$ ∴ An Explosion takes place.

Ex 7

Q A trolley of mass 2kg is moving with a constant velocity when it collides and sticks together with a second stationary trolley.

The graph below shows how the velocity of the 2kg trolley varies with time.



a) Calculate the mass of the second trolley.

b) Show by calculation what type of collision takes place between the trolleys.

A. a) T.M.B.C = T.M.A.C (Providing no external forces are acting)

$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

$\Rightarrow (2 \times 0.5) + 0 = (2 \times 0.2) + (m_2 \times 0.2)$

$\Rightarrow 1 = 0.4 + 0.2m_2$

$\Rightarrow 0.2m_2 = 1 - 0.4 = 0.6$

$\Rightarrow m_2 = \frac{0.6}{0.2} = \underline{\underline{3kg}}$

b) $E_{K\,BEFORE} = \frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2$

$= \frac{1}{2} \times 2 \times (0.5)^2 + 0 = \underline{\underline{0.25J}}$

$E_{K\,AFTER} = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$

$= \frac{1}{2} \times 2 \times (0.2)^2 + \frac{1}{2} \times 3 \times (0.2)^2$

$E_{K\,AFTER} = 0.04 + 0.06 = \underline{\underline{0.10J}}$

AS $E_{K\,BEFORE} > E_{K\,AFTER}$ ∴ INELASTIC COLLISION.

ie E_K IS NOT CONSERVED.

IMPULSE

In the Conservation of Momentum equation we saw that

T.M.B.C = T.M.A.C (No external forces)

However if an external average force, F, acts over a time interval t, then this is referred to as Impulse.

IMPULSE = AVERAGE FORCE ACTING × TIME OF CONTACT.

(Ns) (N) (S)

IMPULSE = Ft

Now

Impulse = Change in momentum

Why?

If a football is slowly rolled in front of a player, then its velocity will increase when it is kicked by the player.

O → The player exerts a force on the ball for a short time → O

$5ms^{-1}$ $20ms^{-1}$

⇓ Small momentum ⇓ Impulse ⇓ Large momentum

∴ Impulse is the difference in the momentum of the ball before and after it is kicked.

∴ Impulse = Change in momentum

$$Ft = mv - mu$$ ⇐ DB!!

Where does this equation come from?

$$F = ma$$

FROM NAT5 ⇒ $a = \frac{v-u}{t}$ (Variation of $v = u + at$)

⇒ $F = m\left(\frac{v-u}{t}\right)$ ⇒ $Ft = m(v-u)$

⇒ $Ft = mv - mu$

Units

Impulse = Ns * not Ns^{-1} *

change in momentum = $kgms^{-1}$

As Impulse = Change in momentum

$1\,Ns = 1\,kgms^{-1}$ ⇒ They are interchangable

Technically Impulse can be measured in $kgms^{-1}$ and change in momentum can be measured in Ns.

Ex 8

Q A bullet of mass 6g has a force of 3kN acting on it for 1·5ms in the barrel of the rifle.

Calculate the velocity of the bullet as it leaves the gun.

A
$F = 3kN = 3\times10^{3}N$
$t = 1{\cdot}5ms = 1{\cdot}5\times10^{-3}s$
$m = 6g = 6\times10^{-3}kg$
$V = ?$
$u = 0ms^{-1}$

$$Ft = mv - mu$$

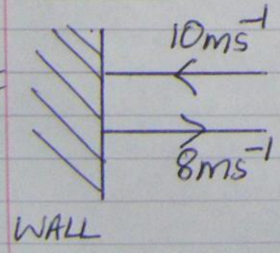
$$\Rightarrow 3\times10^{3}\times1{\cdot}5\times10^{-3} = 6\times10^{-3}\times V - 6\times10^{-3}\times 0$$

$$\Rightarrow \quad 4{\cdot}5 = 6\times10^{-3}V$$

$$\Rightarrow \quad V = \frac{4{\cdot}5}{6\times10^{-3}} = \underline{750ms^{-1}}$$

Ex 9

Q



A tennis ball of mass 60g hits a wall at $10ms^{-1}$ and rebounds from the wall at $8ms^{-1}$.

