

## Unit 1 - Our Dynamic Universe - Part 1

### MOTION - EQUATIONS OF MOTION

1. Equations of motion for objects with constant acceleration in a straight line.
  - Practical experiments to verify the relationships shown in the equations.
2. Motion-time graphs for motion with constant acceleration.
  - Displacement-time graphs. Gradient is velocity.
  - Velocity-time graphs. Area under graph is displacement. Gradient is acceleration.
  - Acceleration-time graphs. Restricted to zero and constant acceleration.
  - Graphs for bouncing objects and objects thrown vertically upwards.
3. Motion of objects with constant speed or constant acceleration.
  - Objects in freefall and the movement of objects on slopes.

### FORCES, ENERGY AND POWER

4. Balanced and unbalanced forces. The effects of friction. Terminal velocity.
  - Forces acting in one plane dimension only.
  - Analysis of motion using Newton's First and Second Laws.
  - Friction force as a vector quantity acting in a direction to oppose motion. No reference to static and dynamic friction.
  - Tension as a pulling force exerted by a string or cable on another object.
  - Velocity-time graph of falling object when air resistance is taken into account, including changing the surface area of the falling object.
  - Analysis of the motion of a rocket may involve a constant force on a changing mass as fuel is used up.
5. Resolving a force into two perpendicular components.
  - Forces acting at an angle to the direction of movement.
  - The weight of an object on a slope can be resolved into a component acting down the slope and a component acting normal to the slope.
  - Systems of balanced and unbalanced forces with forces acting in two dimensions.
6. Work done, potential energy, kinetic energy and power.
  - Work done as transfer of energy.
  - Conservation of energy.

### COLLISIONS AND EXPLOSIONS

7. Elastic and inelastic collisions.
  - Conservation of momentum in one dimension and in which the objects may move in opposite directions.
  - Kinetic energy in elastic and inelastic collisions.
8. Explosions and Newton's Third Law.
  - Conservation of momentum in explosions in one dimension only.
9. Impulse as equivalent to change in momentum.
  - Force-time graphs during contact of colliding objects.
  - Impulse can be found from the area under a force-time graph.

## EQUATIONS OF MOTION

### Equations of motion for objects with constant acceleration in a straight line.

The following are some of the **quantities** you will meet in the Higher Physics course:

**DISTANCE, DISPLACEMENT, SPEED, VELOCITY, TIME, FORCE.**

**Quantities** can be divided into 2 groups:

#### Scalars

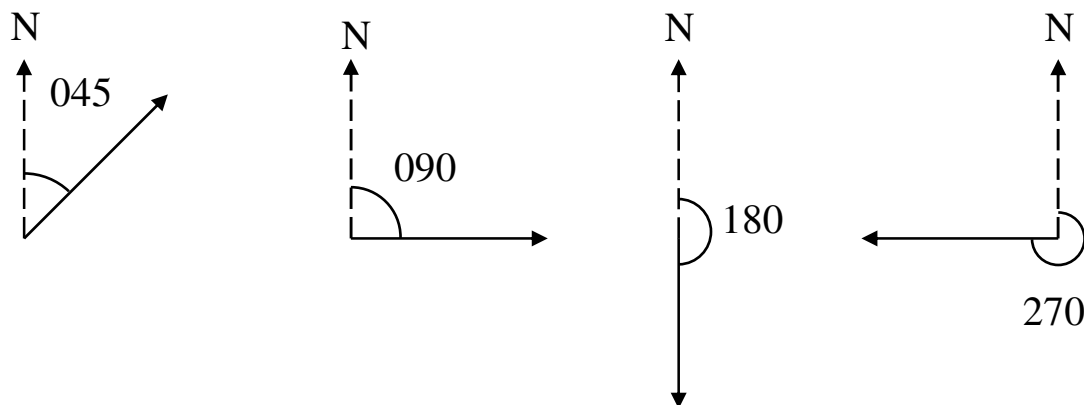
These are specified by stating their magnitude (size) only, with the correct unit.

#### Vectors

These are specified by stating their magnitude (size), with the correct unit, and a direction (often a bearing).

#### Bearings

Compass directions are measured from North which is always taken to be at the top of the page. The angle specified is always a 3-figure bearing. For example:



Some **scalar** quantities have a corresponding **vector** quantity.

Other **scalar** and **vector** quantities are independent.

scalar quantity	corresponding vector quantity
distance (25 m)	displacement (25 m bearing 120)
speed (10 ms <sup>-1</sup> )	velocity (10 m s <sup>-1</sup> bearing 090)
time (12 s)	NONE
NONE	force (10 N bearing 045)
-	acceleration
-	momentum
energy	-

### DISTANCE and DISPLACEMENT

**Distance** (a **scalar** quantity) is **the total length of path travelled.**

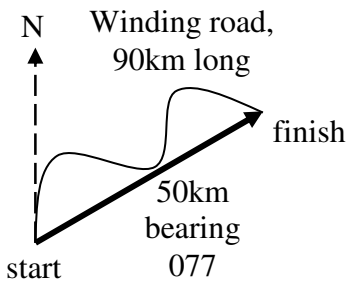
[A **unit** must always be stated.]

**Displacement** (a **vector** quantity) is **the length and direction of a straight line drawn from the starting point to the finishing point.**

[A **unit** and **direction** (often a **3-figure bearing from North**) must always be stated, unless the displacement is zero.]

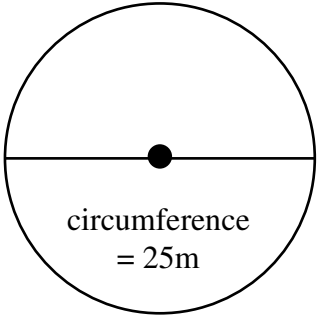
Example:

1) Bill drives 90km along a winding road.



- Distance travelled = 90 km
- Displacement = 50 km bearing 077

2) Ben jogs once around the centre of a football pitch.



- Distance travelled = 25 m
- Displacement = 0 m.  
(He is back where he started, so the length of a straight line drawn from his starting point to his finishing point is 0 m).

**SPEED and VELOCITY**

**Speed** (a scalar quantity) is **the rate of change of distance**.

$$\text{speed} = \frac{\text{distance}}{\text{time}}$$

$$d = \bar{v}t$$

$$\text{average speed} = \frac{\text{total distance travelled}}{\text{total time taken}}$$

**Velocity** (a vector quantity) is **the rate of change of displacement**.

$$\text{velocity} = \frac{\text{displacement}}{\text{total time}}$$

$$s = \bar{v}t$$

Example:

Calculate the average speed and the velocity of Bill and Ben in the cases above. (Bill's journey took 2 hours. Ben's journey took 10 seconds).

<u>Bill</u>	
average speed	velocity
$\bar{v} = \frac{d}{t}$	$\bar{v} = \frac{s}{t}$
$\bar{v} = \frac{90}{2}$	$\bar{v} = \frac{50}{2}$
$\bar{v} = 45\text{kmh}^{-1}$	$\bar{v} = 25\text{kmh}^{-1} (077)$

<u>Ben</u>	
average speed	velocity
$\bar{v} = \frac{d}{t}$	$s = 0$
$\bar{v} = \frac{25}{10}$	therefore,
$\bar{v} = 2.5\text{ms}^{-1}$	$\bar{v} = 0$

Adding Scalar Quantities

**Scalar** quantities can be added arithmetically if they have the same unit, e.g.

$$2 \text{ cm} + 3 \text{ cm} = 5 \text{ cm} \quad \text{but} \quad 2 \text{ cm} + 3 \text{ minutes CANNOT BE ADDED}$$

Adding Vector Quantities

**Vector** quantities can be added together to produce a single vector if they have the same unit - but their directions must be taken into account. We do this using the "**tip to tail**" rule.

The single vector obtained is known as the **resultant vector**.

The "TIP TO TAIL" RULE

Each vector must be represented by a straight line of suitable scale.

The straight line must have an arrow head to show its direction.

The vectors must be joined one at a time so that the tip of the previous vector touches the tail of the next vector.

A straight line is drawn from the starting point to the finishing point.

The scaled-up length and direction of this straight line is the resultant vector.

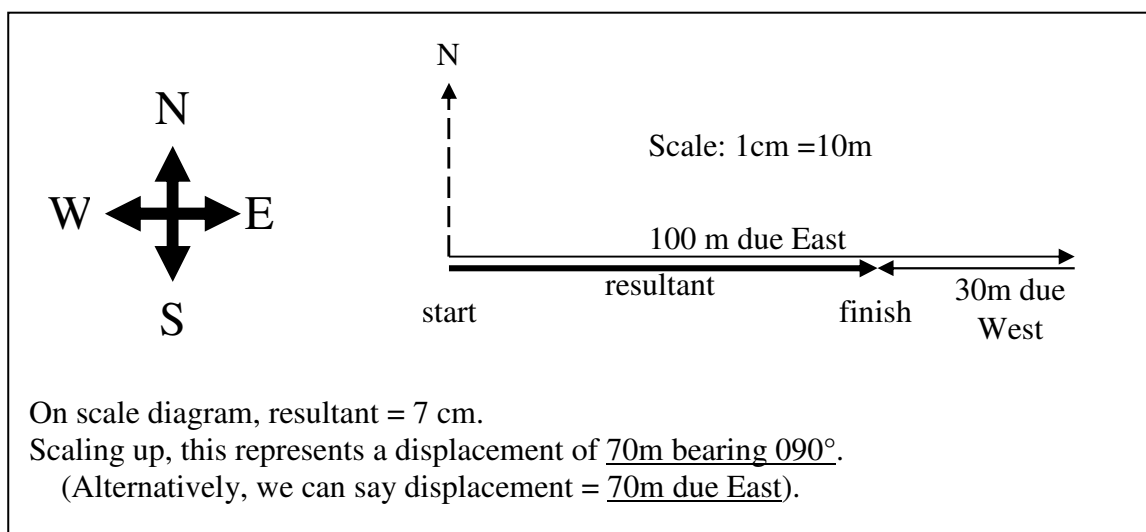
It should have 2 arrow heads to make it easy to recognise.

YOU MUST BE ABLE TO ADD VECTOR QUANTITIES USING BOTH A SCALE DIAGRAM AND MATHEMATICS - Pythagoras theorem, SOHCAHTOA, the Sine Rule and the Cosine Rule.

LARGE SCALE DIAGRAMS GIVE MORE ACCURATE RESULTS THAN SMALLER ONES! - ALWAYS USE A SHARP PENCIL!

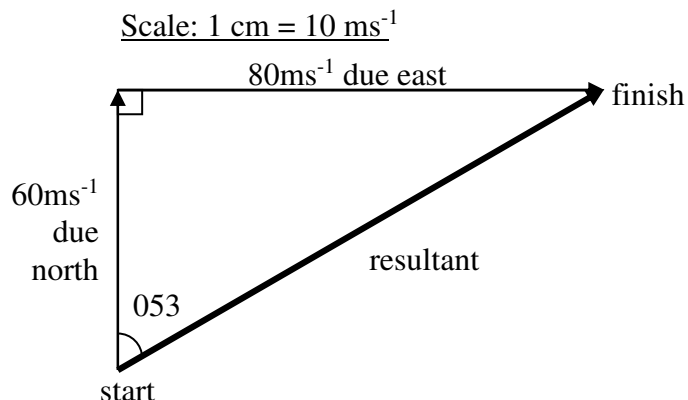
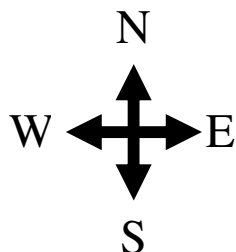
Example 1:

Anna rides her mountain bike 100 m due East along a straight road, then cycles 30 m due West along the same road. Determine Anna's displacement from her starting point using a scale diagram.



