Projectile Motion - B McMULLEN

This involves the motion of an object which moves freely through the air.

There are 2 components of the objects motion: Horizontal and vertical.

These two components of motion are totally independant of each other.

Motion Upwards	Motion Downwards
↑ All quantities are positive	↓ All quantities are negative.

The only force acting on an object in projectile motion is the force of gravity, which acts vertically downwards.

(We usually assume that the frictional forces in air = 0, unless told otherwise!)

In attempting any projectiles questions the following framework must be used:

V	H
$u_V =$	$u_H =$
$v_V =$	$v_H =$
$a_V = -9.8 ms^{-2}$	$a_H = 0$
$s_V =$	$s_H =$
$t =$	$t =$

} Same

NB The time t is the only quantity without a V or H subscript.

Ex 1

Q// A stone is dropped down a water well from rest and hits the water surface 3 seconds later.

Calculate the vertical velocity of the stone as it hits the water surface.

A// V

$u_V = 0$
$v_V = ?$
$a_V = -9.8 ms^{-2}$
$s_V = /$
$t = 3s$

No horizontal column is required in this question !!

$v_V = u_V + a_V t$

$\Rightarrow v_V = 0 + (-9.8) \times 3$

$\Rightarrow \underline{v_V = -29.4 ms^{-1}}$

Vertical velocity is $29.4 ms^{-1}$ downwards.

Ex2

A player kicks a football vertically upwards, with the ball leaving his foot at $18ms^{-1}$.

Calculate or find:

Q a) How long the ball took to reach its maximum height?

b) What the maximum height reached by the ball is?

A

V

$u_V = 18ms^{-1}$
$v_V = 0$ (at max height)
$a_V = -9{\cdot}8ms^{-2}$
$s_V = ?\ 16{\cdot}5m$
$t = ?\ 1{\cdot}84s$

No horizontal column is required in this question.

a) $v_V = u_V + a_V t$

$\Rightarrow 0 = 18 + (-9{\cdot}8)t \quad \Rightarrow 9{\cdot}8t = 18$

$\Rightarrow t = \frac{18}{9{\cdot}8} = \underline{1{\cdot}84s}$

b) $s_V = u_V t + \frac{1}{2}a_V t^2$

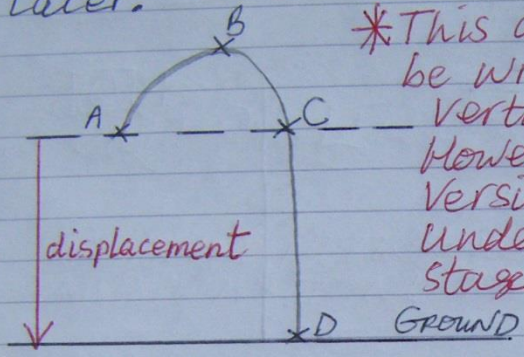
$\Rightarrow s_V = 18 \times 1{\cdot}84 + \frac{1}{2}(-9{\cdot}8) \times 1{\cdot}84^2$

$\Rightarrow s_V = 33{\cdot}1 - 16{\cdot}6 = \underline{16{\cdot}5m}$

$v_V^2 = u_V^2 + 2a_V s_V$ could also be used here !!

Ex 3

A balloon is climbing vertically upwards with a velocity of $5ms^{-1}$. A sandbag is then dropped from the balloon and hits the ground 4s later.



This diagram should be written completely vertical. However this expanded version helps with the understanding at each stage.

A → Sandbag is released with a vertical velocity of $+5ms^{-1}$ (ie $5ms^{-1}$ upwards)

B → Sandbag reaches it maximum height (decelerated to rest at its max height)

C → Sandbag is now accelerating downwards and has a vertical velocity of $-5ms^{-1}$.

D → At the instant before the sandbag hits the ground, the vertical velocity has a large negative value.

Q Calculate or find:

a) The velocity of the sandbag as it hits the ground.

b) The height of the sandbag above the ground at the instant that it is dropped from the balloon.

