Unit 1 - Our Dynamic Universe - Part 2

GRAVITATION

10. Projectiles and Satellites.
    - Resolving the motion of a projectile with an initial velocity into horizontal and vertical components and their use in calculations.
    - Comparison of projectiles with objects in free fall.
    - Newton‘s thought experiment and an explanation of why satellites remain in orbit.

GRAVITY AND MASS

11. Gravitational Field Strength of planets, natural satellites and stellar objects.
12. Calculating the force exerted on objects placed in a gravity field.
13. Newton‘s Universal Law of Gravitation.

SPECIAL RELATIVITY

14. Introduction to special relativity.
    - Relativity introduced through Galilean Invariance, Newtonian Relativity and the concept of absolute space.
    - Experimental and theoretical considerations (details not required) lead to the conclusion that the speed of light is the same for all observers.
    - The constancy of the speed of light led Einstein to postulate that space and time for a moving object are changed relative to a stationary observer.
    - Length contraction and time dilation.

THE EXPANDING UNIVERSE

15. The Doppler Effect and redshift in galaxies
    - The Doppler Effect is observed in sound and light.
    - For sound, the apparent change in frequency as a source moves towards or away from a stationary observer should be investigated.
    - The Doppler Effect causes similar shifts in wavelengths of sound & light. The light from objects moving away from us is shifted to longer wavelengths - redshift.
    - The redshift of a galaxy is the change in wavelength divided by the emitted wavelength.
    - For galaxies moving at non-relativistic speeds, redshift is the ratio of the velocity of the galaxy to the velocity of light.
    - (Note that the Doppler Effect equations used for sound cannot be used with light from fast moving galaxies because relativistic effects need to be taken into account.)

HUBBLE'S LAW

- Hubble's Law shows the relationship between the recession velocity of a galaxy and its distance from us.
- Hubble's Law leads to an estimate of the age of the Universe.
- Evidence for the expanding Universe
- Measurements of the velocities of galaxies and their distance from us lead to the theory of the expanding Universe.
- Gravity is the force which slows down the expansion.
- The eventual fate of the Universe depends on its mass.
- The orbital speed of the Sun and other stars gives a way of determining the mass of our galaxy.
- The Sun's orbital speed is determined almost entirely by the gravitational pull of matter inside its orbit.
- Measurements of the mass of our galaxy and others lead to the conclusion that there is significant mass which cannot be detected - dark matter.
- Measurements of the expansion rate of the Universe lead to the conclusion that it is increasing, suggesting that there is something that overcomes the force of gravity - dark energy.

BIG BANG THEORY

16. The temperature of stellar objects
    - Stellar objects emit radiation over a wide range of wavelengths.
    - Although the distribution of energy is spread over a wide range of wavelengths, each object emitting radiation has a peak wavelength which depends on its temperature.
    - The peak wavelength is shorter for hotter objects than for cooler objects.
    - Also, hotter objects emit more radiation per unit surface area at all wavelengths than cooler objects.
    - Thermal emission peaks allow the temperature of stellar objects to be measured.

17. Evidence for the Big bang
    - The Universe cools down as it expands.
    - The peak wavelength of cosmic microwave background allows the present temperature of the Universe to be determined.
    - This temperature corresponds to that predicted after the Big Bang, taking into account the subsequent expansion and cooling of the Universe.

Gravitation

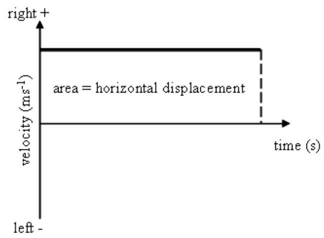
Projectiles and Satellites

Any object that is thrown, launched or falls through the air is known as a **projectile**. The path travelled by the **projectile** is known as its **trajectory**.

In our study of **projectile motion**, we assume that **air resistance** has no effect. In reality, **air resistance** makes the values we obtain from our calculations slightly different from those obtained from real-life situations - but our calculated values are reasonably accurate.

When dealing with **projectile motion** for an object projected **horizontally**, we treat the motion as independent **horizontal** and **vertical** components:

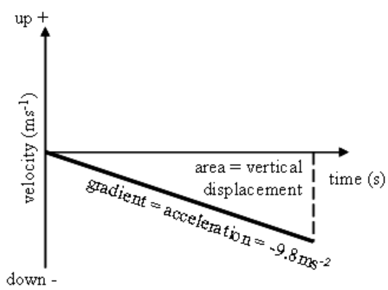
Horizontal Motion



- **Always uniform** (**constant**) **velocity equal to the horizontal projection velocity**, i.e., if a projectile is fired **horizontally** at 5 ms^{-1} to the right, its **horizontal component of velocity** will remain at 5 ms^{-1} to the right, until it hits the ground.
- The **larger** the **horizontal component of** velocity, the **further** the **range** (horizontal distance travelled) before hitting the ground.
- Because there is **no acceleration** in the **horizontal** direction, the **three equations of motion do not apply**. You can only apply the equation:

$$\text{horizontal displacement} = \text{horizontal velocity x time}$$
$$s_h = v_h t$$