Example 2

A helicopter tries to fly due North at  $60 \text{ ms}^{-1}$ . It is affected by a very strong wind blowing due East at  $80 \text{ ms}^{-1}$ . Determine the resultant velocity of the helicopter relative to the ground.

Method 1 - by scale diagram

On scale diagram, resultant = 10 cm. Scaling up, this represents a velocity relative to the ground of  $100 \text{ ms}^{-1}$  bearing (053).

Method 2 - Using mathematics

A rough sketch of the vector diagram (NOT to scale) should be made if you solve such a problem using mathematics.

First, Using PYTHAGORAS THEOREM

$$a^2 = b^2 + c^2$$

$$a^2 = 60^2 + 80^2$$

$$a^2 = 3600 + 6400$$

$$a^2 = 10,000$$

$$a = \sqrt{10,000}$$

$$a = 100 \text{ ms}^{-1}$$

Next, Using SOH CAH TOA

$$\tan \theta = \frac{O}{A} = \frac{80}{60} = 1.33$$

$$\text{so, } \theta = \tan^{-1}(1.33)$$

$$= 53^\circ$$

Resultant velocity relative to the ground =  $100 \text{ ms}^{-1}$  bearing (053).

Note - the mathematical method provides a more accurate answer for the angle. (You can't read a protractor to  $0.1^\circ$  !!!)

Acceleration

**Acceleration** is the change of velocity per unit time. Unit:  $\text{ms}^{-2}$  (vector).

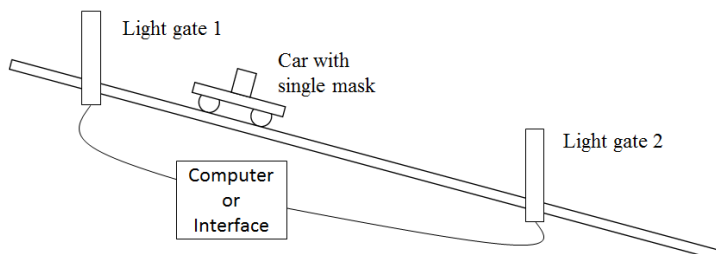
$$\text{acceleration} = \frac{\text{final velocity} - \text{initial velocity}}{\text{time taken for change}}$$

$$a = \frac{v - u}{t}$$

To determine the **acceleration** of a trolley running down a slope, we can use:

Method 1

a **single card (mask)** of known length and **2 light gates** connected to a **computer** (which records times).



Find length of card  $L$

Find  $t_1$  (time taken for card fixed on trolley to pass through first (top) light gate)

Find  $t_2$  (time taken for card fixed on trolley to pass through second (bottom) light gate)

Find  $t_3$  (time taken for card fixed on trolley to pass between the two light gates)

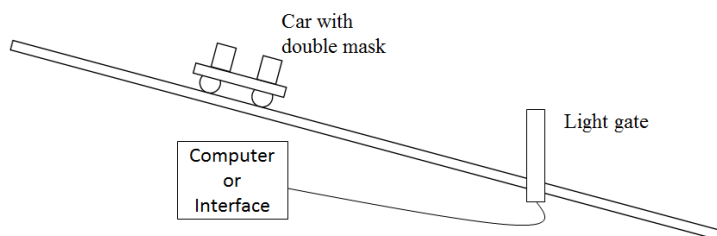
Calculate **initial velocity** of card through first light gate  $u = \frac{L}{t_1}$

Calculate **final velocity** of card through second light gate  $v = \frac{L}{t_2}$

Calculate **acceleration** using  $a = \frac{v - u}{t_3}$

Method 2

a **double card (mask)** (2 known lengths) and **1 light gate** connected to a **computer** (which records times).



Find the length,  $L_1$ , of the right edge of the card (first to pass through the light gate)

Find the length,  $L_2$ , of the left edge of the card (second to pass through the light gate)

Find  $t_1$  (time taken for first edge of card to pass through light gate)

Find  $t_2$  (time taken for second edge of card to pass through light gate)

Find  $t_3$  (time between first and second edge of card passing through light gate)

Calculate **initial velocity** of the first edge through light gate  $u = \frac{L_1}{t_1}$

Calculate **final velocity** of the second edge through light gate  $v = \frac{L_2}{t_2}$

Calculate **acceleration** using  $a = \frac{v - u}{t_3}$

This method can be adapted to measure **gravitational acceleration**.

A **single mask** can be dropped vertically through **2 light gates**, or a **double mask** can be dropped vertically through **1 light gate**.

## The 4 Equations of Motion (for uniform acceleration in a straight line)

**FOUR equations** can be applied to any object moving with uniform (constant) acceleration in a straight line:

$$v = u + at$$

$$s = \frac{(u + v)}{2} t$$

$$s = ut + \frac{1}{2} at^2$$

$$v^2 = u^2 + 2as$$

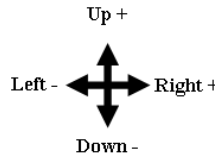
t = time for motion to take place (s)

u = initial velocity ( $\text{ms}^{-1}$ )

v = final velocity after time t ( $\text{ms}^{-1}$ )

a = uniform (constant) acceleration during time t ( $\text{ms}^{-2}$ )

s = displacement (in a straight line) during time t (m)



Because **u**, **v**, **a** and **s** are **vectors**, we must specify their direction by placing a **+** or **-** sign in front of the number representing them:

The equation of motion used to solve a problem depends on the quantities given in the problem.

Often, the term straight line is not mentioned in the problem.

If no direction is specified for the accelerating object, we assume it is travelling to the right - This means we use positive vector values in the equations of motion.

### Examples

$$v = u + at$$

A racing car starts from rest and accelerates uniformly in a straight line at  $12 \text{ ms}^{-2}$  for 5.0 s. Calculate the **final velocity** of car.

$$u = 0 \text{ ms}^{-1} \text{ (rest)}$$

$$a = 12 \text{ ms}^{-2}$$

$$t = 5.0 \text{ s}$$

$$v = ?$$

$$v = u + at$$

$$v = u + at$$

$$v = 0 + (12 \times 5.0)$$

$$v = 0 + 60$$

$$v = 60 \text{ ms}^{-1} \text{ (in direction of acceleration)}$$

$$s = ut + \frac{1}{2} at^2$$

A speedboat travels 400 m in a straight line while accelerating uniformly from  $2.5 \text{ ms}^{-1}$  in 10 s. Calculate the acceleration of the speedboat.

$$s = 400 \text{ m}$$

$$u = 2.5 \text{ ms}^{-1}$$

$$t = 10 \text{ s}$$

$$a = ?$$

$$s = ut + \frac{1}{2} at^2$$

$$400 = (2.5 \times 10) + (0.5 \times a \times 10^2)$$

$$400 = 25 + 50a$$

$$50a = 400 - 25 = 375$$

$$a = 375/50 = 7.5 \text{ m s}^{-2} \text{ (in direction of original velocity)}$$

$v^2 = u^2 + 2as$	A rocket is travelling through outer space with uniform velocity. It then accelerates at $2.5 \text{ ms}^{-2}$ in a straight line in the original direction, reaching $100 \text{ ms}^{-1}$ after travelling 1 875 m. Calculate the rocket's <b>initial velocity</b> ?
$a = 2.5 \text{ ms}^{-2}$ $v = 100 \text{ ms}^{-1}$ $s = 1\,875 \text{ m}$ $u = ?$	$v^2 = u^2 + 2as$ $100^2 = u^2 + (2 \times 2.5 \times 1\,875)$ $u^2 = 10\,000 - 9\,375 = 625$ $u = \sqrt{625} = \underline{25 \text{ ms}^{-1}}$ (in direction of acceleration)

### Decelerating objects and Equations of Motion

When an object **decelerates**, its **velocity decreases**. If the vector quantities in the **equations** of motion are positive, we represent the decreasing velocity by use of a negative sign in front of the acceleration value (and vice versa).

#### Examples

$v = u + at$	A car, travelling in a straight line, decelerates uniformly at $2.0 \text{ ms}^{-2}$ from $25 \text{ ms}^{-1}$ for 3.0 s. Calculate the car's velocity after the 3.0 s.
$a = -2.0 \text{ ms}^{-2}$ $u = 25 \text{ ms}^{-1}$ $t = 3.0 \text{ s}$ $v = ?$	$v = u + at$ $v = 25 + (-2.0 \times 3.0)$ $v = 25 + (-6.0)$ $v = \underline{19 \text{ ms}^{-1}}$ (in direction of original velocity)

$s = ut + \frac{1}{2}at^2$	A greyhound is running at $6.0 \text{ ms}^{-1}$ . It decelerates uniformly in a straight line at $0.5 \text{ ms}^{-2}$ for 4.0 s. Calculate the displacement of the greyhound while it was decelerating.
$u = 6.0 \text{ ms}^{-1}$ $a = -0.5 \text{ ms}^{-2}$ $t = 4.0 \text{ s}$ $s = ?$	$s = ut + \frac{1}{2}at^2$ $s = (6.0 \times 4.0) + (0.5 \times -0.5 \times 4.0^2)$ $s = 24 + (-4.0)$ $s = \underline{20 \text{ m}}$ (in direction of original velocity)

$v^2 = u^2 + 2as$	A curling stone leaves a player's hand at $5.0 \text{ ms}^{-1}$ and decelerates uniformly at $0.75 \text{ ms}^{-2}$ in a straight line for 16.5 m until it strikes another stationary stone. Calculate the velocity of the decelerating curling stone at the instant it strikes the stationary one.
$u = 5.0 \text{ ms}^{-1}$ $a = -0.75 \text{ ms}^{-2}$ $s = 16.5 \text{ m}$ $v = ?$	$v^2 = u^2 + 2as$ $v^2 = 5.0^2 + (2 \times -0.75 \times 16.5)$ $v^2 = 25 + (-24.75)$ $v^2 = 0.25$ $v = \sqrt{0.25} = \underline{0.5 \text{ ms}^{-1}}$ (in direction of original velocity)

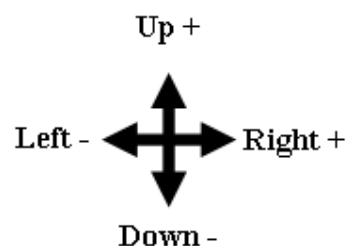


## Motion - time graphs for motion with constant acceleration

### Direction of vector motion

Velocity, acceleration and displacement are all vector quantities - we must specify a direction for them.

We usually do this by placing a + or - sign in front of the number representing the quantity, according to the direction diagram on the right.