A

V

$u_v = +5 ms^{-1}$
$v_v = ? \; -34.2 ms^{-1}$
$a_v = -9.8 ms^{-2}$
$s_v = ?$
$t = 4s$

No horizontal column is required here!!

a) $v_v = u_v + a_v t$

$\Rightarrow v_v = 5 + (-9.8) \times 4$

$\Rightarrow v_v = 5 - 39.2 = \underline{-34.2 ms^{-1}}$

ie $34.2 ms^{-1}$ vertically downwards.

b) $s_v = u_v t + \frac{1}{2} a_v t^2$

$\Rightarrow s_v = 5 \times 4 + \frac{1}{2}(-9.8) \times 4^2$

$\Rightarrow s_v = 20 - 78.4$

$\underline{s_v = -58.4m}$

The sandbag drops 58.4m to hit the ground.

Ex4

A person stands on the edge of a cliff and projects a football horizontally with a velocity of 18ms^{-1}.

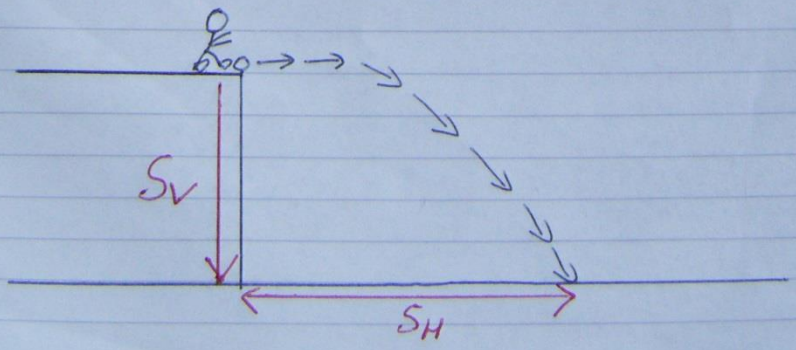
If it takes 2·5seconds for the football to reach the ground then calculate or find:

Q a) The horizontal velocity of the ball just before it hits the ground.

b) The vertical velocity of the ball just before it hits the ground.

c) The horizontal range of the ball.

d) The height of the cliff above the ground.



A a)

V	H
$u_V = 0$	$u_H = 18ms^{-1}$
$v_V = ?$	$v_H = ?\ 18ms^{-1}$ } Same
$a_V = -9.8ms^{-2}$	$a_H = 0$
$s_V = ?$	$s_H = ?$
$t = 2.5s$	$t = 2.5s$

$u_H = v_H = 18ms^{-1}$ ie horizontal component of velocity is always constant.

b) $v_V = u_V + a_V t = 0 + (-9.8) \times 2.5$

$\Rightarrow \underline{v_V = -24.5ms^{-1}}$ ($24.5ms^{-1}$ downwards)

c) $s_H = u_H t + \frac{1}{2} a_H t^2$

$\Rightarrow s_H = 18 \times 2.5 + 0$

$\Rightarrow \underline{s_H = 45m}$

d) $s_V = u_V t + \frac{1}{2} a_V t^2$

$\Rightarrow s_V = 0 + \frac{1}{2} \times -9.8 \times 2.5^2$

$\Rightarrow \underline{s_V = -30.6m}$

(ie 30.6m downwards)

EXTRA

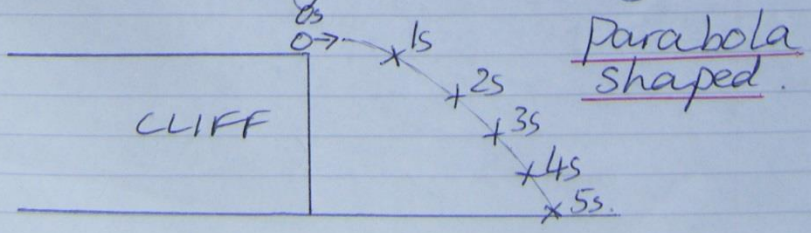
An object is projected from a cliff with a horizontal velocity of $12 ms^{-1}$ and hits the ground 5 seconds later.

