Uncertainties - B McMULLEN

1) Analogue Instruments

Uncertainty in any reading $= \pm \frac{1}{2}$ SSD

(SSD → Smallest Scale division)

eg1 Mercury thermometer ⇒ SSD $= \pm 1°C$

∴ Uncertainty in any reading $= \pm \frac{1}{2}$ SSD $= \pm 0.5°C$

ie Room temperature $= (20.0 \pm 0.5)°C$

NB The reading and the uncertainty must be to the same number of decimal places.

eg2 Ruler ⇒ SSD $= \pm 1mm$

∴ Uncertainty in any reading $= \pm \frac{1}{2}$ SSD $= \pm 0.5mm$

ie for a measured length $= 25.1cm$

then Length $= 25.1cm \pm 0.5mm$

$= 251mm \pm 0.5mm$

$= (251.0 \pm 0.5)mm$

NB Remember the same number of decimal places.

2) Digital Instruments

Uncertainty in any reading = ± 1SSD

eg1 Digital voltmeter ⇒ SSD = ± 0·01V

∴ Uncertainty in any reading = ± 1SSD = ± 0·01V

then for a measured voltage = 8·23V

Voltage = (8·23 ± 0·01)V

eg2 Digital Thermometer ⇒ SSD = ± 0·1°C

Then for a measured temperature = 27·5°C

then Temperature = (27·5 ± 0·1)°C

3) Systematic Uncertainties

Due to instruments not calibrated properly ie They have not been properly zeroed.

Experimentally this can be seen when

a) Trying to measure the half-life of a radioactive source. We need to take into consideration the background activity in the room.

* eg Corrected Activity = Uncorrected Activity − Background Activity.

(Corrected Activity: Source on its own) (Uncorrected Activity: Source in addition to the background activity)

b) Measuring light Irradiance from a lamp and taking into consideration the background light Irradiance.

As above in a)

* Corrected light Irradiance = Uncorrected light Irradiance − Background light Irradiance

4) Random Uncertainty

When an experiment is repeated over a number of runs.

$$* \text{Mean Reading} = \frac{\text{Total of the readings}}{\text{Number of readings}}$$

$$* \text{Random Uncertainty in the mean} = \frac{\text{Max Reading} - \text{Min. Reading}}{\text{Number of readings}}$$

Ex1

Greg runs the 100m sprint six times over the course of the season. The times recorded were: 12·75s, 12·90s, 13·25s, 13·00s, 12·95s and 13·18s.

Calculate or find:

Q a) mean time

b) Random uncertainty in the mean time.

c) mean time ± Random Uncertainty

A a) $\text{mean time} = \frac{\text{Total}}{\text{Number}} = \frac{12{\cdot}75+12{\cdot}90+13{\cdot}25+13{\cdot}00+12{\cdot}95+13{\cdot}18}{6}$

$\text{mean time} = \frac{78{\cdot}03}{6} = \underline{13{\cdot}01\text{s}}$

b) $\text{Random Uncertainty in the mean} = \frac{\text{max Reading} - \text{Min Reading}}{\text{Number of Readings}}$

$= \frac{13{\cdot}25 - 12{\cdot}75}{6}$

$= \frac{0{\cdot}5}{6} = \underline{\pm 0{\cdot}08\text{s}}.$

c) mean time ± Random Uncertainty $= \underline{(13{\cdot}01 \pm 0{\cdot}08)\,\text{s}}$

5) Dominant Uncertainties

* $\% \text{ uncertainty} = \frac{\text{Uncertainty}}{\text{Reading}} \times 100\%$

* Absolute Uncertainty = % uncertainty × Reading

Ex2

Voltage = (12.00 ± 0.05) V
Current = (5.0 ± 0.1) mA

Q: Calculate or find:

a) Resistance

b) % uncertainty in Resistance

c) Absolute uncertainty in the Resistance

d) Resistance ± Absolute Uncertainty

A: a) From $V = IR$

$$R = \frac{V}{I} = \frac{12.00}{5.0 \times 10^{-3}} = 2400\,\Omega$$

b) % uncertainty in R = Dominant % uncertainty between V and I.

% uncertainty in Voltage $= \frac{0.05}{12.00} \times 100\% = \pm 0.4\%$

% uncertainty in current $= \frac{0.1}{5.0} \times 100\% = \pm 2\%$

∴ Dominant % uncertainty is in the current $= \pm 2\%$

∴ % uncertainty in Resistance $= \pm 2\%$

c) Absolute uncertainty in Resistance = % uncertainty x Reading

$= \pm 2\% \times 2400\,\Omega$

$= \pm 48\,\Omega$

d) Resistance ± Uncertainty $= (2400 \pm 48)\,\Omega$

6) Random and Dominant Uncertainties combined.

Ex 3

A trolley runs down a ramp with the following times recorded as the times for the trolley to run from the top to the bottom:
0.176s, 0.182s, 0.179s, 0.184s and 0.174s.

If the length of the ramp is 1.25m, which is measured with a tape measure of SSD = ±1cm, then

Calculate or find:

Q. a) Mean time

b) Random uncertainty in the mean

c) Mean time ± Random uncertainty

d) Average speed

e) % uncertainty in the average speed.

f) Absolute uncertainty in the average speed

g) Average speed ± Absolute uncertainty

A. a) $\text{mean time} = \frac{\text{Total}}{\text{Number}} = \frac{0.176 + 0.182 + 0.179 + 0.184 + 0.174}{5}$

$\therefore$ mean time $= \underline{0.179\text{s}}$

b) $\text{Random Uncertainty in the mean} = \frac{\text{max} - \text{min}}{\text{Number}} = \frac{0.184 - 0.174}{5}$

$= \underline{\pm 0.002\text{s}}$

c) mean time ± Random Uncertainty

$= \underline{(0.179 \pm 0.002)\text{s}}$

* Reading and uncertainty to the same number of decimal places.

d) Average speed, $\bar{v} = \frac{d}{t} = \frac{1.25}{0.179} = \underline{6.98 ms^{-1}}$

e) SSD in distance measurement = ±1cm

As analogue instrument uncertainty = $\pm \frac{1}{2}$ SSD = ± 0.5cm

1.25 m = 125cm.

% uncertainty in distance $= \frac{0.5}{125} \times 100\% = \underline{\pm 0.4\%}$

% uncertainty in time $= \frac{0.002}{0.179} \times 100\% = \underline{\pm 1.1\%}$

* The dominant uncertainty is in the time = ±1.1%

∴ % uncertainty in the average speed = $\underline{\pm 1.1\%}$

f) Absolute uncertainty in the average speed $= \pm 1.1\% \times 6.98 ms^{-1}$

$= \underline{\pm 0.08 ms^{-1}}$ (2dp's).

g) Average speed ± Absolute Uncertainty

$= \underline{(6.98 \pm 0.08) ms^{-1}}$

Units

1) $m/s = ms^{-1}$ 2) $m/s^2 = ms^{-2}$

3) $N/Kg = NKg^{-1}$ 4) $J/Kg = JKg^{-1}$

5) $J/Kg°C = JKg^{-1}°C^{-1}$ 6) $J/s = Js^{-1} = W$

Prefixes

1) $\times 10^{-12}$ → pico (p)
2) $\times 10^{-9}$ → nano (n)
3) $\times 10^{-6}$ → micro (μ)
4) $\times 10^{-3}$ → milli (m)
5) $\times 10^{3}$ → kilo (k)
6) $\times 10^{6}$ → Mega (M)
7) $\times 10^{9}$ → Giga (G)
8) $\times 10^{12}$ → Tera (T)