For example, for horizontal motion,  $+5 \text{ ms}^{-1}$  represents a velocity of  $5 \text{ ms}^{-1}$  to the right and  $-5 \text{ ms}^{-1}$  represents a velocity of  $5 \text{ ms}^{-1}$  to the left. For vertical motion,  $+10 \text{ m}$  represents a displacement of  $10 \text{ m}$  up and  $-10 \text{ m}$  represents a displacement of  $10 \text{ m}$  down.

(WARNING: The + sign is often missed out! and some graphs/questions you may encounter use the opposite sign convention, e.g., up is -, down is +. BE CAREFUL!!!)

### Comparing motion-time graphs

In all areas of science, graphs are used to display information. Graphs are an excellent way of giving information, especially to show relationships between quantities. In this section we will be examining three types of motion-time graphs.

Displacement-time graphs

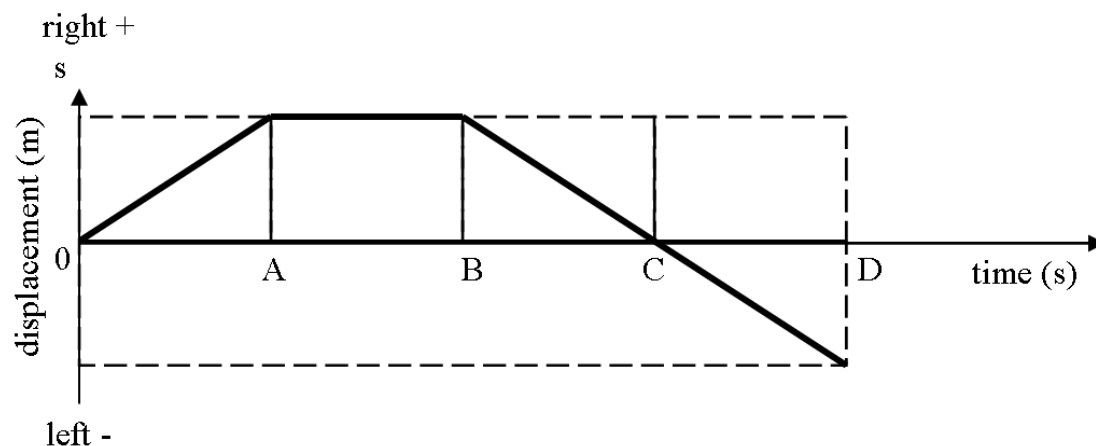
Velocity-time graphs

Acceleration-time graphs

If you have an example of one of these types of graph then it is possible to draw a corresponding graph for the other two factors.

Displacement – time graphs

This graph represents how far an object is from its starting point at some known time. Because displacement is a vector it can have positive and negative values. [+ve and –ve will be opposite directions from the starting point.]



OA – the object is moving away from the starting point. It is moving a **constant displacement each second**. This is shown by the constant gradient.

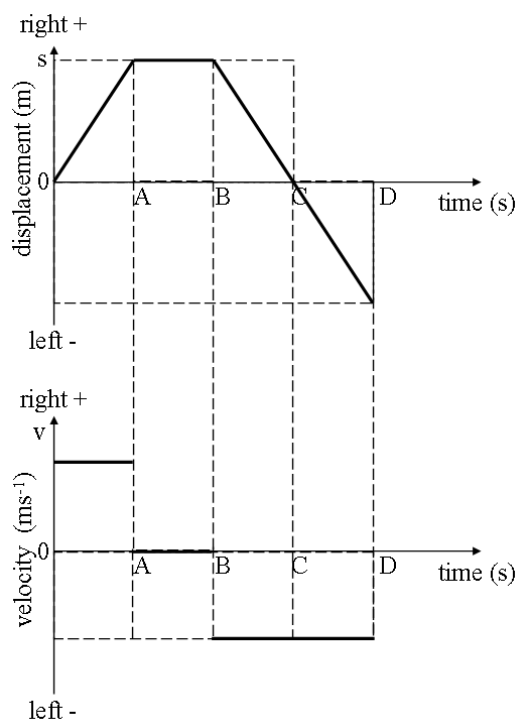
$$\text{gradient} = \frac{\text{change in } y}{\text{change in } x} = \frac{\text{displacement}}{\text{time}} = \text{velocity}$$

We can determine the **velocity** from the **gradient** of a displacement time graph.

AB – the object has a **constant** displacement so is not changing its position, therefore it must be at rest. The gradient in this case is zero, which means the object has a velocity of zero.

BC – the object is now moving back towards the starting point, reaching it at time C. It then continues to move away from the start, but in the opposite direction.

The gradient of the line is **negative**, indicating the **change in direction** of motion.



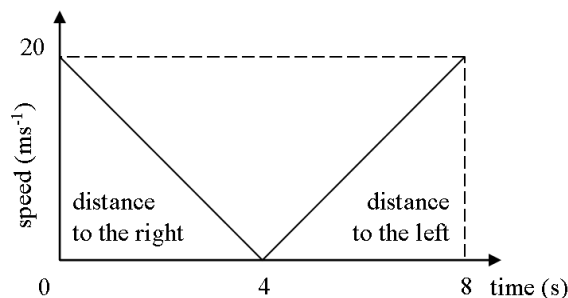
The **velocity time** graph is essentially a graph of the gradient of the displacement time graph.

It is important to take care to determine whether the gradient is **positive** or **negative**.

The gradient gives us the information to determine the **direction** an object is moving.

Comparing speed-time and velocity-time graphs for motion in a straight line

A car, initially travelling at  $20 \text{ ms}^{-1}$  in a straight line to the **right**, brakes and decelerates uniformly (constantly) at  $5.0 \text{ ms}^{-2}$ , coming to **rest** in 4.0 s. Immediately, it reverses, accelerating uniformly (constantly) at  $5.0 \text{ ms}^{-2}$  in a straight line to the **left** for 4.0 s, back to where it started.

speed-time graph of motion

Speed is a scalar quantity. No account is taken of the direction of travel.

The straight lines indicate uniform deceleration and uniform acceleration. (Gradient = acceleration.)

The total area under the graph gives the total distance travelled.

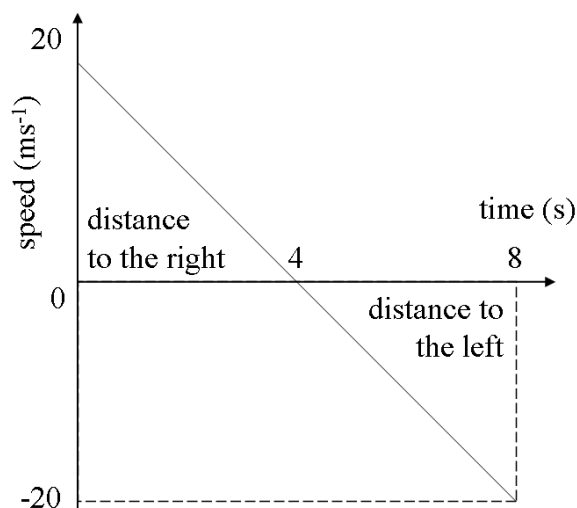
**Determine the total distance travelled:**

The total area under the graph gives the total distance travelled.

$$\text{Area} = \frac{1}{2}(20 \times 4) + \frac{1}{2}(20 \times 4)$$

$$\text{Area} = 80$$

$$\text{Distance travelled} = 80 \text{ m}$$

velocity-time graph of motion

Velocity is a vector quantity, so change in direction is taken into account. This is shown by the line crossing the time axis at 4.0 s.

In this case, the deceleration and acceleration have the same numerical value, so the gradient of the line (which indicates their value) is uniform.

The total mathematical area under the graph gives the displacement.

**Show that the displacement is zero:**

The total mathematical area under the graph gives the displacement.

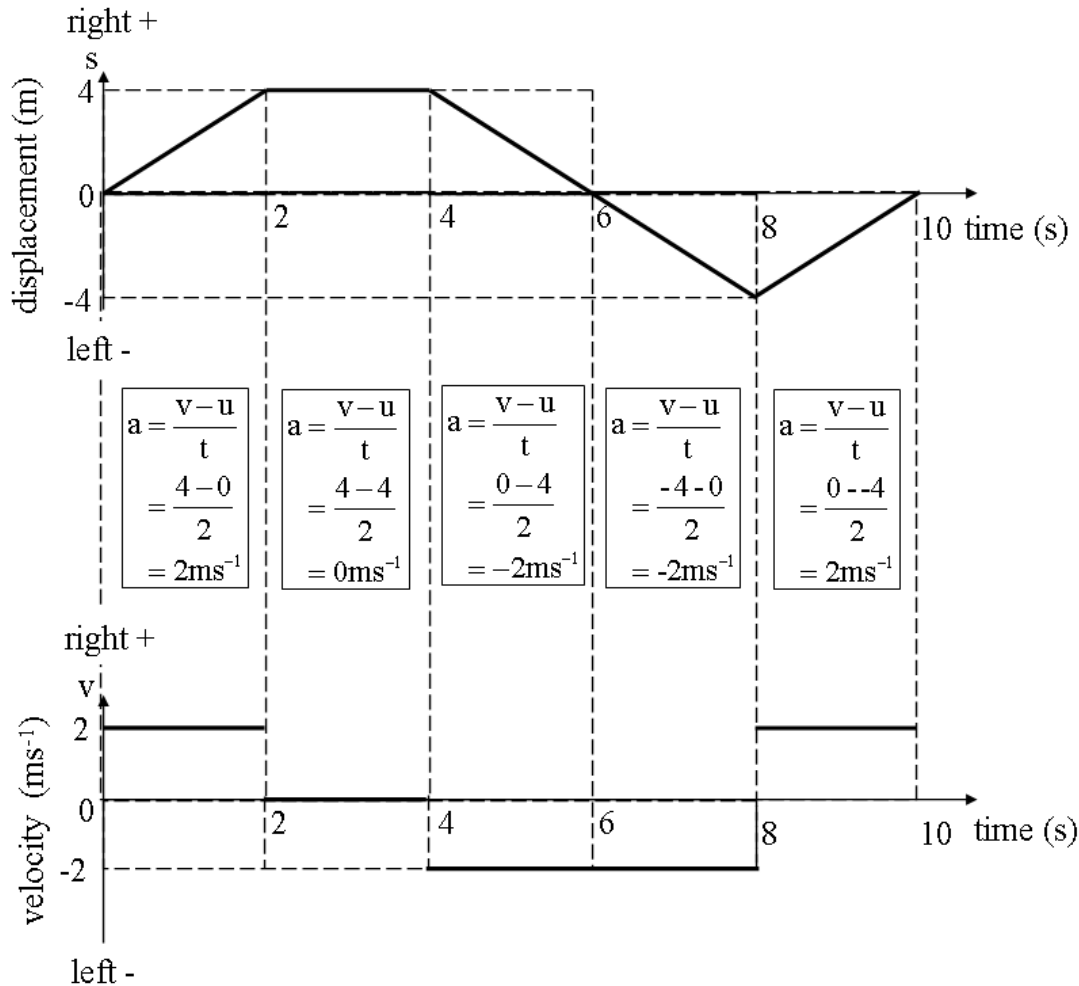
$$\text{Area} = \frac{1}{2}(20 \times 4) + (-)\frac{1}{2}(20 \times 4)$$

$$\text{Area} = 40 + -40$$

$$\text{Displacement} = 0 \text{ m}$$

### Obtaining an acceleration-time graph from a velocity-time graph for motion in a straight line

The velocity-time graph for an object moving in a straight line over horizontal ground is shown. Calculate the acceleration for each part of the graph, then use your values to draw the corresponding acceleration-time graph below:



Describe fully, the motion of the object - You must include all accelerations, times and directions:

- 0-2s, accelerates right at  $2\text{ ms}^{-2}$
- 2-4s, constant velocity right at  $4\text{ ms}^{-1}$
- 4-6s, decelerates right at  $-2\text{ ms}^{-2}$
- 6-8s, accelerates left at  $2\text{ ms}^{-2}$
- 8-10s, decelerates left to a stop at  $-2\text{ms}^{-2}$

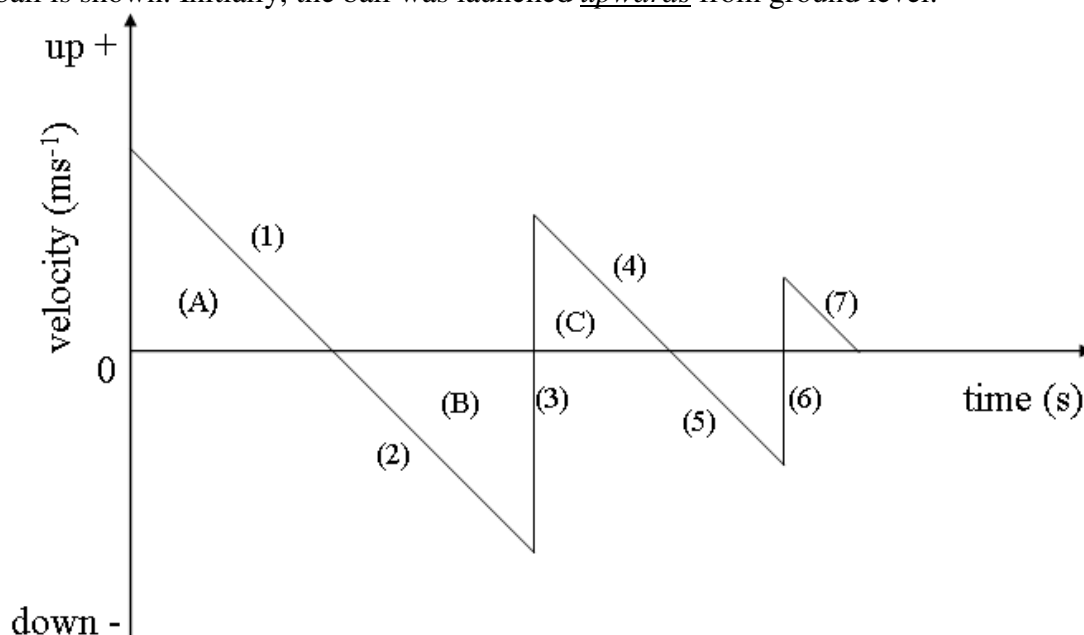
Determine the displacement of the object:

The total mathematical area under the graph gives the displacement.

$$\begin{aligned}
 0-2\text{s}: & \frac{1}{2}(4 \times 2) &= 4\text{m} \\
 2-4\text{s}: & (2 \times 4) &= 8\text{m} \\
 4-6\text{s}: & \frac{1}{2}(4 \times 2) &= 4\text{m} \\
 6-8\text{s}: & \frac{1}{2}(-4 \times 2) &= -4\text{m} \\
 8-10\text{s}: & \frac{1}{2}(-4 \times 2) &= -4\text{m} \\
 \text{Total displacement} &= \underline{8\text{m right}}
 \end{aligned}$$

Velocity-time graph for a bouncing ball

The velocity-time graph for the *vertical (up and down) motion* of a bouncing ball is shown. Initially, the ball was launched upwards from ground level.



What is happening in each section of the graph:

- (1) Ball is launched with positive upwards velocity which **decreases until it reaches its highest point** where velocity becomes zero.
- (2) Ball's **velocity increases in negative direction (downwards)** until it reaches maximum velocity and then hits the ground.
- (3) Ball hits ground and changes direction
- (4) Ball leaves the ground in positive direction at a slightly lower speed (as some energy is lost in collision) and **decelerates** to top of motion.
- (5) - (7) Ball repeats motion.

The graph proves that the **acceleration** of the ball is **constant**, and is the **acceleration due to gravity**, at  $-9.8 \text{ ms}^{-2}$ .

This is evident by the **constant negative gradient** at each section of the graph.

The **areas** represent the distance travelled in each section of the motion.

Area A and area B are the same as the vertical distance travelled upwards is the same as the vertical distance travelled downwards.

**Area A and B** are larger than **Area C** as on rebound some energy is lost as heat and so the **height gained is smaller after each rebound**.

## Motion of objects with constant speed or constant acceleration

### Equations of Motion Applied to Objects Dropped or Launched Upwards

Any object moving **freely** through the air is **accelerated** towards the ground under the influence of **gravity**.

It does not matter if the object is **falling** or **moving upwards**

**Gravity** always provides a **downward acceleration** of **9.8 ms<sup>-2</sup>**.

If we adopt the sign convention shown on the right for the **three equations of motion**, we must use the value of **-9.8 ms<sup>-2</sup>** for the **acceleration** of any object moving **freely** through the air.



### (1) Dropped Objects

At the instant an object is dropped, it is stationary - It is not moving downwards, so **initial downward velocity (u) = 0 ms<sup>-1</sup>**.

The object will **accelerate** towards the ground under the influence of **gravity**  
**a = -9.8 ms<sup>-2</sup>**.

#### Example:

A helicopter is hovering at a constant height. A wheel falls off and hits the ground below 4.0 s later. Calculate:

(a) the downward vertical velocity of the wheel at the instant it hits the ground;

$s = ?$	$v = u + at$
$u = 0 \text{ ms}^{-1}$	$v = 0 + (-9.8 \times 4.0)$
$v = ?$	$v = 0 - 39.2$
$a = -9.8 \text{ ms}^{-2}$	$v = \underline{-39 \text{ ms}^{-1}}$
$t = 4.0 \text{ s}$	(i.e., $39 \text{ ms}^{-1}$ downwards)

(b) the height of the hovering helicopter.

$s = ut + \frac{1}{2}at^2$	$v^2 = u^2 + 2as$
$s = (0 \times 4.0) + (0.5 \times -9.8 \times 4.0^2)$	$-39.2^2 = 0^2 + (2 \times -9.8 \times s)$
$s = 0 + (-78.4)$	$1536.6 = 0 + (-19.6 s)$
$s = \underline{-78 \text{ m}}$	-OR- $1536.6 = -19.6 s$
(i.e., wheel falls <u>78 m downwards</u> ,	$s = 1536.6 / -19.6 = \underline{-78 \text{ m}}$
so height = <u>78 m</u> )	(i.e., wheel falls <u>78 m downwards</u> ,
	so height = <u>78 m</u> )

**(2) Objects Launched Upwards**

At the instant an object is launched upwards, it is travelling at **maximum velocity**.  
 **$u = \text{maximum upward velocity at launch}$** .

As soon as the object starts to travel upwards, **gravity** will **accelerate** it towards the ground at  **$-9.8 \text{ ms}^{-2}$** . ( **$a = -9.8 \text{ ms}^{-2}$** ).

As a result, the **upward velocity** of the object will eventually become  **$0 \text{ ms}^{-1}$** .  
 This happens at its **maximum height**. ( **$v = 0 \text{ ms}^{-1}$  at maximum height**).

**Example**

A spring-powered toy frog is launched vertically upwards from the ground at  $4.9 \text{ ms}^{-1}$ .

(a) What will be the velocity of the toy frog at its maximum height?

$s = ?$	
$u = 4.9 \text{ ms}^{-1}$	
$v = ?$	At maximum height, <u><math>v = 0 \text{ ms}^{-1}</math></u>
$a = -9.8 \text{ ms}^{-2}$	
$t = ?$	

(b) Calculate:

(i) the time taken for the toy frog to reach its maximum height;

$s = ?$	$v = u + at$
$u = 4.9 \text{ ms}^{-1}$	$0 = 4.9 + (-9.8 \times t)$
$v = 0 \text{ ms}^{-1}$	$0 = 4.9 - 9.8 t$
$a = -9.8 \text{ ms}^{-2}$	$9.8 t = 4.9$
$t = ?$	$t = 4.9/9.8 = 0.5 \text{ s}$ .

(ii) the maximum height.

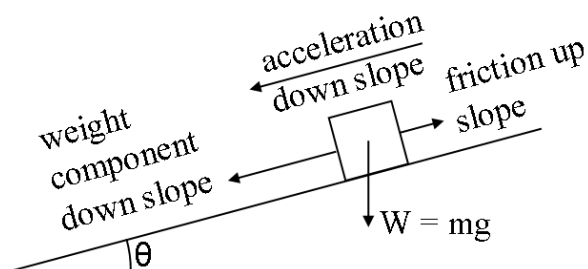
$s = ut + \frac{1}{2}at^2$	$v^2 = u^2 + 2as$
$s = (4.9 \times 0.5) + (0.5 \times -9.8 \times 0.5^2)$	$0^2 = 4.9^2 + (2 \times -9.8 \times s)$
$s = 2.45 + (-1.225)$	$0 = 24.01 + (-19.6 s)$
$s = \underline{1.2 \text{ m}}$	$19.6 s = 24.01$
(i.e., <u>1.2 m upwards</u> ,	-OR- $s = 24.01/19.6 = \underline{1.2 \text{ m}}$
so height = <u>1.2 m</u> )	(i.e., <u>1.2 m upwards</u> ,
	so height = <u>1.2 m</u> )

**(3) Objects on a Slope**

As we shall see later, when an object is placed on a slope, the **weight** of the object causes a force down the slope which is constant. Even if there is a **constant friction force** acting up the slope, the object will have a **uniform acceleration down the slope**.

(this acceleration could be zero, in which case the block may be stationary or move at constant speed)

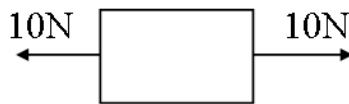
The **Equations of Motion** can also be applied to objects on slopes.



## FORCES, ENERGY AND POWER

### Balanced and unbalanced forces. The effects of friction. Terminal velocity

#### Balanced and Unbalanced Forces



#### Balanced Forces

The forces acting on this object cancel each other out - The **resultant force** is **0 N**. The forces are **balanced**.

#### Unbalanced Forces

The forces acting on this object **do not** cancel each other out - The **resultant force** is **4 N to the right**. The forces are **unbalanced**.

#### NEWTON'S FIRST LAW OF MOTION

"An object will remain at rest, or continue to travel with constant speed and in the same direction, unless acted upon by an unbalanced force"

- A situation where the forces acting on an object are **balanced** results in **constant speed**, where constant speed may be  $0\text{ms}^{-1}$ .
- A situation where the forces acting on an object are **unbalanced** results in an **acceleration**, where the acceleration may be negative (deceleration).

