

# Monitoring and measuring a.c.





# **Problems**



Q = Ixt, Q = charge in coulombs (C),

I = current in amperes (A),

t = time in seconds (s).

# Current:

Current is defined as the **rate** of flow of charge. It is the number of coulombs passing per second.

# Potential difference (voltage):

The potential difference between two points, is a measure of the **work done** in moving **one coulomb of charge** between the two points in a circuit e.g. across a lamp. W = Q x V , W = work done in joules (J) Q = charge in coulombs (C) V = potential difference in volts (V)

Remember: work is required to move electric charges through components of a circuit. The work done comes from the energy supplied to the charges as they pass through the battery/cell or any other source.

### Electromotive force (e.m.f.):

The e.m.f. is the energy supplied by the cell to each coulomb of charge. The e.m.f. is measured in volts (or JC<sup>-1</sup>)

# **Resistance:**

Ohm's Law holds for resistors at a constant temperature:

$$R = \frac{V}{I}$$

where R is a constant, known as the resistance (in ohms,  $\Omega$ ).

### Conservation of energy:

As charges pass through the source, the energy supplied per coulomb of charge must **equal** to the energy dissipated per coulomb of charge in that circuit.

In other words, the sum of the e.m.f.s round a circuit, is **equal** to the sum of the p.d.s round that circuit.



By re-arranging the formula:

$$I = \frac{Q}{t}$$

A current of **1** ampere is **1** coulomb per second  $(1 \text{ A} = 1 \text{ Cs}^{-1})$ 

In a resistor, the work done becomes heat.

$$V = \frac{W}{Q}$$

From the re-arranged formula (above), the voltage is a measure of the energy given out per coulomb of charge.

1 volt is 1 joule per coulomb: 1V = 1 JC<sup>-1</sup>.

### Power:

Power is the **rate** of doing **work**. **Power** is the amount of **work done** per **second**.

$$P = \frac{E}{t}$$

where P is the power measured in watts (W).

Power is related to potential difference (voltage), current and resistance by the following:

$$P = \frac{E}{t} = \frac{QV}{t} = \frac{ItV}{t} = IV$$
$$P = IV = I(IR) = I^2R$$
$$P = IV = \left(\frac{V}{R}\right)V = \frac{V^2}{R}$$

 $V_{supply} = V_1 + V_2 + V_3$ 

The current, *I*, is the same in each resistor. If *R* is the total resistance,

$$IR = IR_{1} + IR_{2} + IR_{3}$$
  

$$IR = I(R_{1} + R_{2} + R_{3})$$
  

$$R = R_{1} + R_{2} + R_{3}$$

If the voltage supply is constant and *R* increases then the current, *I*, decreases.

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From the circuit rules: the current in each branch adds up to the supply current:  $I = I_1 + I_2 + I_3$ 

If *R* is the total resistance,

$$\frac{V_{supply}}{R} = \frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3}$$
  
which is the same as  $V_{supply}\left(\frac{1}{R}\right) = V\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$   
Since  $V_{supply} = V_1 = V_2 = V_3$   
then  $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$ 

When you add resistive branches in parallel the total resistance decreases.

The total resistance of a parallel network is always less than that of a resistor with the smallest resistance .



### Worked example: Calculate the total resistance in the circuit shown



Loop 1, loops around three 2  $\Omega$  resistors, which are connected in **series**. The combined resistance in that loop is 6  $\Omega$ .

Loop 2, loops around a 2  $\Omega$  resistor, which is connected **across** the 6  $\Omega$  resistor.

The total resistance is given as

Method 1	1 1 1	Method 2
	$\overline{R} = \overline{R_1} + \overline{R_2}$ $\frac{1}{R} = \frac{1}{6} + \frac{1}{2}$ $1  1 + 3  4$	$R = \frac{6 \times 2}{6+2}$ $R = 1.5 \Omega$
	$\frac{R}{R} = \frac{1.5 \Omega}{6} = \frac{1}{6}$	

## Potential divider calculations:

 $R_1$ 

 $R_2$ 



$$V_1 = \left(\frac{R_1}{R_1 + R_2}\right) V_s \qquad \text{and} \qquad \frac{V_1}{R_1} = \frac{V_2}{R_2}$$

You should know which equation to use for a given potential divider problem.

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Remember: the bigger the component's resistance, the bigger the p.d. across that component (i.e. the bigger the share of the supply voltage).