If the ball is in contact with the wall for 40ms, then calculate or find:

a) Change in momentum of the ball

b) The average contact force between the tennis ball and the wall.

A// a) $m = 60g = 60 \times 10^{-3} kg$
$u = 10 ms^{-1}$
$v = -8 ms^{-1}$ * (Direction change!!) *
$t = 40 ms = 40 \times 10^{-3} s$

Change in momentum $= mv - mu$

$= 60 \times 10^{-3} \times -8 - 60 \times 10^{-3} \times 10$

$= -0.48 - 0.60$

Change in momentum $= \underline{\underline{-1.08 kg m s^{-1}}}$

b) $Ft = mv - mu$

Impulse = Change in momentum

$\Rightarrow F \times 40 \times 10^{-3} = -1.08$

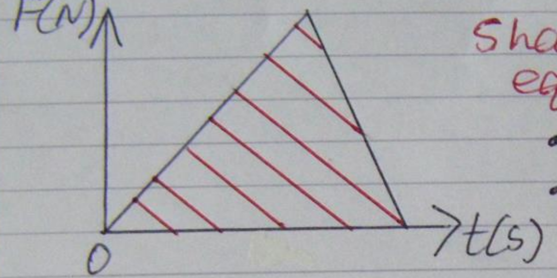
$\Rightarrow \quad F = -\frac{1.08}{40 \times 10^{-3}} = \underline{\underline{-27N}}$

* Take care with the signs, as velocity is a vector quantity. *

Why a negative force? This is due to the force of the wall on the ball.

Impulse Graphs

The Impulse and the Change in momentum can be found from the area under a force-time graph.

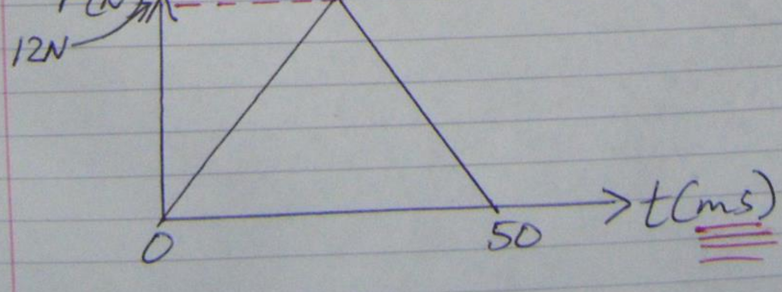


Shaded area is equal to either
- Impulse or
- Change in momentum.

* Take care as the time axis is often measured in milliseconds ($\times 10^{-3}$ s) *

Ex10

A stationary snooker ball of mass 200g is struck by a cue. The graph below shows how the force varies over the 50ms time of contact between the ball and the cue.



Q Calculate or find:

a) The change in momentum of the ball from the graph.

b) The velocity of the snooker ball as it leaves contact with the cue.

A a) Change in momentum = Area under the F-t graph

(Change ms → s !!)

$= \frac{1}{2} \times 50 \times 10^{-3} \times 12$

change in momentum $= 0.3 \text{ Kgms}^{-1}$

b) Impulse = Change in momentum

$\Rightarrow Ft = mv - mu$

$\Rightarrow 0.3 = 0.2 \times V - 0.2 \times 0$

$\Rightarrow 0.3 = 0.2V \Rightarrow V = \frac{0.3}{0.2} = 1.5 \text{ms}^{-1}$

Why should you wear a crash helmet on a bike?

The crash helmet will soften the blow on your skull.

Although it may surprise you that the skull will be in contact with the road surface for a longer time.

In Physics terms

- The change in momentum of your skull when hitting the ground is constant.
- The impulse on your skull hitting the ground is also constant.