#### NEWTON'S SECOND LAW OF MOTION

The acceleration ( $a$ ) of an object is directly proportional to the unbalanced force ( $F_{\text{un}}$ ) in newtons acting on it and inversely proportional to its mass ( $m$ ) in kilograms.

$$\text{combining } \boxed{a \propto F_{\text{un}}} \text{ and } \boxed{a \propto \frac{1}{m}} \text{ gives } \boxed{a = \text{constant} \times \frac{F_{\text{un}}}{m}}$$

#### Defining the Newton

When the **unbalanced force** ( $F_{\text{un}}$ ) is measured in newtons and the **mass** ( $m$ ) is measured in kilograms, the value of the constant is **1**.

So,  $a = 1 \times \frac{F_{\text{un}}}{m}$  or  $a = \frac{F_{\text{un}}}{m}$       Rearranging gives  $\boxed{F_{\text{un}} = ma}$

**This shows that 1 newton is the value of the unbalanced force which will accelerate a mass of 1 kg at  $1\text{ms}^{-2}$ .**



Solving  $F_{\text{un}} = ma$  problems

The following technique should be applied to  $F_{\text{un}} = ma$  problems involving either **single objects** or **objects connected together** (like a train with carriages.)

1. Draw a free body diagram showing the magnitude (size) and direction of all the forces acting on the object/objects.
2. Use the free body diagram to determine the magnitude (size) and direction of the unbalanced force ( $F_{\text{un}}$ ) and draw this on the diagram.
3. Apply  $F_{\text{un}} = ma$ .

If the objects are **connected together** and the problem asks about the **whole system**, use the **total mass** of the system in the equation  $F_{\text{un}} = ma$ .

If the problem asks about only **part of the system** (like one carriage of a long train), only show the **single object** on your free body diagram. Use this **unbalanced force** and the **mass of the single object** (not the mass of the whole system) in the equation  $F_{\text{un}} = ma$ .

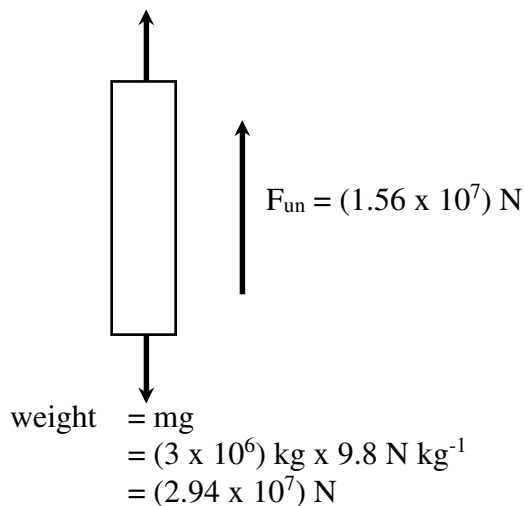
Example 1

A space rocket of mass  $3 \times 10^6$  kg is launched from the earth's surface when its engine produces an upward thrust of  $4.5 \times 10^7$  N. Calculate the rocket's acceleration at launch.

Free body diagram

(Represent the rocket by a box.)

thrust =  $(4.5 \times 10^7)$  N



$$F_{\text{un}} = \text{thrust} - \text{weight}$$

$$F_{\text{un}} = (4.5 \times 10^7) - (2.94 \times 10^7)$$

$$= 1.56 \times 10^7 \text{ N (upwards)}$$

$$a = \frac{F_{\text{un}}}{m}$$

$$= \frac{1.56 \times 10^7}{3 \times 10^6}$$

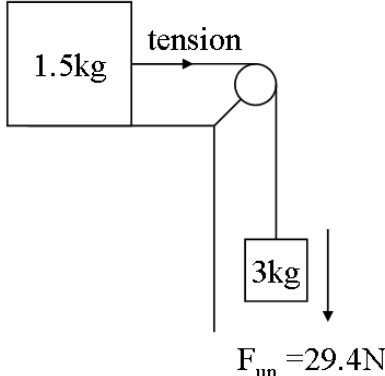
$$= \underline{5.2 \text{ ms}^{-2} \text{ (upwards)}}$$

Example 2

2 wooden blocks are tied together by piece of weightless string. One block (of mass 1.5 kg) sits on a horizontal table. There is no force of friction between the block and table. The other block (of mass 3 kg) is passed over a frictionless pulley. This block falls to the floor, dragging the 1.5 kg block across the table.

Calculate: (a) the acceleration of both wooden blocks;

Free body diagram  
(Represent the blocks by boxes.)



weight of 3 kg block =  $mg$   
 $= 3 \text{ kg} \times 9.8 \text{ N kg}^{-1}$   
 $= 29.4 \text{ N (downwards)}$

The weight of the 3 kg block is the unbalanced force which produces the acceleration.

$$a = \frac{F_{\text{un}}}{m_{\text{total}}}$$

$$= \frac{29.4 \text{ N}}{1.5 \text{ kg} + 3 \text{ kg}}$$

$$= \frac{29.4 \text{ N}}{4.5 \text{ kg}}$$

$$= 6.5 \text{ ms}^{-2} \text{ (down and right)}$$

(b) the tension (pulling force) in the string.

The tension in the string is the pulling force on the 1.5 kg block.

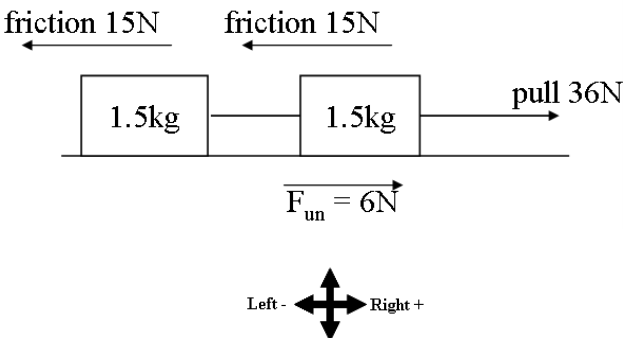
$$\begin{aligned} \text{tension} &= m_{1.5 \text{ kg}} \times a \\ &= 1.5 \text{ kg} \times 6.5 \text{ ms}^{-2} \\ &= 9.8 \text{ N} \end{aligned}$$

Example 3

Adam pulls 2 metal blocks (both of mass 1.5 kg), joined by string of zero mass, along a horizontal bench top with a constant force of 36 N. The force of friction acting on each block is 15 N.

Calculate: (a) the acceleration of the metal blocks;

Free body diagram  
(Represent the blocks by boxes.)



$F_{\text{un}} = \text{pull} - \text{friction}$   
 $F_{\text{un}} = 36 \text{ N} - (2 \times 15) \text{ N}$   
 $= 36 \text{ N} - 30 \text{ N}$   
 $= 6 \text{ N (to the right)}$

$$a = \frac{F_{\text{un}}}{m_{\text{total}}}$$

$$= \frac{6 \text{ N}}{1.5 \text{ kg} + 1.5 \text{ kg}}$$

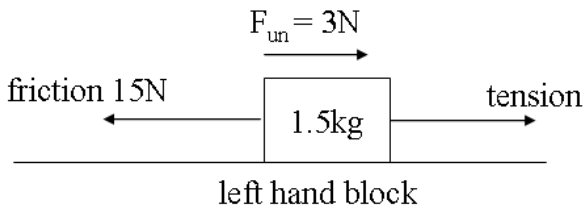
$$= \frac{6 \text{ N}}{3 \text{ kg}}$$

$$= 2 \text{ ms}^{-2} \text{ (to the right)}$$

(b) the tension (force) in the string between the 2 metal blocks.

**Free body diagram**  
**(Represent the left hand block by a box.)**

acceleration =  $2\text{ms}^{-2}$



friction 15N

1.5kg

tension

left hand block

Unbalanced force acting on left block  
 $(F_{\text{un}}) = m_{1.5\text{kg}} a$   
 $= 1.5\text{ kg} \times 2\text{ ms}^{-2}$   
 $= 3\text{ N (to the right)}$

$F_{\text{un}} = \text{tension} - \text{friction}$   
 $3\text{ N} = \text{tension} - 15\text{ N}$   
 $\text{tension} = 3\text{ N} - (-15)\text{ N}$   
 $= 3\text{ N} + 15\text{ N}$   
 $= 18\text{ N}$

The tension force  
 overcomes the friction  
 and causes the  
 acceleration.

The following examples relate to Hannah, mass 60 kg, who is standing on a set of scales in a lift.

Two forces act:

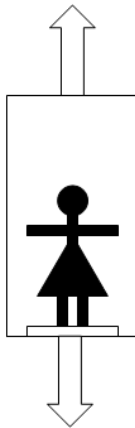
Weight downwards (W)

Value does not change.

Reaction upwards (R)

Value changes as motion of lift changes.

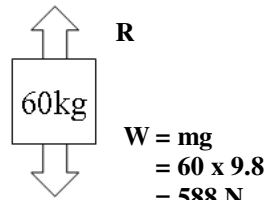
**(R is the reading on the scales.)**



**1) Lift cable breaks**

Determine the reading on the scales (R) if the lift cable breaks, causing the lift, scales and Hannah to accelerate downwards at  $9.8 \text{ ms}^{-2}$ .

Free body diagram



**Both Hannah and the scales accelerate downwards at the same rate**

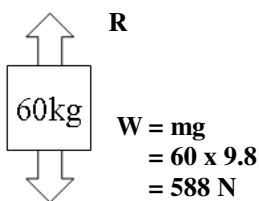
**There is no reaction force upwards**

**$\therefore R = 0 \text{ N}$**

**2) Lift stationary**

Determine the reading on the scales (R) if the lift is stationary.

Free body diagram



Lift is stationary

$\Rightarrow$  balanced forces

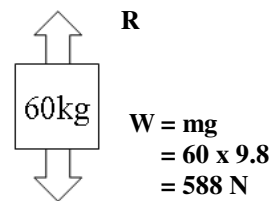
$\Rightarrow R = W$

$\Rightarrow R = 588 \text{ N}$

**3) Lift travelling at constant velocity**

Determine the reading on the scales (R) if the lift is travelling at constant velocity.

Free body diagram



Constant velocity

$\Rightarrow$  balanced forces

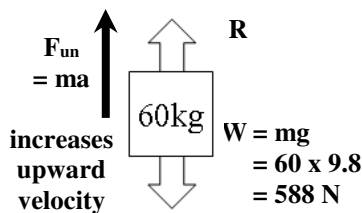
$\Rightarrow R = W$

$\Rightarrow R = 588 \text{ N}$

**4) Lift accelerating upwards**

Calculate the reading on the scales (R) if the lift is accelerating upwards at  $2.5 \text{ ms}^{-2}$ .

Free body diagram



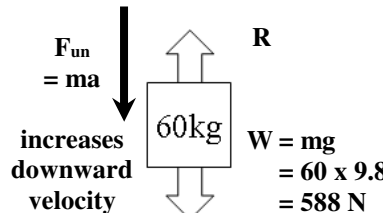
Unbalanced force ( $F_{un} = ma$ ) and R act in same direction:

$$\begin{aligned} ma &= R - W \\ 60 \times 2.5 &= R - 588 \\ 150 &= R - 588 \\ R &= 150 + 588 \\ R &= 738 \text{ N} \end{aligned}$$

**5) Lift accelerating downwards**

Calculate the reading on the scales (R) if the lift is accelerating downwards at  $1.5 \text{ ms}^{-2}$ .