Look at the example, next page, that involves two potential dividers in parallel.





# 1. Calculate the potential at point A.

The potential at point A is the same as the voltage (p.d.) across 3000  $\Omega$  resistor.

First step is to calculate the potential difference across 3000  $\Omega$  resistor.

Use the formula

 $V_1 = \left(\frac{R_1}{R_1 + R_2}\right) V_s$ 

where  $V_1$  is the voltage at point A,  $R_1 = 3000 \Omega$ ,  $R_2 = 1000 \Omega$ and  $V_s = 20 V$ .

You should be able to work out that  $V_1 = 15 V$ . So  $V_A = 15 V$ .

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### 2. Calculate the potential at point B.

This time  $V_1$  is the voltage at point B,  $R_1 = 6000 \Omega$ ,  $R_2 = 4000 \Omega$  and  $V_s = 20 V$ .

You should be able to work out that  $V_1 = 12 V$ . So  $V_B = 12 V$ .

3. Calculate the potential difference between points A and B.

 $V_{AB} = V_A - V_B = 15 - 12 = 3V.$ 

The above circuit is called a **Wheatstone bridge**. A Wheatstone bridge is made up of two potential dividers connected in parallel.

Generally, for any Wheatstone bridge, if the potential difference between points A and B ( $V_{AB}$ ) is equal to **zero volts**, then the bridge **balanced**.

From the example above, the potential difference  $V_{AB}$  is equal to 3 V and not 0 V. We would say the bridge is **out-of-balance.** 

The Wheatstone bridge circuit is very useful for monitoring changes in voltages between points A and B. A variable resistor, or a thermistor or LDR could be used to replace one of the four resistors. This special type of circuit would be used to show changes in temperature or light intensity etc.

# Electrical sources and internal resistance.

# Internal resistance:

Real cells and batteries have internal resistance.

An electrical source is equivalent to a source of e.m.f., E, with a resistor in series known as the internal resistance (r).





# e.m.f (E):

The e.m.f. is the energy supplied by the cell to each coulomb of charge. The e.m.f. is measured in volts (or JC<sup>-1</sup>).

### lost volts:

The p.d. across the internal resistance. The internal resistance is considered to be a constant.

load resistance (R) increases?

V<sub>lost</sub> = Ir

### terminal potential difference:

The terminal potential difference is the voltage across the terminals of the supply. In a closed circuit, as shown above, the terminal potential difference is the same as the p.d. across the load resistor (R).

For a closed circuit:  $V_{tpd} = IR$ where R is the load resistance, which can be a combination of resistors connected to the terminals of the source.

 $V_{tpd} = E - V_{lost}$ 

### Worked example:

A 10 V cell has an internal resistance of 3  $\Omega$ . Calculate the terminal potential difference when the current drawn is 2 A?

$$V_{tpd} = E - V_{lost}$$
$$V_{tpd} = 10 - (2 \times 3)$$
$$V_{tpd} = 4 V$$

Recap on expressions: $E = V_{tpd} + V_{lost}$  $V_{tpd} = IR$  $V_{lost} = Ir$  $I = \frac{E}{R+r}$  $r = \frac{E-V}{I}$ Vibration of the page, explain what would happen to  $V_{lost}$  and  $V_{tpd}$  if the would happen to  $V_{lost}$  and  $V_{tpd}$  if the would happen to  $V_{lost}$  and  $V_{tpd}$  if the would happen to  $V_{lost}$ From this expression, as the circuit resistance R

\* The lost volts (*V*<sub>lost</sub> = *Ir*) decreases.

\* The terminal potential difference,  $(V_{tpd} = E - V_{lost})$ , increases.



A short circuit occurs when two or more terminals of a source of electrical energy are connected through a path of low resistance.

The circuit (on the right) has a wire of negligible resistance connected across the source. The maximum current available will take the route of low resistance (i.e. in the wire) and the wire itself will overheat. The size of the short circuit current is worked out as follows:

$$I = \frac{E}{R+r} \qquad \begin{array}{c} \text{Total load} \\ \text{resistance} \\ R = 0 \ \Omega \end{array} \qquad I = \frac{E}{0+r} \qquad \begin{array}{c} I_{short} = \frac{E}{r} \\ I_{short} = \frac{5}{0.5} \end{array}$$



The heater element has a resistance of 1 ohm.

In this set-up, the element's output power is 25 watts.