Vertical Motion



- At the instant the projectile is launched **horizontally**, it is not moving downwards, so **initial downward velocity** (**u**) = **0 ms^{-1}**.
- The projectile **accelerates** towards the ground under the influence of **gravity**. Using the sign convention from before, **a = -9.8 ms^{-2}**.
- The higher the starting point above the ground, the greater the **final vertical velocity** (**v**) just before hitting the ground. (**v** is **downward**, so should be given a **negative** value).

The **three equations of motion** apply:

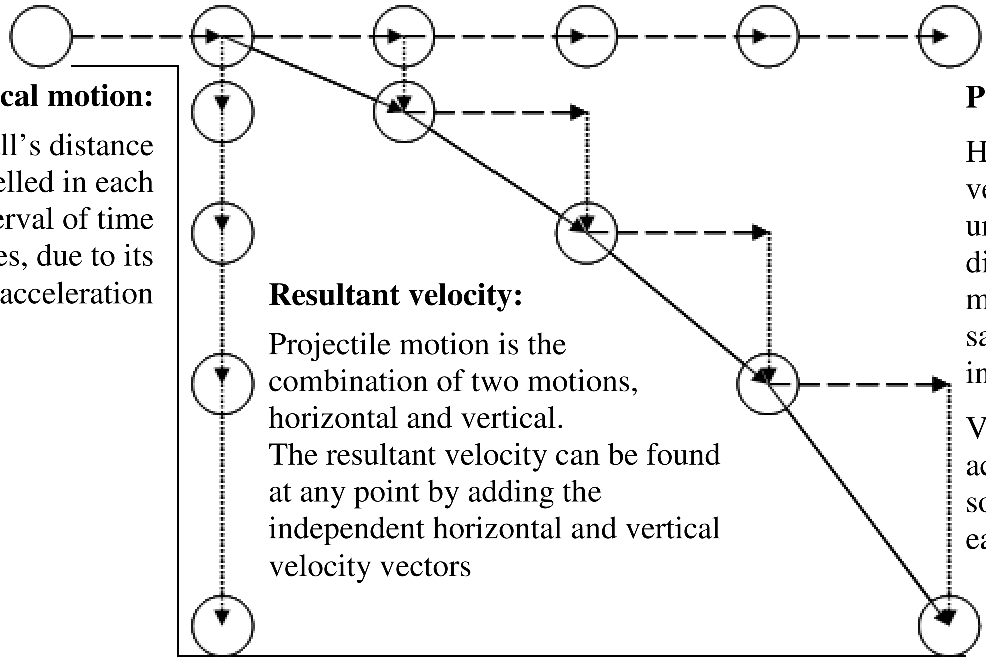
$$v = u + at \qquad s = ut + \frac{1}{2}at^2 \qquad v^2 = u^2 + 2as$$

Projectiles vs objects in freefall

It is helpful to compare the **path of a projectile** with that of an object **dropped vertically** downward. The diagram shows the position of a ball released from one position at **equal intervals of time**.

Horizontal motion:

Ball travels the same distance for each time interval



Vertical motion:

The ball's distance travelled in each interval of time increases, due to its acceleration

Resultant velocity:

Projectile motion is the combination of two motions, horizontal and vertical.
The resultant velocity can be found at any point by adding the independent horizontal and vertical velocity vectors

Projectile motion:

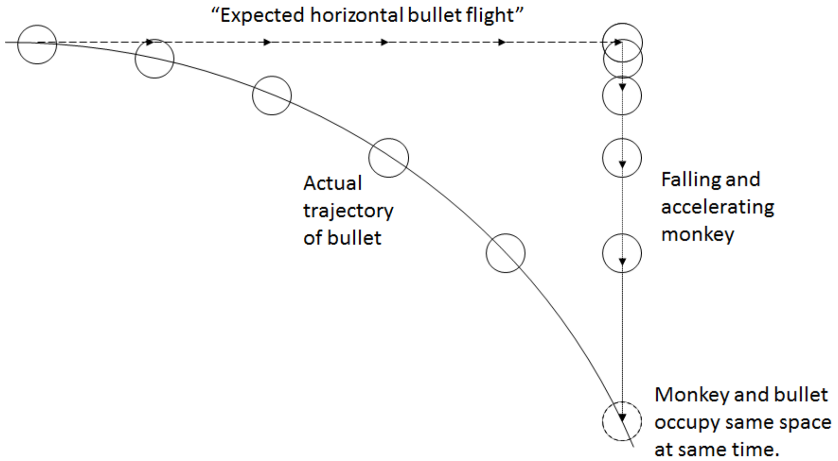
Horizontally, the ball's velocity remains unchanged, so like the diagram for horizontal motion, the ball travels the same distance for each time interval.