The following table shows the horizontal and vertical velocity of the object over the 5 seconds.

Time (s)	Horizontal velocity (ms^{-1})	Vertical Velocity (ms^{-1})
0	12	0
1	12	−9.8
2	12	−19.6
3	12	−29.4
4	12	−39.2
5	12	−49.0

Conclusion

- Hor. component of velocity is always constant
- Vertical component of velocity increases by $-9.8 ms^{-1}$ every second.



Parabola shaped.

Ex5

A person is standing on a roof 12m above ground level. He then horizontally projects a ball with his arms which are 0.5m above roof level. The ball then covers a horizontal displacement of 35m from where it was projected.

Q Calculate or find:

a) The time of flight of the ball.

b) The horizontally velocity that the ball is projected with.

c) The vertical velocity that the ball hits the ground with.

d) The resultant velocity of the ball on impact with the ground.

A

FRAMEWORK

V	H	
$u_V = 0$	$u_H = ?\ 21.9 ms^{-1}$	} Same
$v_V = ?$	$v_H = ?\ 21.9 ms^{-1}$	
$a_V = -9.8 ms^{-2}$	$a_H = 0 ms^{-2}$	
$s_V = -12.5m$	$s_H = 35m$	
$t = ?\ 1.60s$	$t = ?\ 1.60s$	

(10)

a) $S_V = u_V t + \frac{1}{2} a_V t^2$

$\Rightarrow -12.5 = 0 + \frac{1}{2} \times (-9.8) \times t^2$

$\Rightarrow -12.5 = -4.9t^2 \Rightarrow t^2 = \frac{-12.5}{-4.9}$

$\Rightarrow t^2 = 2.55 \Rightarrow \underline{t = 1.60s}$

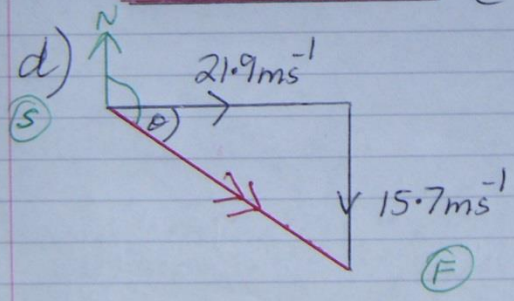
b) $S_H = u_H t + \frac{1}{2} a_H t^2$

$\Rightarrow 35 = u_H \times 1.6 + 0 \Rightarrow u_H = \frac{35}{1.6} = \underline{21.9 ms^{-1}}$

c) $V_V = u_V + a_V t = 0 + (-9.8) \times 1.60$

$\Rightarrow \underline{V_V = -15.7 ms^{-1}}$ ($15.7 ms^{-1}$ downwards)

d)



Resultant Velocity
$\Rightarrow$ M+D's

M $\Rightarrow SF^2 = 21.9^2 + 15.7^2 = 726.1$

$\Rightarrow \underline{SF = 26.9 ms^{-1}}$

D $\Rightarrow$ Bearing $= 090° + \theta°$ $\Rightarrow \tan\theta = \frac{O}{A} = \frac{15.7}{21.9}$

$\Rightarrow$ Bearing $= 090° + 36° = \underline{126°}$ $\Rightarrow \tan\theta = 0.717$

M + D's $\Rightarrow \theta = 35.6°$

$\therefore \theta = 36°$ (Bearings)

$\underline{V_R = 26.9 ms^{-1} @ 126°}$

Ex6

An athlete projects the javelin at 35° to the horizontal with a velocity of $28ms^{-1}$.

Q Calculate or find:

a) i) Initial horizontal component of velocity of the javelin.

