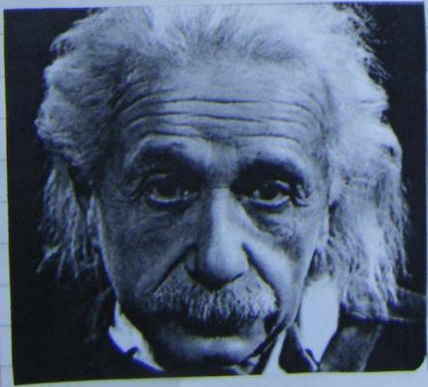
Special Relativity — B McMULLEN

Relativity looks at the motion of an object relative to another object.

The development of the Theories of Relativity through the ages is due in no small part to the following:

Sir Isaac Newton
(England 1642-1727)

James Clerk Maxwell
(Edinburgh 1831-1879)



Albert Einstein
(Germany 1879-1955)

- Newtonian relativity is based on the motion of objects which are moving relative to a fixed point.
  This theory performs well for low velocity motion and forms the basis of rocket science.

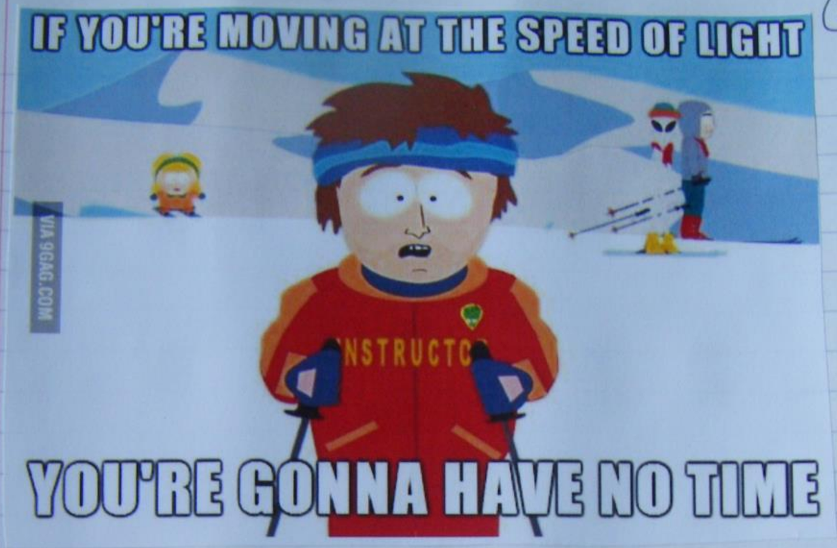
- James Clerk Maxwell then cast doubt over Newtonian relativity as he concluded that light was a wave and that the speed of light in a vacuum was a fixed value.

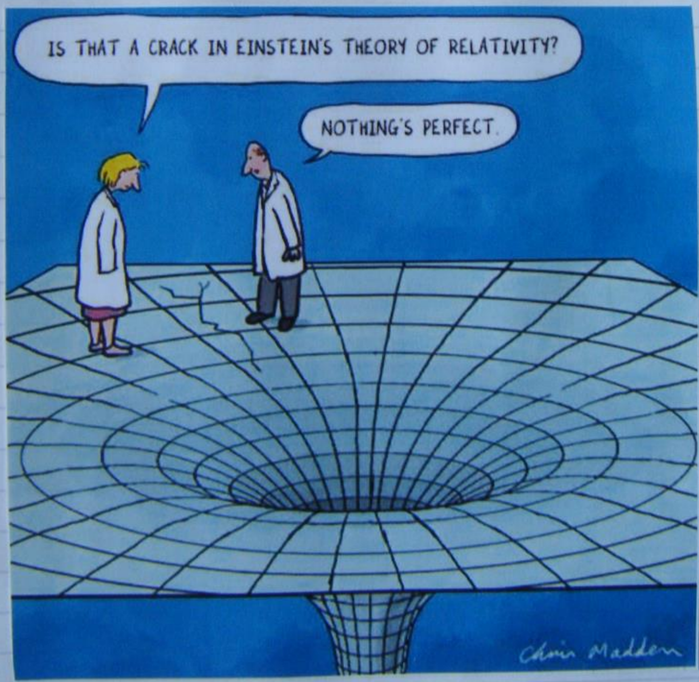
- Einstein then took the thoughts of Newton and Clerk-Maxwell forward and arrived at his Theory of Special Relativity. As the speed of light is a universal constant the nature of space and time for a moving object are changed relative to a stationary observer.