Free body diagram



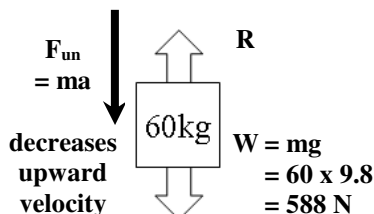
Unbalanced force ( $F_{un} = ma$ ) and W act in same direction:

$$\begin{aligned} ma &= W - R \\ 60 \times 1.5 &= 588 - R \\ 90 &= 588 - R \\ R &= 588 - 90 \\ R &= 498 \text{ N} \end{aligned}$$

**6) Lift decelerating upwards**

Calculate the reading on the scales (R) if the lift is decelerating upwards at  $1.5 \text{ ms}^{-2}$ .

Free body diagram



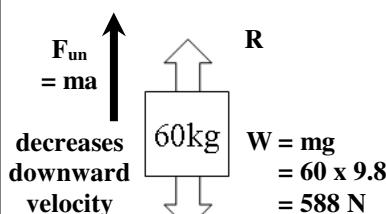
Unbalanced force ( $F_{un} = ma$ ) and W act in same direction:

$$\begin{aligned} ma &= W - R \\ 60 \times 1.5 &= 588 - R \\ 90 &= 588 - R \\ R &= 588 - 90 \\ R &= 498 \text{ N} \end{aligned}$$

**7) Lift decelerating downwards**

Calculate the reading on the scales (R) if the lift is decelerating downwards at  $2.5 \text{ ms}^{-2}$ .

Free body diagram



Unbalanced force ( $F_{un} = ma$ ) and R act in same direction:

$$\begin{aligned} ma &= R - W \\ 60 \times 2.5 &= R - 588 \\ 150 &= R - 588 \\ R &= 150 + 588 \\ R &= 738 \text{ N} \end{aligned}$$

Velocity-time graphs and air resistance

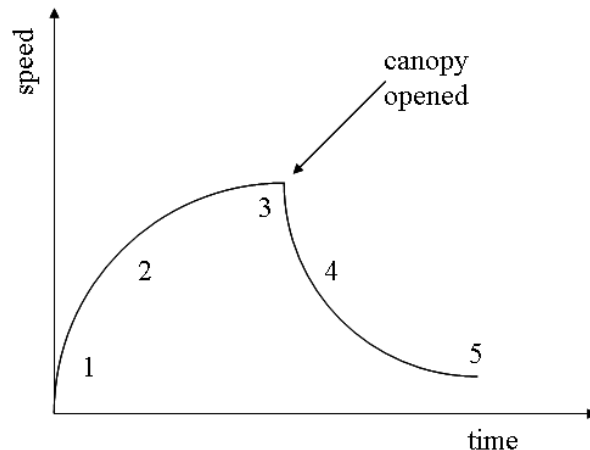
Earlier, we looked at **velocity-time graphs** for **constant acceleration**, but often situations are more complex, for example when jumping out of a plane. *Some really crazy people do this for fun.*

**Before the parachute opens**

1. When the skydiver jumps out of the plane he **accelerates** because there is a force due to gravity (weight) pulling him down.

2. As he speeds up the **upwards air resistance force increases**. He carries on accelerating as long as the air resistance is less than his weight.

3. Eventually, he reaches his **terminal speed** when the air resistance and weight become **equal**. They're said to be balanced.

**After the parachute opens**

4. When the canopy opens it has a **large surface area** which **increases the air resistance**. This unbalances the forces and causes the parachutist to slow down.

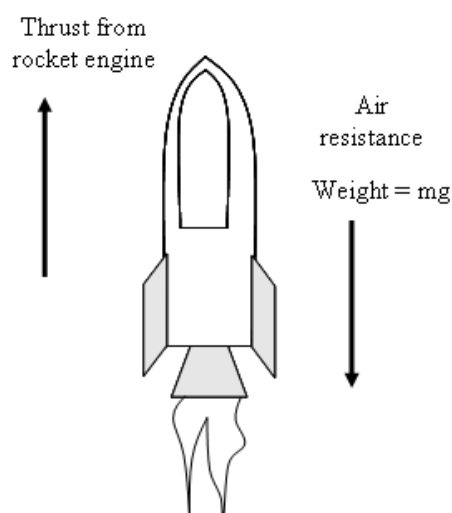
5. As the parachutist **slows down**, his **air resistance gets less** until eventually it **equals** the downward force of gravity on him (his weight). Once again the **two forces balance** and he falls at terminal speed. This time it's a much slower terminal speed than before.

To infinity and beyond...

## Changing speed with time

As a rocket **rises** through the atmosphere, despite a **constant upward thrust** due to the rocket engines, the acceleration **is not constant**.

**Three** things contribute to an **increase** in the rocket's acceleration:

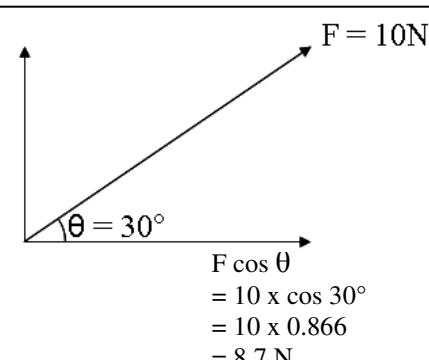


1. Gravitational field strength "g" decreases with distance from the Earth's surface.  
**Weight force decreases**
2. The rocket fuel is being used up as the rocket rises, decreasing the overall mass of the rocket.  
**Weight force decreases**
3. The atmosphere becomes thinner as the rocket rises.  
**Air resistance decreases**

## Resolving a force into two perpendicular components

Any **vector** can be replaced by **2 vectors** of the correct magnitude (size) acting at right-angles ( $90^\circ$ ) to each other.

For example:

$  \begin{aligned}  F \sin \theta &= 10 \times \sin 30^\circ \\  &= 10 \times 0.500 \\  &= \underline{5.0 \text{ N}}  \end{aligned}  $		<p>The <b>10 N</b> vector can be replaced by the 2 vectors: <b>5.0 N</b> acting <b>vertically</b> and <b>8.7 N</b> acting <b>horizontally</b>. (These 2 vectors are at right-angles to each other).</p> <p>The <b>5.0 N</b> and <b>8.7 N</b> vectors are known as <b>components</b> of the <b>10 N</b> vector.</p>
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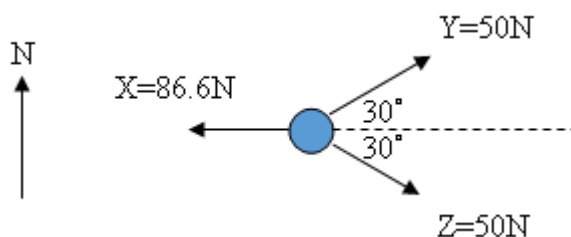
The 5.0 N and 8.7 N forces acting together have exactly the same effect as the 10 N force acting on its own. Acting together, the 5.0 N and 8.7 N forces would move an object in exactly the same direction as the 10 N force would, resulting in exactly the same acceleration.

### Systems of balanced and unbalanced forces with forces acting in two dimensions

When **two or more forces** are acting on an object, in two dimensions, the skills of resolving forces and investigating whether forces are balanced or unbalanced are essential.

#### Example

An object of 28 kg is held stationary by the forces acting in ropes X, Y and Z. The sizes of the forces are shown. (Friction can be ignored).



(a) What is the resultant of the 50 N forces which act in ropes Y and Z?

The resultant force of Y and Z is 86.6 N East, since the object is stationary and so forces Y and Z must balance force X.

Alternatively:

Horizontal component of Y is  $F = 50 \cos 30^\circ = 43.3 \text{ N}$ .

Horizontal component of Z is  $F = 50 \cos 30^\circ = 43.3 \text{ N}$ .

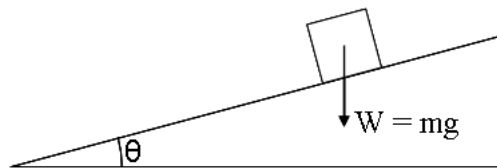
Total horizontal component = 86.6 N

(b) Rope X snaps. Calculate the initial acceleration of the mass

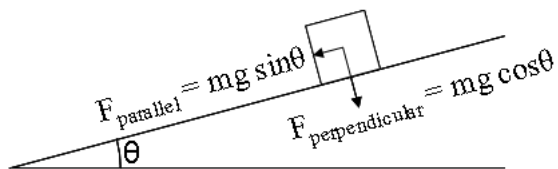
$$a = \frac{F}{m} = \frac{86.6}{28} = 3.09 \text{ ms}^{-1}$$

Objects on a Slope

When an object is placed on a slope, the **weight** of the object acts **downwards** towards the centre of the earth.



The **weight** of the object can be resolved into **right-angle components** acting **down (parallel to)** the slope and **perpendicular to** the slope.

Example 5

A 2 kg metal block is placed on a wooden slope which is at an angle of  $30^\circ$  to the horizontal. The block accelerates down the slope. A constant friction force of 1.2 N acts up the slope.

Determine:

- (a) (i) the component of weight acting down (parallel to) the slope;

$$\begin{aligned} W_{\text{parallel}} &= mg \sin \theta \\ &= 2 \text{ kg} \times 9.8 \text{ N kg}^{-1} \times \sin 30^\circ \\ &= \underline{9.8 \text{ N}} \end{aligned}$$

- (ii) the component of weight acting perpendicular to the slope.

$$\begin{aligned} W_{\text{perpendicular}} &= mg \cos \theta \\ &= 2 \text{ kg} \times 9.8 \text{ N kg}^{-1} \times \cos 30^\circ \\ &= \underline{17 \text{ N}} \end{aligned}$$

- (b) the unbalanced force acting on the metal block down (parallel to) the slope.

$$\begin{aligned} F_{\text{un}} &= W_{\text{parallel}} - \text{Friction} \\ &= 9.8 \text{ N} - 1.2 \text{ N} \\ &= \underline{8.6 \text{ N}} \text{ (down the slope)} \end{aligned}$$

- (c) the acceleration of the metal block down the slope.

$$a = \frac{F_{\text{un}}}{m} = \frac{8.6}{2} = 4.3 \text{ ms}^{-2} \text{ (down the slope)}$$

## Work done, potential energy, kinetic energy and power

$$E = Pt$$

Energy   Power   time  
(J)   (W)   (s)

$$E_p = mgh$$

Potential   mass   gravitational   height  
Energy   (kg)   field strength   (m)  
(J)   (N/kg)

$$E_k = \frac{1}{2}mv^2$$

Kinetic   mass   velocity  
Energy   (kg)   ( $\text{ms}^{-1}$ )  
(J)

$$E_w = Fd$$

Work   Force   distance  
done   (N)   (m)  
(J)

### **Law of Conservation of Energy**

Energy cannot be created or destroyed, it can only be changed from one form to another (or other forms).

#### Example

(a) Thomas drops a football of mass 0.2 kg from a height of 2.25 m. Calculate the velocity of the ball at the instant before it hits the ground.