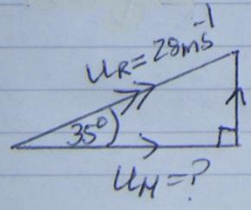
ii) Initial vertical component of velocity of the javelin

b) The maximum height reached by the javelin

c) The horizontal displacement of the javelin.

d) The resultant velocity of the javelin after 2.20s.

A



Cos to go across and Sine to climb.

a) i) $\cos 35^\circ = \frac{u_H}{28} \Rightarrow u_H = 28\cos 35^\circ = \underline{22.9ms^{-1}}$

ii) $\sin 35^\circ = \frac{u_V}{28} \Rightarrow u_V = 28\sin 35^\circ = \underline{16.1ms^{-1}}$

FRAMEWORK

b)

V	H
$u_V = 16.1 ms^{-1}$	$u_H = 22.9 ms^{-1}$ } Same
$v_V = 0 ms^{-1}$ (max height)	$v_H = 22.9 ms^{-1}$ } Same
$a_V = -9.8 ms^{-2}$	$a_H = 0$
$s_V = ? 13.2m$	$s_H = ?$
$t = ? 1.64s$	$t = ? 3.28s$

Max height reached $\Rightarrow v_V^2 = u_V^2 + 2a_V s_V$

$\Rightarrow 0^2 = 16.1^2 + 2 \times -9.8 \times s_V$

$\Rightarrow 0 = 259.2 - 19.6 s_V \Rightarrow 19.6 s_V = 259.2$

$\Rightarrow s_V = \frac{259.2}{19.6} = \underline{13.2m}$

c) Horizontal displacement $\Rightarrow$ 2 stages as time needs to be found first.

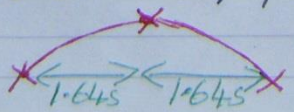
- Time can be found using the vertical components

$v_V = u_V + a_V t \Rightarrow 0 = 16.1 + (-9.8)t \Rightarrow 9.8t = 16.1$

$\Rightarrow t = \frac{16.1}{9.8} = \underline{1.64s}$.

Time to reach max height = 1.64s

$\therefore$ Total time of flight = $1.64s \times 2 = \underline{3.28s}$



• $s_H = u_H t + \frac{1}{2} a_H t^2$

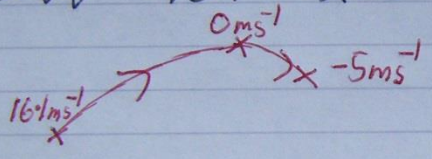
$\Rightarrow s_H = 22.9 \times 3.28 + 0 = \underline{75.1\,m}$

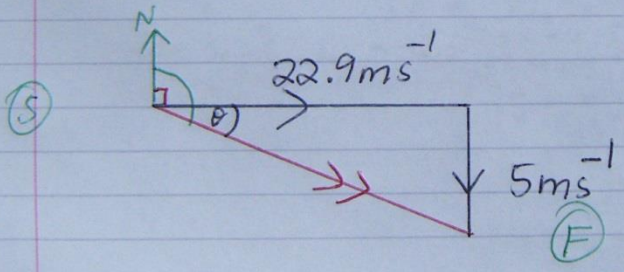
d) Resultant velocity after 2.20s.

$u_H = v_H = \underline{22.9\,ms^{-1}}$

$v_V = u_V + at = 16.1 + (-9.8) \times 2.20$

$\Rightarrow v_V = 16.1 - 21.6 = \underline{-5\,ms^{-1}}$ ($5\,ms^{-1}$ downwards)





Resultant velocity after 2.20s $\Rightarrow$ M + D's.

M $\Rightarrow SF^2 = 22.9^2 + 5^2 = 549.41$

$\Rightarrow \underline{SF = 23.4\,ms^{-1}}$

D $\Rightarrow \tan\theta^\circ = \frac{O}{A} = \frac{5}{22.9} = 0.218 \quad \therefore \theta = 12.3^\circ$

12° for bearings

Bearing $= 090^\circ + 12^\circ = 102^\circ$

M + D's.