Vertically, the ball is accelerating due to gravity, so it's distance travelled in each time interval is

Monkey and Hunter Demonstration

The Monkey and the Hunter is a classic Physics demonstration to show that a projectile fired horizontally and an objected dropped vertically fall at the same rate.

The monkey, shocked by the sudden "bang" of the gun lets go of his branch at the instant the hunter pulls his trigger. By falling from the tree that the hunter was aiming at the monkey now assumes that the bullet will fly over his head.



The question is, did the monkey attend Higher Physics classes, and was he right?

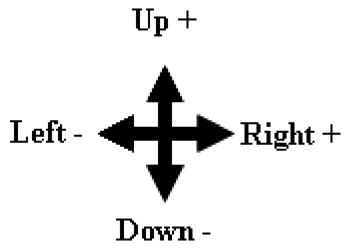
<u>Example</u>

A projectile is fired horizontally from the top of a 45 m high wall at 8.0 ms^{-1}.

(a) What time does the projectile take to hit the ground?

For vertical motion, s = -45 m, u = 0 ms^{-1}, a = -9.8 ms^{-2}

$$s = ut + \frac{1}{2}at^2$$
$$-45 = (0 \times t) + (0.5 \times -9.8 \times t^2)$$
$$-45 = 0 + (-4.9t^2)$$
$$t^2 = \frac{-45}{-4.9} = 9.2$$
$$t = 3s$$



(b) What is the projectile's range (horizontal distance travelled)?

For horizontal motion, v_h = 8.0 ms^{-1}, t = 3.0 s

$$s_h = v_h \times t$$
$$= 8.0 \times 3.0$$
$$= 24\text{m right}$$

(c) What is the projectile's horizontal component of velocity just before hitting the ground?

Horizontal component of velocity remains constant, so v_h = 8.0 ms^{-1} right

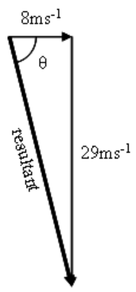
(d) What is the projectile's vertical component of velocity just before hitting the ground?

For vertical motion, u = 0 ms^{-1}, a = -9.8 ms^{-2}, t = 3.0 s

$$v = u + at$$
$$= 0 + (-9.8 \times 3.0)$$
$$= 0 - 29.4$$
$$= -29ms^{-1}\ (29ms^{-1}\text{ downwards})$$

(e) What is the projectile's resultant velocity just before hitting the ground?

Resultant velocity of the projectile just before it hits the ground is a combination of the horizontal and vertical components of velocity at that instant:



$$a^2 = b^2 + c^2$$
$$= 8^2 + 29^2$$
$$= 64 + 841^2$$
$$= 905$$
$$a = \sqrt{905} = 30ms^{-1}$$

$$\tan\theta = \frac{O}{A} = \frac{29}{8} = 3.6$$
$$\theta = \tan^{-1}(3.6) = 74°$$

Resultant velocity of projectile just before hitting ground is <u>30 ms^{-1} at 74° below the horizontal</u>.

Example

A long-range artillery shell is fired from level ground with a velocity of 500 ms^{-1} at an angle of 30° to the horizontal. Determine:

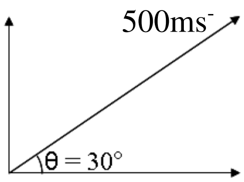
a. the vertical component of the initial velocity

Vertical component of velocity

= 500 sin 30°

= 500 x 0.5

= 250 ms^{-1}.



b. the horizontal component of the initial velocity

Horizontal component of velocity

= 500 cos 30°

= 500 x 0.866

= 433 ms^{-1}.

c. the greatest height the shell reaches;

$$v^2 = u^2 + 2as$$
$$0^2 = 250^2 + (2 \text{ x } -9.8 \text{ x } s)$$
$$0 = 62\,500 - 19.6\,s \qquad \text{(use vertical velocity)}$$
$$19.6\,s = 62\,500$$
$$s = 62\,500/19.6 = 3\,200 \text{ m}$$

d. the time taken to reach that height;

$$v = u + at$$
$$0 = 250 + (-9.8 \text{ x } t)$$
$$0 = 250 - 9.8\,t \qquad \text{(use vertical velocity)}$$
$$9.8\,t = 250$$
$$t = 250/9.8 = 26 \text{ s}$$

e. the total time the shell is in the air;

Total time shell is in air = 2 x 26 s = 52 s

f. the horizontal distance the shell travels (i.e., its range).

$$s_h = v_h t$$
$$= 433 \text{ x } 52 \qquad \text{(use horizontal velocity)}$$
$$= 23\,000 \text{ m right}$$

Newton's Thought Experiment - Newton's Cannon

As well as giving us the three laws, Isaac Newton also came up with an ingenious thought experiment for satellite motion that predated the first artificial satellite by almost 300 years.