#### Ignore air resistance

Before the ball is dropped, it possesses only gravitational potential energy. At the instant before the ball hits the ground, all the gravitational potential energy has been converted to kinetic energy.

$$E_p \text{ lost} = E_k \text{ gained}$$

$$mgh = \frac{1}{2}mv^2$$

$$gh = \frac{1}{2}v^2 \text{ ('m' appears on both sides of equation, so can be cancelled out).}$$

$$9.8 \times 2.25 = 0.5 v^2$$

$$22.1 = 0.5 v^2$$

$$v^2 = 22.1/0.5 = 44.2$$

$$v = \sqrt{44.2} = 6.6 \text{ ms}^{-1}$$

(b) Does the mass of the ball affect the velocity?

Mass does not appear in equation used for calculation, so has no effect on the velocity of the ball



## COLLISIONS AND EXPLOSIONS

### Elastic and inelastic collisions

#### Momentum

The **momentum** of an object is the product of its **mass** and **velocity**

Unit: **kg ms<sup>-1</sup> (Vector)**.

**momentum = mass x velocity**

$$\boxed{p=mv}$$

#### Example

Calculate the momentum of a 70 kg ice skater when she is:

(a) moving to the right at 5 ms<sup>-1</sup>;

(b) moving to the left at 6 ms<sup>-1</sup>

$\begin{aligned} \text{momentum} &= mv \\ &= 70 \text{ kg} \times 5 \text{ ms}^{-1} \\ &= 350 \text{ kg ms}^{-1} \\ &\text{i.e., } 350 \text{ kg ms}^{-1} \text{ to the right} \end{aligned}$
---

$\begin{aligned} \text{momentum} &= mv \\ &= 70 \text{ kg} \times -6 \text{ ms}^{-1} \\ &= -420 \text{ kg ms}^{-1} \\ &\text{i.e., } 420 \text{ kg ms}^{-1} \text{ to the left} \end{aligned}$
--

#### The Law of Conservation of Linear Momentum

The **law of conservation of linear momentum** applies to collisions between 2 objects in a straight line and to an object that explodes into 2 parts that travel in opposite directions along the same straight line.

#### The Law of Conservation of Linear Momentum

In the absence of external forces, the total momentum just before a collision/explosion is equal to the total momentum just after the collision/explosion.

#### I. Elastic Collisions

In an **elastic collision**:

- the 2 colliding objects **bounce apart** after the collision.
- **momentum is conserved.** (The total momentum just before the collision = the total momentum just after the collision.)
- **kinetic energy is conserved.** (The total kinetic energy just before the collision = the total kinetic energy just after the collision.)

#### II. Inelastic Collisions


In an **inelastic collision**:

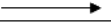
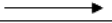
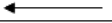
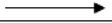
- the 2 colliding objects **stick together** due to the collision.
- **momentum is conserved.** (The total momentum just before the collision = the total momentum just after the collision.)
- **kinetic energy decreases.** (The total kinetic energy just after the collision is less than the total kinetic energy just before the collision.) Some **kinetic energy** is changed into **sound, heat** and **energy of deformation** (which changes the shape of the objects) during the collision.

Example Momentum Problem

A 2 kg trolley moving to the right at  $10 \text{ ms}^{-1}$  collides with a 10 kg trolley which is also moving to the right at  $1 \text{ ms}^{-1}$ . Immediately after the collision, the 2 kg trolley rebounds to the left at  $5 \text{ ms}^{-1}$ .

(a) Calculate the velocity of the 10 kg trolley immediately after the collision.

DIRECTION IS VITAL      -  +

<p style="text-align: center;"><u>Before collision</u></p> <div style="display: flex; justify-content: space-around; align-items: center;"> <div style="text-align: center;"> <math>10\text{ms}^{-1}</math>    <div style="border: 1px solid black; padding: 2px; width: 60px; margin: 0 auto;">2kg</div> </div> <div style="text-align: center;"> <math>1\text{ms}^{-1}</math>    <div style="border: 1px solid black; padding: 2px; width: 60px; margin: 0 auto;">10kg</div> </div> </div>	<p style="text-align: center;"><u>After Collision</u></p> <div style="display: flex; justify-content: space-around; align-items: center;"> <div style="text-align: center;"> <math>-5\text{ms}^{-1}</math>    <div style="border: 1px solid black; padding: 2px; width: 60px; margin: 0 auto;">2kg</div> </div> <div style="text-align: center;"> <math>v = ?</math>    <div style="border: 1px solid black; padding: 2px; width: 60px; margin: 0 auto;">10kg</div> </div> </div>
<p>Total momentum = <math>m_1u_1 + m_2u_2</math>  <math>= (2 \text{ kg} \times 10 \text{ ms}^{-1}) + (10 \text{ kg} \times 1 \text{ ms}^{-1})</math>  <math>= 20 + 10</math>  <math>= 30 \text{ kg ms}^{-1}</math></p>	<p>Total momentum = <math>m_1v_1 + m_2v_2</math>  <math>= (2 \text{ kg} \times -5 \text{ ms}^{-1}) + (10 \text{ kg} \times v)</math>  <math>= -10 + 10v \text{ kg ms}^{-1}</math></p>
<p>Total momentum just before collision = Total momentum just after collision</p> $30 = (-10 + 10v)$ $10v = 30 - (-10)$ $10v = 40$ $v = 40/10 = \underline{4 \text{ ms}^{-1}} \text{ (i.e., } 4 \text{ ms}^{-1} \text{ to the right)}$	

(b) Show that the collision is elastic.

<p>Total kinetic energy before collision</p> $= (1/2 \times 2 \times 10^2) + (1/2 \times 10 \times 1^2)$ $= 100 + 5$ $= 105 \text{ J}$	<p>Total kinetic energy after collision</p> $= (1/2 \times 2 \times 5^2) + (1/2 \times 10 \times 4^2)$ $= 25 + 80$ $= 105 \text{ J}$
<p>Total kinetic energy just before collision = Total kinetic energy just after collision          So, <u>collision is elastic.</u></p>	

Note

You should set out all your momentum problems like this - This makes it easier for you (and anybody marking your work) to see exactly what you are doing.

- Always include a sketch to show the masses of the colliding objects and their velocities just before and just after the collision.
- Take plenty space on your page - Some people take a new page for every problem.
- Take care with your calculations and be careful with directions. Remember:

DIRECTION IS VITAL      -  +

## Explosions

In an **explosion**:

- **There is only 1 stationary object at the start.** This object **explodes (splits up)** into **2 parts** which **travel in opposite directions in a straight line.**
- **Momentum is conserved.** (The total momentum just before the explosion = the total momentum just after the explosion.)
- **Kinetic energy increases.** At the start, the object is stationary, so has **zero kinetic energy.** It has **potential (stored) energy.** When the object explodes, this **potential energy** is changed into **kinetic energy** - the 2 parts move in opposite directions.

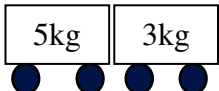
### Example Momentum Problem

Two trolleys, initially at rest and touching on a smooth, level surface, explode apart when a spring loaded pole is released on one trolley. Immediately after the explosion, the 5 kg trolley rebounds to the left at  $0.6 \text{ ms}^{-1}$ .

(a) Calculate the velocity of the 3 kg trolley immediately after the collision.

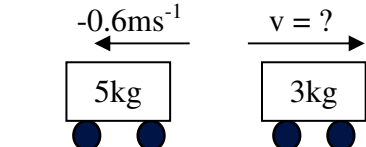
**DIRECTION IS VITAL**

Before collision



Total momentum =  $m_1u_1 + m_2u_2$   
 $= (5 \text{ kg} \times 0 \text{ ms}^{-1}) + (3 \text{ kg} \times 0 \text{ ms}^{-1})$   
 $= 0 + 0$   
 $= 0 \text{ kg ms}^{-1}$

After Collision



Total momentum =  $m_1v_1 + m_2v_2$   
 $= (5 \text{ kg} \times -0.6 \text{ ms}^{-1}) + (3 \text{ kg} \times v)$   
 $= -3 + 3v \text{ kg ms}^{-1}$

Total momentum just before collision = Total momentum just after collision

$$0 = (-3 + 3v)$$

$$-3v = -3$$

$$v = -3/-3 = \underline{1 \text{ ms}^{-1}} \text{ (i.e., } 1 \text{ ms}^{-1} \text{ to the right)}$$

(b) Show that in the collision kinetic energy increases.

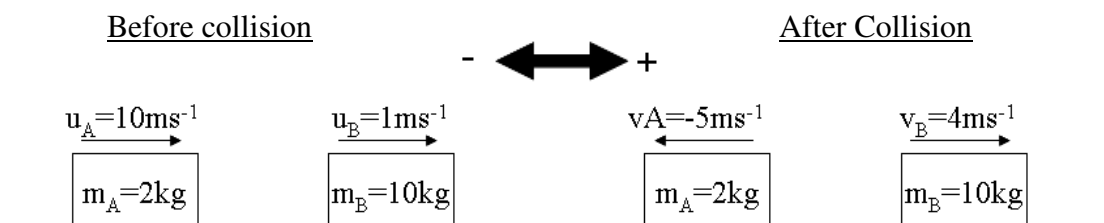
<p>Total kinetic energy</p> $= 0 + 0$ $= 0 \text{ J}$	<p>Total kinetic energy</p> $= (1/2 \times 5 \times 0.6^2) + (1/2 \times 3 \times 1^2)$ $= 0.9 + 1.5$ $= 2.4 \text{ J}$
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Total kinetic energy just before collision < Total kinetic energy just after collision  
 So, kinetic energy is gained in explosion

Momentum and Newton's Third LawNEWTON'S THIRD LAW

If object A exerts a force on object B, then object B exerts a force on object A which is equal in magnitude (size) but in the opposite direction.

We can infer "Newton's Third Law" using the "Law of Conservation of Linear Momentum."



Change in momentum of A =  $m_A(v_A - u_A) = 2 \times (-5 - 10) = -30 \text{ kg ms}^{-1}$

Change in momentum of B =  $m_B(v_B - u_B) = 10 \times (4 - 1) = 30 \text{ kg ms}^{-1}$

The change in momentum of A is **equal** in magnitude (size) but **opposite** in direction to the change in momentum of B.

Assume A and B are in contact for time  $t = 0.1$  seconds:

$$\text{Force acting on A} = m_A a = \frac{m_A(v_A - u_A)}{t} = \frac{-30}{0.1} = -300\text{N}$$

$$\text{Force acting on B} = m_B a = \frac{m_B(v_B - u_B)}{t} = \frac{30}{0.1} = 300\text{N}$$

The forces acting on A and B are **equal** in magnitude (size) but **opposite** in direction.

Sir Isaac Newton

Sir Isaac Newton (25 December 1642 – 20 March 1727) an English physicist, mathematician, astronomer, natural philosopher, alchemist, and theologian, who has been considered by many to be *the greatest and most influential scientist who ever lived*.

His monograph *Philosophiæ Naturalis Principia Mathematica*, published in 1687, lays the foundations for most of classical mechanics. In this work, Newton described **universal gravitation** and **the three laws of motion**, which dominated the scientific view of the physical universe for the next two centuries.

## Impulse and Change of Momentum

When a **force** acts on an object, the **force** is said to give the object an **impulse**. The **impulse** of a force is equal to the **force** (**F**) multiplied by the **time** (**t**) over which the force acts:

$$\text{impulse of force} = Ft$$

(Unit: Ns, Vector.)