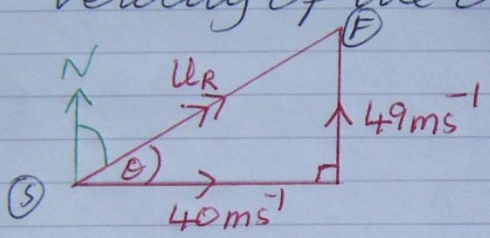
$\underline{V_R = 23.4\,ms \text{ @ } 102^\circ}$

Ex7

An object is projected from the ground at an angle θ° with $u_H = 40ms^{-1}$ and $u_V = 49ms^{-1}$.

Q a) i) Calculate the angle of projection θ°.

ii) Calculate the initial resultant velocity of the object.



A a) i) $\tan\theta^\circ = \frac{O}{A} = \frac{49}{40} = 1.225$

$\therefore \theta^\circ = \tan^{-1}(1.225) = \underline{50.7^\circ}$

(51° for bearings).

ii) Resultant Velocity $\Rightarrow$ M+D's.

M $\Rightarrow SF^2 = 40^2 + 49^2 = 4001$

$\Rightarrow \underline{SF = 63.3ms^{-1}}$

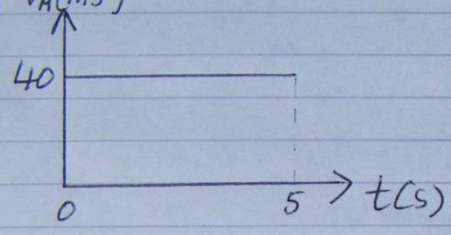
D $\Rightarrow$ Bearing $= 090^\circ - 51^\circ = \underline{039^\circ}$

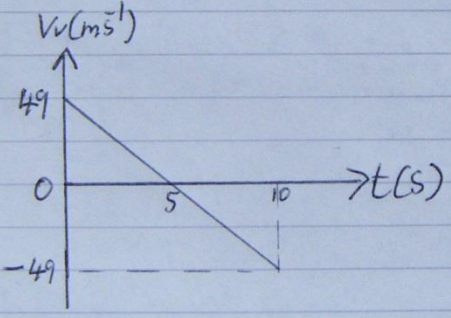
$\underline{V_R = 63.3ms^{-1} @ 039^\circ}$

Q// b) Draw the graphs of:

i) Horizontal velocity against time
and
ii) Vertical velocity against time

if it takes the object 5 seconds to reach its maximum height.

A// b) i) 

ii) 

Q// c) i) Calculate the max height reached by the object.

ii) Calculate the horizontal range of the object.

FRAMEWORK

A c) i)

V	H
$u_V = 49ms^{-1}$	$u_H = 40ms^{-1}$ }
$V_V = 0$ (max height)	$V_H = 40ms^{-1}$ } Same
$a_V = -9.8ms^{-2}$	$a_H = 0$
$S_V = ?$	$S_H = ?$
$t = 5s$	$t = 10s$.

i) $S_V = u_V t + \frac{1}{2} a_V t^2$

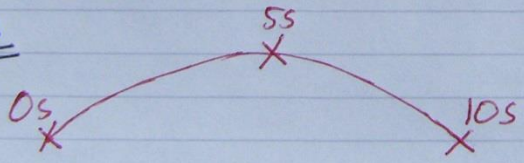
$\Rightarrow S_V = 49 \times 5 + \frac{1}{2} \times -9.8 \times 5^2$

$\Rightarrow S_V = 245 - 122.5 = \underline{122.5m}$

ii) $S_H = u_H t + \frac{1}{2} a_H t^2 = 40 \times 10 + 0$

$\Rightarrow \underline{S_H = 400m}$

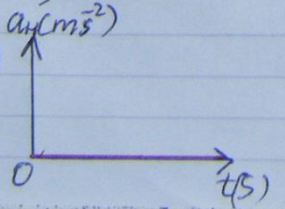
NB



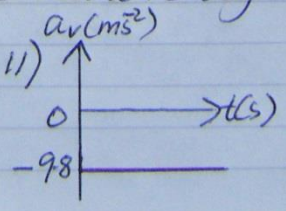
Vertical → 5s to reach max height
horizontal → 10s to cover hor. range.