Essentially Newton suggested that if a cannon fired a cannonball **it would fall towards the Earth**. If it was fired at ever higher speeds then at ***some speed*** it would fall towards the Earth but never land since the **curvature of the Earth would be the same as the flight path** of the cannonball.

This would then be a satellite. The object remains in orbit because **it is being pulled to the Earth by gravity**, **NOT** because it has ***escaped gravity***.

This is how satellites remain in orbit.

If **gravity** was suddenly **switched off**, the object would **continue in a straight line**.

Newton's Thought Experiment became a reality when the first artificial satellite (Sputnik) was launched in 1957.

The world's first artificial satellite was about the size of a beach ball (58 cm in diameter), weighed only 83.6 kg, and took about 98 minutes to orbit the Earth on its elliptical path.

The launch ushered in new political, military, technological, and scientific developments. While the Sputnik launch was a single event, it marked the start of the space age and the US vs USSR space race.

On January 31, 1958, the tide changed, when the United States successfully launched Explorer I. This satellite carried a small scientific payload that eventually discovered the magnetic radiation belts around the Earth, named after principal investigator James Van Allen.

Gravity and Mass

Formation of the solar system

Gravity is responsible for the **formation of the solar system**.

At some point, generally believed to be between 4 and 5 billion years ago, a huge cloud of gas collapsed due to its own gravitational attraction. Most of this gas collapsed into the gravitational centre forming the Sun. However small amounts of mass formed a disk that circled the newly formed star. The gravitational attraction between the particles that made up this disk then resulted in the formation of the planets we can observe today.

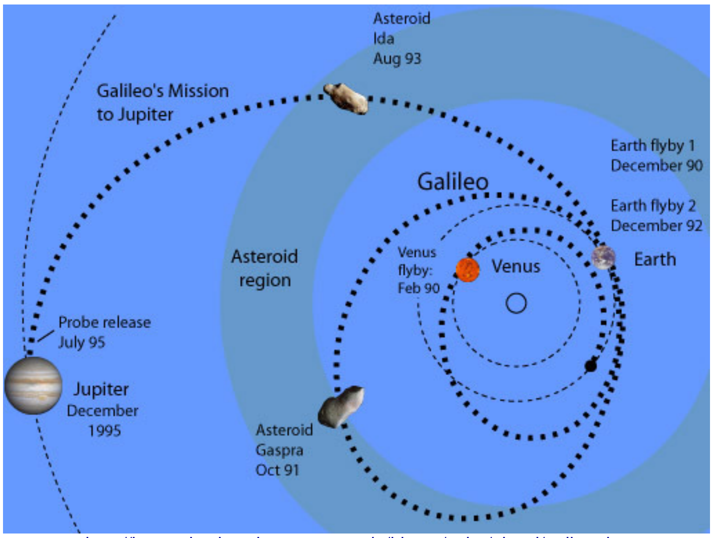
This process is known as the **aggregation of matter**.

Gravity and orbits

As we know the Earth follows a curved path around the Sun. This means that, according to the **first law of motion**, there must be an **unbalanced force** acting on the Earth. This force is provided by the **gravitational pull of the Sun**. The same is true for the orbit of the Moon. The Earth's gravity is exerting a force on the Moon. These forces are due to the masses of both bodies (see Newton's Law of Universal Gravitation)

Gravity assist

Gravity has also been used to assist exploration in the solar system. During the 1990s NASA sent a probe called Galileo to investigate the planet Jupiter. They found that it would be very difficult to give the probe enough energy to reach Jupiter directly so they devised an ingenious method of boosting the probe's speed during its flight.



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This was done by flying the probe past Venus and using the **planet's gravitational field** to ***sling shot*** it with increased speed. The process was then repeated using the Earth's gravitational field (twice) before the probe had enough speed to reach Jupiter.

<u>Gravitational field strength</u>

What is a **<u>field</u>** in Physics?
A FIELD is an area or volume in which a **force** is exerted on a **particle** or **mass**, at a **<u>distance</u>** from the **<u>cause</u>** of that field.