  This development from Einstein lead to conclusions that he made from thought experiments.

  This involved

  - Time dilation
  - Length contraction.


IF YOU'RE MOVING AT THE SPEED OF LIGHT
VIA 9GAG.COM
YOU'RE GONNA HAVE NO TIME


IS THAT A CRACK IN EINSTEIN'S THEORY OF RELATIVITY?
NOTHING'S PERFECT.
Chris Madden

Reference Frames

Relativity involves measuring physical quantities such as distance and time from different frames of reference.

The results however will be slightly different if you compare objects which are moving with:

- low speeds
- speeds approaching the speed of light.

CASE 1 → You are reading your tablet on a train travelling at $100 kmh^{-1}$.

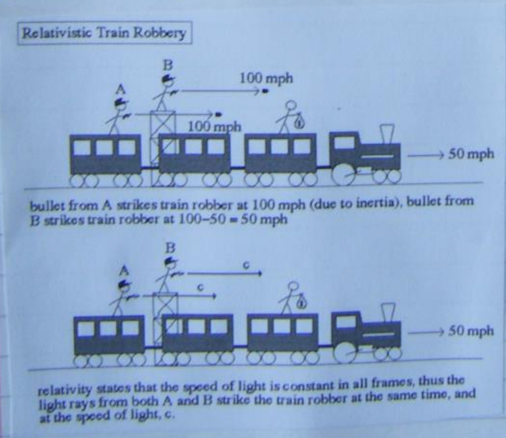
OBSERVER	POSITION	WHAT THEY OBSERVE
A	SITTING BESIDE YOU	YOU ARE STATIONARY
B	STANDING ON THE PLATFORM	YOU ARE TRAVELLING TOWARDS THEM AT $100 kmh^{-1}$
C	PASSENGER ON A TRAIN AT $100 kmh^{-1}$ IN THE OPPOSITE DIRECTION	YOU ARE TRAVELLING TOWARDS THEM AT **$200 kmh^{-1}$**. $(100 - -100 = 200)$

These three scenarios seem to be full of common sense at low speeds.

CASE 2 → You are reading your tablet on a train travelling at $2.5 \times 10^{8} ms^{-1}$

OBSERVER	POSITION	WHAT THEY OBSERVE
A	SITTING BESIDE YOU	YOU ARE STATIONARY.
B	STANDING ON THE PLATFORM	YOU ARE TRAVELLING TOWARDS THEM AT $2.5 \times 10^{8} ms^{-1}$
C	PASSENGER ON TRAIN AT $2.5 \times 10^{8} ms^{-1}$ IN THE OPPOSITE DIRECTION	YOU ARE TRAVELLING TOWARDS THEM AT $5 \times 10^{8} ms^{-1}$ $(2.5 \times 10^{8} - - 2.5 \times 10^{8})$

The result for OBSERVER C is impossible as no object can travel at a speed great than $3 \times 10^{8} ms^{-1}$ ie the speed of light in a vacuum. This applies to any frame of reference.



bullet from A strikes train robber at 100 mph (due to inertia), bullet from B strikes train robber at 100−50 = 50 mph

relativity states that the speed of light is constant in all frames, thus the light rays from both A and B strike the train robber at the same time, and at the speed of light, c.

The robbers A and B are at different frames of reference.

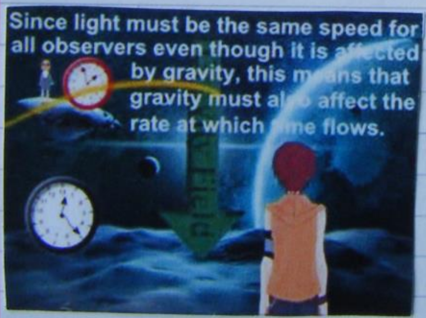
The outcomes again for low speed and the speed of light c are markedly different!!

special relativity conclusions.

1) When two observers are moving at constant speeds relative to one another, they will observe the same laws of physics.

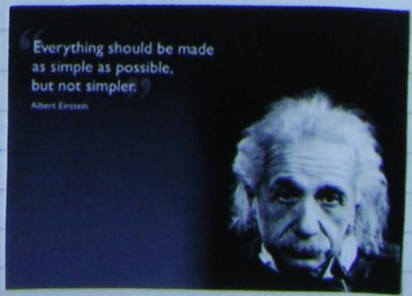
2) The speed of light in a vacuum ie $3 \times 10^8 ms^{-1}$ is the same for all observers.