If a **force** acts on an object of **mass m** travelling with **velocity u**, giving it a new velocity **v**, the **velocity** of the object changes by **(v-u)**, so the **momentum** of the object changes by **m(v-u)**.

The **impulse** of a **force (Ft)** changes the **momentum** of an object by **m(v-u)**, so:

$$\begin{aligned} \text{impulse} &= \text{change in momentum} \\ Ft &= m(v - u) \\ Ft &= mv - mu \end{aligned}$$

### Example 1

Calculate the impulse a force of 5 N exerts on an object which it pushes for 3 seconds.

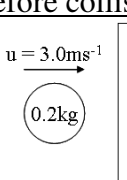
$$\begin{aligned} \text{impulse} &= Ft \\ &= 5 \text{ N} \times 3 \text{ s} \\ &= \underline{15 \text{ Ns}} \end{aligned}$$

### Example 2


A ball of mass 0.2 kg is thrown against a brick wall. The ball is travelling horizontally to the right at  $3.0 \text{ ms}^{-1}$  when it strikes the wall. It rebounds horizontally to the left at  $2.5 \text{ ms}^{-1}$ .

(a) Calculate the ball's change in velocity.

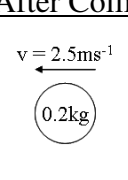
Before collision



- ← → +



After Collision



Change in velocity =  $v - u$   
 $= (-2.5) - 3.0$   
 $= \underline{-5.5 \text{ ms}^{-1}}$  (i.e.,  $5.5 \text{ ms}^{-1}$  to the left)

(b) Calculate the ball's change in momentum.

$$\begin{aligned} \text{Change in momentum} &= m(v - u) \\ &= 0.2 \times [(-2.5) - 3.0] \\ &= 0.2 \times -5.5 \\ &= \underline{-1.1 \text{ kg ms}^{-1}} \text{ (i.e., } 1.1 \text{ kg ms}^{-1} \text{ to the left)} \end{aligned}$$

(c) What is the impulse the wall exerts on the ball?

$$\text{Impulse} = \text{change in momentum} = \underline{-1.1 \text{ N s}} \text{ (i.e., } 1.1 \text{ N s to the left)}$$

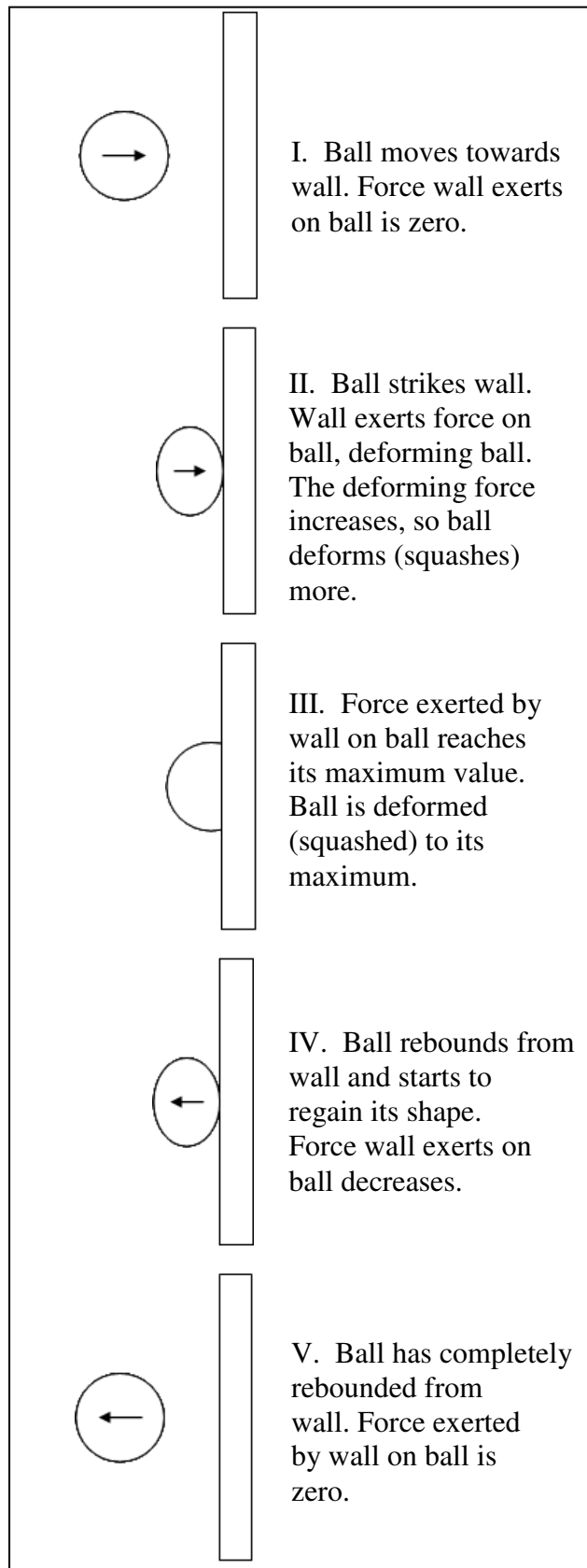
Example 3

A golf ball of mass 0.1 kg, initially at rest, was hit by a golf club, giving it an initial horizontal velocity of  $50 \text{ ms}^{-1}$ . The club and ball were in contact for 0.002 seconds. Calculate the average force that the club exerted on the ball.

$$\begin{aligned} Ft &= m (v - u) \\ F \times 0.002 &= 0.1 \times (50 - 0) \\ 0.002 F &= 5 \\ F &= 5/0.002 = \underline{2\,500 \text{ N}} \end{aligned}$$

## The Average Force Exerted During An Impact

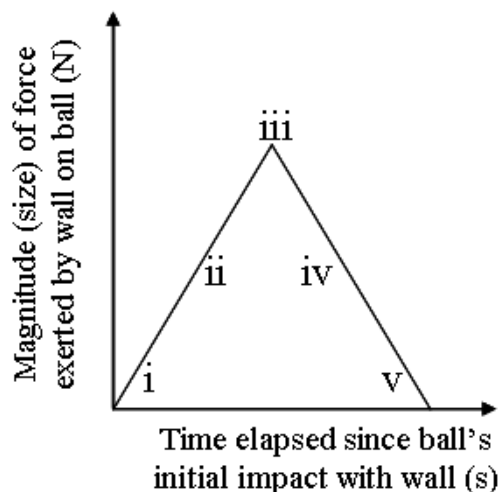
You will notice that the term **average force** has been used in connection with impulse. This is because the magnitude (size) of the force that acts during an impact changes during the impact - so we are only able to determine an average value for the force. For example, imagine a ball striking a wall. The force the wall exerts on the ball is zero before the impact, rises to a maximum as the ball strikes the wall and is deformed (squashed), then decreases to zero as the ball rebounds from the wall, regaining its shape.



This can be represented on a force-time graph.

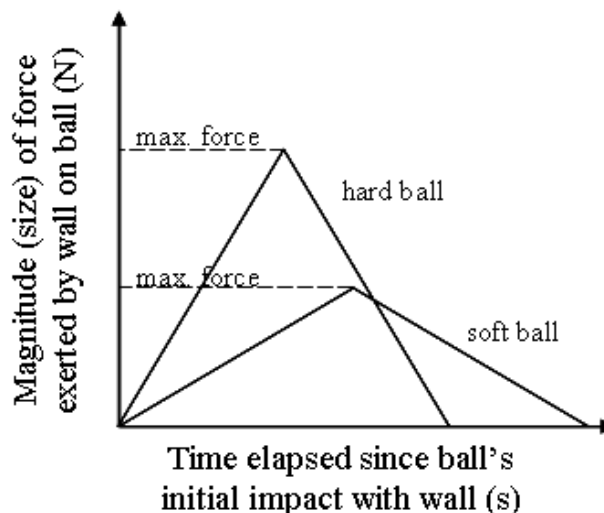
The area under the force-time graph represents:

- The **impulse** of the force exerted by the wall on the ball during its time of contact.
- The **change in momentum** experienced by the ball during its time of contact with the wall.



If the ball is **hard (rigid)**, like a golf ball, the **time of contact** between the ball and wall will be **small** and the **maximum force** exerted by the wall on the ball will be **large** (see graph below).

If the ball is **softer**, like a tennis ball, the **time of contact** between the ball and wall will be **longer** and the **maximum force** exerted by the wall on the ball will be **smaller** (see graph below).



<b>Impulse</b> <b>= Ft</b>	<b><u>Hard object</u></b>	<b><u>Shorter time of contact during impact</u></b>	<b><u>Larger maximum force</u></b>
	<b><u>Soft object</u></b>	<b><u>Longer time of contact during impact</u></b>	<b><u>Smaller maximum force</u></b>

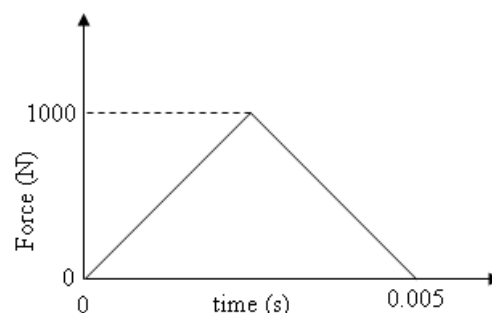
Boxers wear soft (padded) boxing gloves to reduce the damage their punches do to their opponents. A punch with a hard, bare fist will be in contact with the opponent's body for a very short time - so the maximum force exerted by the fist on the opponent will be large - so the damage caused will be large.

A punch with a soft, padded glove will be in contact with the opponent's body for a longer time - so the maximum force exerted by the glove on the opponent will be smaller - so the damage caused to the opponent will be less.

Helmets worn by American football players and motor cyclists contain soft foam padding which is in contact with the head. With no helmet on, a blow to the head during a collision will last for a very short time - so the maximum force exerted on the head will be large - so the damage caused to the head will be large. With a helmet on, a blow to the head during a collision will last for a longer time (due to the soft foam padding) - so the maximum force exerted on the head will be smaller - so the damage caused to the head will be less.

### Example

A ball of mass 0.2 kg is initially at rest. It is acted upon by a changing force, as shown on the graph to the right.



Determine:

(a) the impulse the force gives to the ball;

$$\begin{aligned}
 \text{Impulse} &= \text{Area under force-time graph} \\
 &= 1/2 \times \text{base} \times \text{height} \\
 &= 1/2 \times 0.005 \times 1\,000 \\
 &= 2.5 \text{ N s}
 \end{aligned}$$

(b) the change in momentum of the ball;

$$\begin{aligned}
 \text{Change in momentum} &= \text{Impulse} \\
 &= \text{Area under force-time graph} \\
 &= 2.5 \text{ kg ms}^{-1}
 \end{aligned}$$

(c) the velocity of the ball once the force has acted on it.

$$\begin{aligned}
 \text{Change in momentum} &= m(v - u) \\
 2.5 &= 0.2(v - 0) \\
 2.5 &= 0.2v \\
 v &= 2.5/0.2 \\
 v &= 12.5 \text{ ms}^{-1}
 \end{aligned}$$