<u>Types of fields</u>

Gravitational Field: A place where a mass experiences a force
Electric Field: A place where a charge experiences a force
Magnetic field: A place where a moving charge experiences a force.

We will go on to learn about all three types of field.

Gravitational field strength is defined as the gravitational force acting on a unit mass located at that point.

$$g = \frac{F}{m}$$

g is gravitational field strength (Nkg^{-1})
F is the force (N)
m is the mass (kg)

$$W = mg$$

We refer to the gravitational force as the weight, W. Gravitational field strength, *g* is a VECTOR quantity, having the same direction as the weight.

For masses near a planet, the direction of *g* (and therefore W) is towards the centre of the planet.

<u>When does a = g?</u>

If a mass is allowed to fall freely, (ignoring air resistance), the unbalanced force is the W, and it accelerates downward with a constant acceleration, a.

Acceleration due to gravity in free fall is EQUAL to gravitational field strength.

If we can measure the acceleration of a falling object, we can determine g.

<u>Example</u>

During the Apollo 15 moon landing, the astronauts dropped a hammer on the moon. The hammer fell 1.10 m, taking 1.15s to do so. Calculate the gravitational field strength on the moon?

(Dropped from rest so u = 0)

$$s = ut + \frac{1}{2}at^2$$

$$1.10 = 0 + 0.5 \text{ x a x } 1.15^2$$

$$a = 1.66 \text{ ms}^{-2}$$

$$\text{so, } g = 1.66 \text{ N kg}^{-1}$$

<u>Note</u>: The value of *g* can be very different for different celestial objects e.g.

Sun: 270 N kg^{-1} **Jupiter:** 25 N kg^{-1} **Pluto:** 0.7 N kg^{-1}

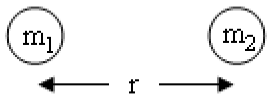
Newton's Law of Universal Gravitation

Newton's law of universal gravitation proposed that **every body with mass** will exert a force on **every other body with mass.**

The theory states that the force of gravitational attraction is dependant on the **masses of both objects** and is inversely proportional to the **square of the distance** that seperates them.

$$F = \frac{Gm_1m_2}{r^2}$$

F is the force (N)
m_1 and m_2 are the two masses (kg)
r is the distance between them (m)
G is the gravitational constant (Nm^2kg^{-2})



Note: The value G is one of the most difficult constants to measure accurately. Cavendish determined it in the late 1700's, and Boys (1855-1944) improved on its accuracy. The value we use in Higher Physics is **$G = 6.67 \times 10^{-11} Nm^2kg^{-2}$**.

Examples

The formula allows us to calculate the force of gravity between point masses or spherical objects (or any object that we assume to be spherical!)

Everyday objects

Calculate the gravitational force between a folder of mass 0.3 kg and a pen of mass 0.05 kg on a desk, 0.25 m apart.

$$F = \frac{Gm_1m_2}{r^2}$$
$$= \frac{6.67 \times 10^{-11} \times 0.3 \times 0.05}{(0.25)^2}$$
$$= 1.60 \times 10^{-11}\,N$$

Subatomic objects

Calculate the gravitational force between a proton and a neutron in the nucleus of an atom. The separation between a proton and neutron in a nucleus is 0.84×10^{-13}m.

$$F = \frac{Gm_1m_2}{r^2}$$
$$= \frac{6.67 \times 10^{-11} \times 1.67 \times 10^{-27} \times 1.67 \times 10^{-27}}{(0.84 \times 10^{-15})^2}$$
$$= 2.64 \times 10^{-34}\,N$$

Note the force of gravity is clearly insignificant except when dealing with very large mass!

Gravitational force and Weight

For the same pen of mass 0.05 kg and the Earth of mass 5.97×10^{24} kg, calculate the gravitational force and weight. The Earth has a radius of 6.38×10^6 m.

$$F = \frac{Gm_1m_2}{r^2}$$
$$= \frac{6.67 \times 10^{-11} \times 5.97 \times 10^{24} \times 0.05}{(6.38 \times 10^6)^2}$$
$$= 0.489N$$

$$W = mg$$
$$= 0.05 \times 9.8$$
$$= 0.49N$$

Notice that these values are the same!

INTRODUCTION TO RELATIVITY

Useful Definitions and ideas

inertial reference frames: Simply two places that are moving at constant speeds relative to one another

absolute reference frame: A unique, universal frame of reference from which everything could be defined or measured. Einstein's theories prove no such reference frame exists.

the ether: Early theories suggested that electromagnetic waves (light) required a medium (a space-filling substance or field) to travel through. This ether was believed to be an absolute reference frame. Modern theories have no requirement for this idea, and indeed the Michelson–Morley experiment performed in 1887 provided no evidence for such a field.