You would never be able to catch up with a beam of light no matter how fast that you were travelling. Light would always come towards or move away from you at $3 \times 10^8 ms^{-1}$. This is due to you being stationary in your own reference frame.



⇐ This is enough to 'blow your mind', even for a teacher!!

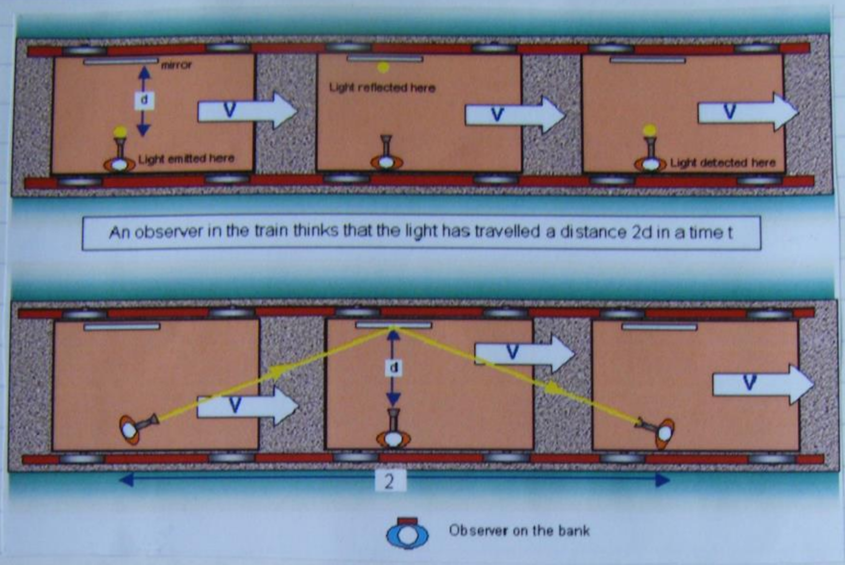
I would not want to argue with this guy!! ⇒



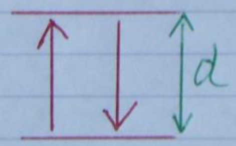
Time Dilation

Ref 1.

Ref 2.



Ref 1. To the observer in the train the light appears to go straight up and down.



Period of the pulse of light $t = \frac{2d}{v}$

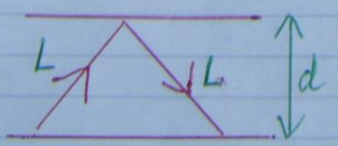
OR $t = \frac{2d}{c}$ (When $v = c$ = Speed of light)

$t' > t$

Why?

As $L > d$.

Ref 2. To the stationary observer on the bank



Period of the pulse of light $t' = \frac{2L}{v}$

OR $t' = \frac{2L}{c}$ (When $v = c$ = Speed of light)

Time dilation Equation

$$t' = t\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

$\boxed{t' = \text{dilated time}}$

$t' = t$ prime $\Rightarrow$ time measured by a stationary observer in a different frame of reference

t = time measured in the same reference frame as the moving object.

$\therefore \boxed{t' > t}$

$\boxed{t = \text{proper time.}}$

Ex1

A space traveller embarks on a mission at a speed of $2{\cdot}7 \times 10^8 ms^{-1}$. If the space traveller measures the time taken as 5 years then calculate the time recorded for the mission by a stationary observer on Earth.

$t' = ?$
$t = 5$ years
$v = 2{\cdot}7 \times 10^8 ms^{-1}$
$c = 3 \times 10^8 ms^{-1}$

$$t' = \frac{t}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$t' = \frac{5}{\sqrt{1-\frac{(2{\cdot}7\times10^8)^2}{(3\times10^8)^2}}} = \frac{5}{\sqrt{1-0{\cdot}81}}$$

$$\Rightarrow t' = \frac{5}{\sqrt{0{\cdot}19}} = \underline{\underline{11{\cdot}5 \text{ years}}}$$

ie $t' > t$, the time recorded by the stationary observer on Earth is greater than the time recorded by the spaceman.