(Impulse = Change in momentum)

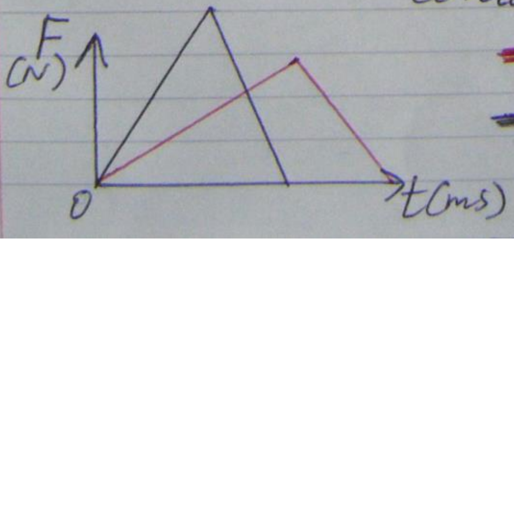
- The time of contact with your skull on the ground increases with the helmet on.
- As Impulse (↔) = Average force (↓) x Time of contact (↑).

then the average force exerted on the skull will decrease with the helmet on.

∴ Less damage to the skull.

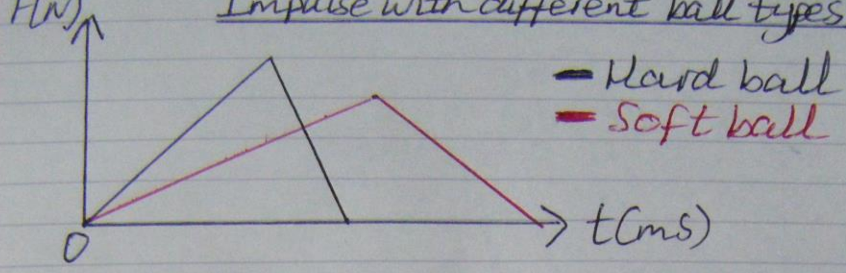
Summary

- Helmet on ⇒ Time of Contact ↑ + Force on skull ↓
- Helmet off ⇒ Time of Contact ↓ + Force on skull ↑



— Helmet on
— Helmet off.

Impulse with different ball types.



Briefly

- change in momentum is constant
- Impulse is constant
- Harder ball exerts a greater force on the ground.
- Since Impulse = Ft
  then the harder ball is in contact with the ground for a shorter time.

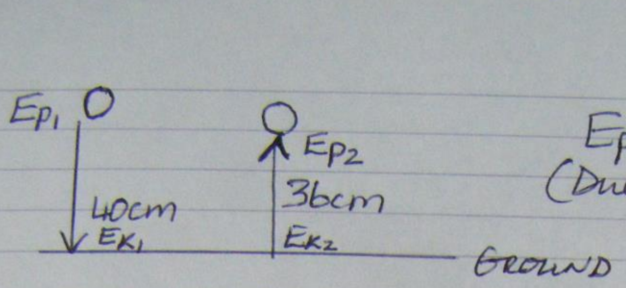
Ex 11

A ball of mass 50g falls from a height of 40cm on to the ground and rebounds to a height of 36cm. If the time of contact between the ball and the ground is 24ms, then calculate or find:

Q a) The change in momentum of the ball when in contact with the ground.

b) The average force exerted on the ball by the ground.

A



$E_{P_1} > E_{P_2}$
(Due to the greater height.)

a) • Firstly find the velocity of the ball as it hits the ground.

• Secondly find the velocity of the ball as it leaves the ground.

• Firstly ↓

E_{P_1} at the top $= E_{K_1}$ when hitting the ground.

$\Rightarrow mgh_1 = \frac{1}{2}mv_1^2 \Rightarrow 2mgh_1 = mv_1^2$

$\Rightarrow v_1^2 = 2gh_1 = 2 \times 9.8 \times 0.4 = 7.84$

$\Rightarrow v_1 = \sqrt{7.84} = 2.8ms^{-1}$ ↓ ($-2.8ms^{-1}$)

• Secondly ↑

E_{K_2} at the bottom $= E_{P_2}$ at the top

$\Rightarrow \frac{1}{2}mv_2^2 = mgh_2 \Rightarrow mv_2^2 = 2mgh_2$

$\Rightarrow v_2^2 = 2gh_2 = 2 \times 9.8 \times 0.36 = 7.056$

$\Rightarrow v_2 = \sqrt{7.056} = 2.66ms^{-1}$ ↑ ($+2.66ms^{-1}$)