Background definitions

Galilean Invariance

Galileo was one of the first scientists to consider the idea of relativity.

He stated that the laws of Physics should be the same in all **inertial frames of reference**.

He first described this principle in 1632 using the example of a ship, travelling at **constant velocity**, without rocking, on a smooth sea; any **observer** doing experiments below the deck would not be able to tell whether the ship was **moving** or **stationary**.

In other words, the **laws of Physics** are the same whether **moving at constant speed** or when at **rest**.

Newtonian Relativity

Newton followed this up by expanding on Galileo's ideas.

He introduced the idea of **absolute**, or **universal space time**.

He believed that it was the same time at all points in the **universe** as it was on **Earth**, not an unreasonable assumption.

According to Newton, **absolute time** exists independently of any perceiver and progresses at a consistent rate throughout the universe. **Absolute space**, in its own nature, without regard to anything external, remains always **similar** and **immovable**.

Example on relativity (at slow speeds)

You are standing in the back of a jeep moving at 30 mph directly toward a monkey in a tree, and you fire an arrow from a bow, which leaves the bow at a speed of 60 mph.

Relative to the monkey, what speed is the arrow traveling?

Relative speed of arrow = 60 + 30
= 90 mph

Example on relativity (at high speeds)

You are standing in the front of a train moving at 90 mph ($40 ms^{-1}$) and you shine a torch ahead of you.

What is the speed of the light

(a) Relative to you?

Relative to you: Speed of light = $3 \times 10^8 ms^{-1}$

(b) Relative to a stationary observer?

Relative to observer: Speed of light = $3 \times 10^8 + 40$ exceeds speed of light!

Einstein's *Special* Relativity
The experiments carried out by Albert Michelson and Edward Morley in 1887 led to the understanding that there is no underlying ether. There is **no absolute frame of reference** and so, either Maxwell's Equations of electromagnetism are different in every frame of reference or they are absolute (and the speed of light too, as a result) and we have to adapt other aspects of our understanding.

This led to **Albert Einstein** publishing **the theory of special relativity** in 1905. This explains how to interpret motion between different *inertial frames of reference*.

Thanks to Michelson and Morley, Einstein did not appeal to the *ether* as an absolute frame of reference. Instead, he explained observations in terms of the relative motion between two objects.

In essence, say for example, you and another astronaut, Amber, are moving in different spaceships and want to compare observations, all that matters is how fast you and Amber are moving with respect to each other.

Why SPECIAL relativity?
It is only *special* because this is a special case, where the motion between observers is uniform.

UNIFORM MOTION - Traveling in a straight line at a constant speed

Einstein's Postulates

The postulates on which Einstein based his theory of Special Relativity are:

1. When two observers are moving at constant speeds relative to one another, they will observe the same laws of physics within their own frames of reference.

You cannot do any experiment to tell if you are in a stationary frame of reference or one moving at constant speed, since every observer is "stationary" within their own reference frame.

.

2. The speed of light (in a vacuum) is the same for all observers, regardless of their motion relative to the light source.

Since Maxwell's Equations (determining the constant for the speed of light) are consistent in all inertial reference frames. Evidence continued to support this postulate even though at the time it was just a postulate.

Speed of light depends on the medium in which it is travelling, but cannot exceed 3×10^8 ms^{-1}, the speed of light in a vacuum.

This means that if you were:

- at rest then speed of light is 3×10^8 ms^{-1}
- in a moving frame of reference then the speed of light remains at 3×10^8 ms^{-1} when viewed by the person in that frame of reference

No particular frame of reference is any more 'stationary' than any other.

As a consequence of Einstein's theories, measured time and length will change for a moving system depending on who is observing the system. Speed of light, c, cannot change so from $s = vt$ the time and the distances must change.

For an external observer **time will appear to dilate** for the traveller and **length will appear to contract** for the traveller. The traveller, however, will not notice being "contracted".

The traveller will make the same observations of the stationary observer, since from their point of view the stationary observer is moving!

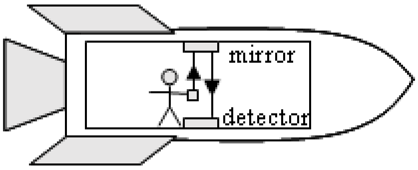
Understanding time dilation

Einstein's theory of Special Relativity created a fundamental link between **space** and **time**.
The universe can be viewed as having:

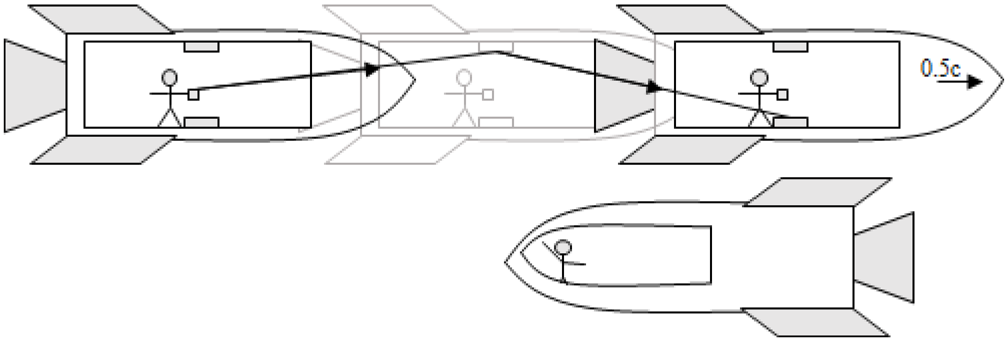
Three space dimensions
- up/down
- left/right
- forward/backward

One time dimension

Einstein's Thought Experiment



Imagine that you're on a spaceship and holding a laser so it shoots a **beam of light** directly up, striking a mirror you've placed on the ceiling. The **light beam** then comes back down and strikes a detector. We shall call this an **event**.



However, the spaceship is traveling at a constant speed of half the speed of light, 0.5c. According to Einstein, this **makes no difference** to you — you can't even tell that you're moving. However, if astronaut Amber were spying on you, it would be a different story.

Amber would see your **beam of light** travel upward along a **diagonal path**, strike the mirror, and then travel downward along a **diagonal path** before striking the detector. In other words, you and Amber would see **different paths** for the light and, more importantly, those paths **aren't** even the **same length**.

Since Einstein's 2nd postulate states that the speed of light is the same for all observers, because the beam Amber observes travels a greater distance, **it would appear to happened in a longer time** (if the speed of light is constant).

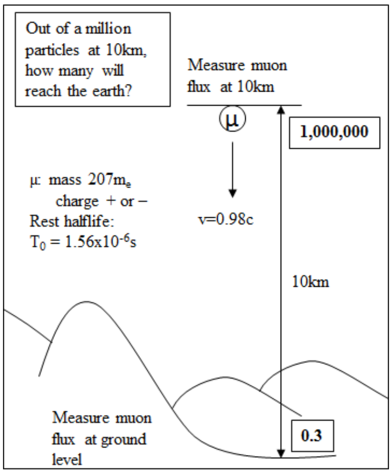
In other words, in Amber's case, it appears that **more time has passed** for the event to happen. More time has not passed and so on a ship moving very quickly time appears to pass slower. **Moving clocks run slow!**

This phenomenon is known as **time dilation**.

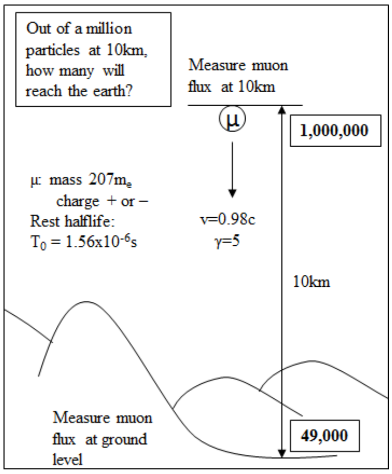
Top Tips **What will you need to be able to do?** - examples lie ahead…

- State that stationary and moving observers will perceive light to travel at 3 x 10^8 ms^{-1}.
- Determine the apparent contracted length of a moving object given its velocity relative to a stationary observer, and the length of the object.
- Determine the stationary length of a space craft given its contracted length and its velocity.
- Determine the apparent dilated time for a moving object given its velocity relative to a stationary observer and the time measured by the stationary observer.
- Determine the unaffected time of a stationary clock given the contracted time of a moving clock and its velocity.

Evidence for time dilation - Muons



There is experimental evidence to support Einstein's theory. A particle known as a **muon** (see Particles and Waves unit) is created in the upper atmosphere. It only exists for a short time, having a half-life of 1.56 x 10^{-6} s. This means that for every **million muons** created at a height of 10 km, only **0.3** should reach the surface of the Earth.



However, around **50 000** are detected. This is because the muon is traveling **very fast toward Earth**, and so its **clock runs slowly** (time is dilated) compared to an observer on Earth, and so the muon reaches the ground!

The time dilation formula

Moving objects run slower clocks as observed by a stationary observer. GPS satellites have to adjust their clocks to match the ones on Earth.

The key to using the time dilation formula is understanding the terms proper time (t) and dilated time (t').

Proper time t: The time measured in the frame in which the clock is at rest relative to the event is called the "proper time". The time will always be shorter in the rest frame.