Calculate:

Q (i) the current in the heater (ii) the terminal potential difference 1Ω (iii) the internal resistance of the supply. Another 1 ohm heater element is connected in parallel **Q** with the original heater element. Calculate the terminal potential difference for this setup. (Remember: the internal resistance has not changed, so you cannot use the output power above for this problem).

> Answers: (i) 5 A (ii) 5 V (iii) 0.1 Ω extended:  $R_{parallel} = 0.5 \Omega$ ,  $R + r = 0.6 \Omega$ , method 1:  $V = 0.5 \times 5.5/0.6 = 4.58 V$ method 2: I = E/(R+r) = 5.5/0.6 = 9.17 A,  $V_{tod} = IR = 9.17 \times 0.5 = 4.58 V.$

5.5V

н

r

#### Maximum power:

The maximum power in a load resistor, R, occurs when R = r.



### Voltmeters:

Voltmeters have nearly infinite resistance (actually it is around 1 M $\Omega$ ).

This means the total resistance in the circuit is very high.

The current in the circuit is negligible (close to 0 A), so Ir = 0 volts (i.e. there are no lost volts). This means the voltmeter reads the e.m.f., which is 4 V.



# Capacitors





voltage / V

energy stored.

# Stored energy:

Work has to be done to charge a capacitor. The first electron placed on one plate repels the next. The more electrons already in place on the metal plate, the stronger the force of repulsion which has to be overcome to add more charge and so the greater the work that has to be done.

The work becomes energy stored in the electric field between the plates of the capacitor.

$$\boldsymbol{E} = \frac{1}{2}\boldsymbol{Q}\boldsymbol{V} \qquad \boldsymbol{E} = \frac{1}{2}\boldsymbol{Q}\boldsymbol{V} = \frac{1}{2}\boldsymbol{Q}\left(\frac{\boldsymbol{Q}}{\boldsymbol{C}}\right) = \frac{1}{2}\frac{\boldsymbol{Q}^2}{\boldsymbol{C}}$$

$$E = \frac{1}{2}QV = \frac{1}{2}(CV)V = \frac{1}{2}CV^{2}$$

### Charging up a capacitor:

- The moment switch S is closed,  $V_R = V_s$ .
- Current reading is high (*I=V<sub>s</sub>*÷*R*)
- As charge builds up on the plates, a potential difference is produced across the capacitor (i.e. V<sub>c</sub> increases).
- Since  $V_R + V_c = V_{s_i} V_R$  decreases and *I* decreases as the capacitor charges up.
- Eventually,  $V_R = 0 V$ , I = 0 A,  $V_c = V_s$ .



### The current/time and voltage/time graphs for an RC charging circuit are shown:



# Discharging a capacitor:

- The moment switch S is closed,  $V_R = -V_s$  and  $V_c = V_s$ .
- Current reading is high (I = V<sub>s</sub>÷R), but negative in sign as the current is in opposite direction when charging.
- The capacitor is discharging through resistor R. The charge stored in the capacitor decreases and p.d. across it also decreases (i.e. V<sub>c</sub> decays to 0 V)
- Since  $V_R + V_C = 0$ ,  $V_R$  decreases and *I* decreases as the capacitor discharges.
- Eventually,  $V_R = 0 V$ , I = 0 A,  $V_c = 0 V$ .



### The current/time and voltage/time graphs for an RC discharging circuit are shown:



# Charging and discharging time:

The time to charge or discharge depends on the resistance and the capacitance.

Increasing the capacitance will cause the time for charging (or discharging) to increase. More charge will be stored.



Increasing the resistance will cause the time for charging (or discharging) to increase. The same charge will be stored.

The current starts smaller but takes longer to reduce to 0 A.



# Q

A 500  $\mu\text{F}$  capacitor is initially uncharged. Switch S is now closed.

- (a) State the p.d. across 2 k $\Omega$  after S is closed.
- (b) State the p.d. across 500  $\mu$ F after S is closed.
- (c) Calculate the initial current.
- (d) State the p.d. across 2 k  $\Omega$  when the capacitor is fully charged.
- (e) State the p.d. across 500  $\mu$ F when the capacitor is fully charged.
- (f) Calculate the charge stored on the fully charged capacitor.
- (g) Calculate the energy stored on the fully charged capacitor.