Change in momentum $= mv - mu$

$= mv_2 - mv_1$

$= (50\times10^{-3}\times -2.8) - (50\times10^{-3}\times2.66)$

$= -0.14 - 0.133$

Change in momentum $= -0.273\,\text{Kgms}^{-1}$

b) Impulse = Change in momentum

$\Rightarrow \quad F\times24\times10^{-3} = -0.273$

$\Rightarrow F = \dfrac{-0.273}{24\times10^{-3}} = -11.4\,\text{N}$

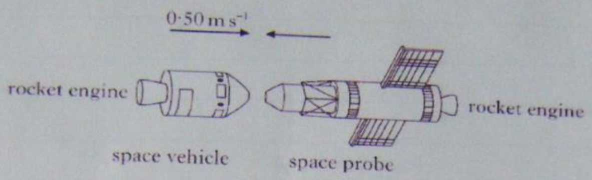
(The force is negative as it acts in the opposite direction to the original motion of the ball.)

EX12 – 2002 HIGHER PAPER Q23

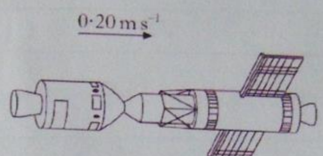
(20)

Q

23. (*a*) A space vehicle of mass 2500 kg is moving with a constant speed of $0{\cdot}50\,m\,s^{-1}$ in the direction shown. It is about to dock with a space probe of mass 1500 kg which is moving with a constant speed in the opposite direction.



After docking, the space vehicle and space probe move off together at $0{\cdot}20\,m\,s^{-1}$ in the original direction in which the space vehicle was moving.



Calculate the speed of the space probe before it docked with the space vehicle. **2**

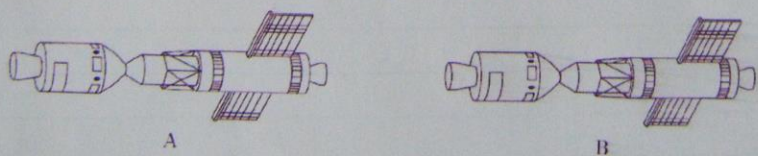
(*b*) The space vehicle has a rocket engine which produces a constant thrust of 1000 N. The space probe has a rocket engine which produces a constant thrust of 500 N.

The space vehicle and space probe are now brought to rest from their combined speed of $0{\cdot}20\,m\,s^{-1}$.

(i) Which rocket engine was switched on to bring the vehicle and probe to rest?

(ii) Calculate the time for which this rocket engine was switched on. You may assume that a negligible mass of fuel was used during this time. **3**

(*c*) The space vehicle and space probe are to be moved from their stationary position at A and brought to rest at position B, as shown.



Explain clearly how the rocket engines of the space vehicle and the space probe are used to complete this manoeuvre.

Your explanation must include an indication of the relative time for which each rocket engine must be fired.

You may assume that a negligible mass of fuel is used during this manoeuvre. **2**

(7)

A a) T.M.B.C = T.M.A.C (Providing no external forces are acting)

$\Rightarrow m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

$(v_1 = v_2 = v)$

$\Rightarrow (2500 \times 0.5) + (1500u_2) = (2500 \times 0.2) + (1500 \times 0.2)$

$\Rightarrow 1250 + 1500u_2 = 500 + 300$

$\Rightarrow 1500u_2 = 800 - 1250 = -450$

$\Rightarrow u_2 = \frac{-450}{1500} = \underline{-0.3ms^{-1}}$

b) i). Space Probe

ii) $Ft = mv - mu$

$\Rightarrow -500t = 4000 \times 0 - 4000 \times 0.2$

Direction !!!

$\Rightarrow -500t = -800$

$\Rightarrow t = \frac{-800}{-500} = \underline{1.6s}.$

c) • Fire rocket engine of space vehicle

• Then fire the probe engine for twice as long (As it only has half of the force exerted.)