Lorentz Factor

As $t' = \dfrac{t}{\sqrt{1-\frac{v^2}{c^2}}}$

The Lorentz Factor γ is

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

and so $\boxed{t' = \gamma t}$

The Lorentz Factor γ is the scale factor which shows how much the time increases.

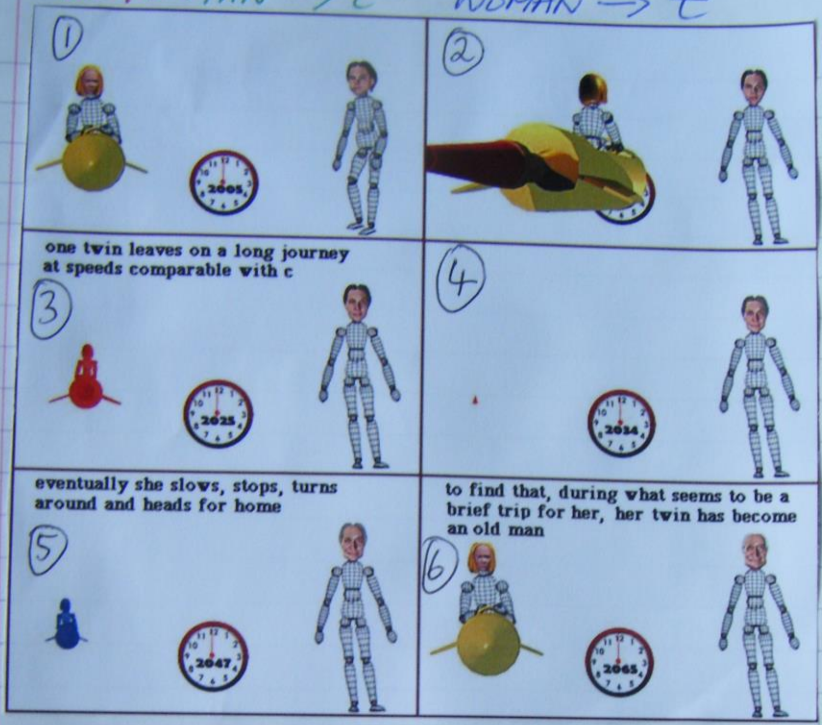
ie $t' > t$

Time dilation means that time increases.
(This can be thought about in terms of the pupil of an eye dilating. This will increase the diameter of the pupil. eg In the dark.)

t = space traveller on a 10 year mission travelling at $2.7 \times 10^8 ms^{-1}$.
t' = The twin observing from Earth with 23 years passing.

$\boxed{t' > t}$

KEY: MAN → t' WOMAN → t



Lorentz Factor Graph

- For small speeds the Lorentz Factor ≈ 1
- The Lorentz Factor increases beyond 2 at 0·9C.
- >0·9C the Lorentz Factor increases markedly on a steep incline.

Length Contraction

$$\boxed{l' = l\sqrt{1-\frac{v^2}{c^2}}}$$

Length contraction means a shortening in length where

$l' < l$ ie contracted length < proper length

The proper length (l) is measured in the same frame of reference where the object is at rest.

The contracted length (l') is measured in a different frame of reference to the object. ie when there is relative movement between the observer and the object.

The Lorentz factor involved here is given by

$$l' = \frac{l}{\gamma} \quad \text{where} \quad \gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

Lengths measured by someone travelling at speeds approaching the speed of light (l') will be less than someone who is stationary on the ground measuring the length of the object (l).

Conclusions

- $t' > t$ ⇒ Time measured in the same frame of reference ie proper time(t) is less than the dilated time (t') which is measured in another frame of reference.

- $l' < l$ ⇒ Length measured in the same frame of reference ie proper length (l) is greater than the contracted length (l') which is measured from another frame of reference.

Ex2

A spacecraft is travelling at a constant speed of 0·70C relative to the Earth.

An observer on Earth measures the length of the moving spacecraft to be 210m.

Calculate the length of the spacecraft as measured by an astronaut on the spacecraft.

$$l' = l\sqrt{1-\frac{v^2}{c^2}} \Rightarrow 210 = l\sqrt{1-\frac{(0{\cdot}70c)^2}{c^2}}$$

$$\Rightarrow 210 = l\sqrt{1-0{\cdot}49}$$

$$\Rightarrow l = \frac{210}{\sqrt{0{\cdot}51}} \Rightarrow \underline{l = 294m}$$