Answer: (a) 20 V (b) 0 V (c) 10 mA (d) 0 V (e) 20 V (f) 10 mC (g) 100 mJ

# Q

A 500  $\mu\text{F}$  capacitor is charged and has a potential difference of 10 V. Switch S is now closed.

- (a) Calculate the initial current after S is closed.
- (b) While the capacitor is discharging, the voltage across it is 8 V.Calculate the current in the circuit at that time.
- (c) The voltage across the capacitor has now dropped to 2 V. Calculate the charge and energy stored on the capacitor.



In a d.c. circuit, when a capacitor is fully charged, no more current flows. The capacitor blocks d.c.

In an **a.c. circuit**, the capacitor opposes the alternating current but does not block it completely. This opposition becomes less as the frequency increases. Because of this, the **current increases as the frequency increases**.

To do this as an experiment, you need an a.c. type voltmeter so that the voltage of the signal generator remains constant. The frequency of the signal generator is varied and the current is measured.  $I_c / A$ 0 f / Hz

An  $I_c/f$  graph is a straight line through the origin. This means **current** in the circuit is **directly proportional to the frequency of the signal generator**.

### **Uses of capacitors:**

- Capacitors block d.c. but allow a.c. to pass.
- Capacitors can be used to tune radio circuits.
- Capacitors store charge, which can be used to smooth the output of a rectified power supply. Rectification means changing a.c. to d.c.
- Capacitors are also used to direct high and low frequencies to the appropriate speakers in a hi-fi system.

# Conductors, insulators and semi-conductors.

# Materials:

Materials belong roughly to three categories depending on their electrical resistance: **conductors** (e.g. copper), **insulators** (e.g. polythene) and **semiconductors** (e.g. silicon).



During manufacturing, the conductivity of semiconductors can be controlled by means of **doping**. Doping means **adding a very few "impurity" atoms to a pure semiconductor**. This reduces its resistance. As a result, there are two types of semiconductors that can be produced from doping: **p-type** and **n-type**.



The majority of charge carriers, in n-type semiconductors, are negative (electrons).

Group V doping agents result in n-type extrinsic semiconductors, which contain extra electrons.



The majority of charge carriers, in p-type semiconductors, are positive ("holes").

Group III doping agents result in p-type extrinsic semiconductors, which contain extra holes.

- Semiconductors may be **doped** with impurities that add either extra electrons or holes to the lattice.
- Doping of semiconductors can significantly reduce the width of the gap between the conduction and valence band.
- The energy band gap in semiconductors is small enough that thermal excitation is sufficient for significant numbers of electrons to be able to move up from the valence to the conduction band.
- Semiconductors allow conduction by means of negative charge carriers (electrons) or positive charge carriers ("holes").

## Fermi level:

At this level, the Fermi level represents a point where it is equally likely that an electron is, or is not, present. The Fermi level (i.e. the energy of the Fermi level) must remain constant through any semiconductor device if there is no applied field.

Conduction

band

Fermi level ----

Valence

band

band

since



# p-n junction.

A p-n junction diode is made from a crystal of semiconductor material, half of which is p-type and half n-type.



semiconductors, before they are interfaced.

# Band model of a non-biased p-n junction:



The region where the conduction and valence bands bend is called the **depletion layer.** 

The Fermi level is flat in the depletion layer.

The slope in the conduction band acts as a potential barrier as it would require work, of value  $eV_D$ , to get electrons to move against the barrier.

When the two types are interfaced, some electrons and holes diffuse across the border and combine.



Since the electrons and holes have combined, there are no free charges carriers around the junction. This is called the **depletion layer**.



As shown above, there are fixed charged atoms in the depletion layer that stop any further movement of charge carriers across the junction. This is equivalent to having a voltage (p.d.) across the junction, which opposes further movement of electrons (from n-type to p-type) and holes (ntype to p-type). (See the band model to the left.)

Applying a voltage to a semiconductor device is called biasing.

This is very useful, as this represents a diode. A diode is a device which conducts electricity in one direction much more than in the other.









# The photodiode:

Electron-hole pairs are produced when light falls on the junction (depletion layer). Each photon gives up its energy and produces one electron-hole pair.



### Solar cells:

The photodiode is operating in **photovoltaic mode**. This means the photodiode can be used to supply power to a load. When the photon is incident on the junction, its energy is absorbed, freeing electrons and creating electron-hole pairs. This creates a voltage. The more light, the more photons are absorbed and the higher the voltage.