Q d) Draw graphs of:
i) Horizontal acceleration against time
ii) Vertical acceleration against time.

A i)



ii)



Ex8 – 2003 HIGHER PAPER Q21

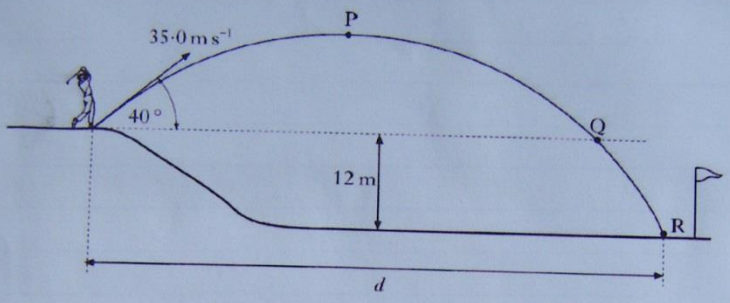
Q

21. A golfer on an elevated tee hits a golf ball with an initial velocity of $35{\cdot}0\,\text{m s}^{-1}$ at an angle of $40°$ to the horizontal.

The ball travels through the air and hits the ground at point R.

Point R is 12 m below the height of the tee, as shown.

diagram not to scale



The effects of air resistance can be ignored.

(*a*) Calculate:

(i) the horizontal component of the initial velocity of the ball;

(ii) the vertical component of the initial velocity of the ball;

(iii) the time taken for the ball to reach its maximum height at point P. **4**

(*b*) From its maximum height at point P, the ball falls to point Q, which is at the same height as the tee.

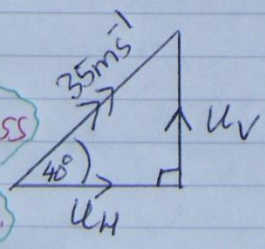
It then takes a further 0·48 s to travel from Q until it hits the ground at R.

Calculate the total horizontal distance *d* travelled by the ball. **3**

(7)

A a) i)

Cos to go across and Sin to climb.



$\cos 40° = \frac{u_H}{35}$

$\Rightarrow u_H = 35\cos 40°$

$\Rightarrow u_H = 26{\cdot}8\,\text{ms}^{-1}$

ii) $\sin 40° = \frac{u_V}{35}$

$\Rightarrow u_V = 35\sin 40°$

$\Rightarrow u_V = 22{\cdot}5\,\text{ms}^{-1}$

iii). V

$u_V = 22.5 ms^{-1}$
$v_V = 0 ms^{-1}$
$a_V = -9.8 ms^{-2}$
$s_V = ?$
$t = ?$

$v_V = u_V + a_V t$

$\Rightarrow 0 = 22.5 + (-9.8)t$

$\Rightarrow 9.8t = 22.5$

$\Rightarrow t = \frac{22.5}{9.8} = \underline{2.3s}$

b). START $\rightarrow$ P $= 2.3s$
P $\rightarrow$ Q $= 2.3s$
Q $\rightarrow$ R $= 0.48s$

$\therefore$ START $\rightarrow$ R $= 2.3 + 2.3 + 0.48 = \underline{5.08s}$

H

$u_H = 26.8 ms^{-1}$
$v_H = 26.8 ms^{-1}$ } SAME
$a_H = 0 ms^{-2}$
$s_H = ?$
$t = 5.08s$

$s_H = u_H t + \frac{1}{2} a_H t^2$

$\Rightarrow s_H = 26.8 \times 5.08 + 0 = \underline{136m}$