Dilated time t': If you are watching from somewhere else and you look at the clock on the moving object you will measure t', the dilated time. The clock will be seen to be running slow.

The equation linking these two times is given as

v = speed object is moving at
c = speed of light
t = time measured by the observer at rest **with respect to the event**
t' = time measure by another observer moving **relative to the event**

$$t' = \frac{t}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Examples

1. The lifetime of a muon is 2.2μs. Muons travel at 99% the speed of light. How long do muons last for here on Earth?

$$t = 2.2\times10^{-6}\,s$$
$$v = 0.99c = 0.99\times3.0\times10^{8}\,ms^{-1} = 2.97\times10^{8}\,ms^{-1}$$
$$c = 3.0\times10^{8}\,ms^{-1}$$
$$t' = \frac{t}{\sqrt{1-\frac{v^2}{c^2}}}$$
$$t' = \frac{2.2\times10^{-6}}{\sqrt{1-\frac{(2.97\times10^{8})^2}{(3.0\times10^{8})^2}}}$$

Or,

$$t' = \frac{2.2\times10^{-6}}{\sqrt{1-(0.99)^2}}$$
$$t' = 1.6\times10^{-5}\,s = 16\text{ micro seconds}$$

2. A rocket is traveling at a constant 2.7×10^{8} ms^{-1} compared to an observer on Earth. The pilot measures the journey as taking 240 minutes. How long did the journey take when measured from Earth?

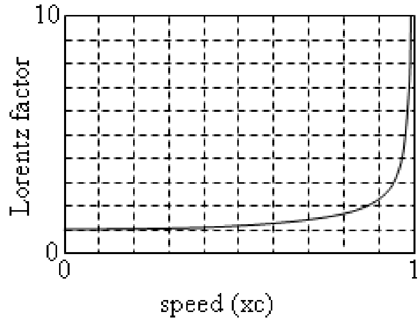
$$t = 240\text{ minutes}$$
$$v = 2.7\times10^{8}\,ms^{-1}$$
$$t' = \frac{t}{\sqrt{1-\frac{v^2}{c^2}}}$$
$$t' = \frac{240}{\sqrt{1-(0.9)^2}}$$
$$t' = 551\text{ minutes}$$

We do not notice this time difference in every day life because for speeds of 0.1c or smaller the *Lorentz factor* is approximately equal to 1.

The **Lorentz Factor** (γ) is part of the time dilation equation. It takes into account the speed of the object.

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

We can see that for small speeds (i.e. less than 0.1 times the speed of light) the Lorentz factor is approximately 1 and relativistic effects are negligibly small.



The Twin Paradox

The Twin Paradox helps us to understand the limitations of Special Relativity.

In this thought experiment, one of a pair of twins leaves on a high speed space journey during which he travels at a large fraction of the speed of light (let's say 0.995c) while the other remains on the Earth (stationary, relatively speaking). Because of time dilation, time is running more slowly in the spacecraft (by a specific factor, 10) as seen by the earthbound twin and the traveling twin will find that the earthbound twin will be older at the end of the journey, let's say 1 year, from the travelers point of view.

Is this real?

The basic question about whether time dilation is real is settled by the muon experiment, as well as experiments using atomic clocks and long haul aircraft, as well as satellites.

Would one twin really be younger?

The clear implication is that the traveling twin would indeed be younger, but the scenario is complicated by the fact that to meet again the traveling twin must be accelerated up to traveling speed, turned around, and decelerated again upon return to Earth.

Accelerations are outside the realm of special relativity and require general relativity. Despite the experimental difficulties, an experiment on a commercial airline confirms the existence of a time difference between ground observers and a reference frame moving with respect to them.

Worse still, the traveler will not experience this time dilation happening to them while travelling. Instead the space through which they travel is contracted (by the same factor, 10). As a result they have less far to travel and, believe it or not, they arrive sooner than expected after, you guessed it, just 1 year!

Stationary Twin	Travelling Twin
t' = time measured by observer	t = time measured by traveller
$t = 1\text{ year}$ $v = 0.995c$ $t' = \frac{t}{\sqrt{1-\frac{v^2}{c^2}}}$ $t' = \frac{1\text{ year}}{\sqrt{1-\frac{(0.995c)^2}{(c)^2}}}$ $t' = \frac{1\text{ year}}{\sqrt{1-(0.995)^2}}$ $t' = 10\text{ years}$ observer experiences 10 years	traveller experiences 1 year

Length Contraction

Another implication of Einstein's theory is the observed decrease in length of an object which is moving at high speeds. This is called length contraction, or more formally Lorentz contraction, and is only noticeable at a substantial fraction of the speed of light, and only in the direction parallel to the direction in which the observed body is travelling.

v = speed object is moving at
c = speed of light
l = length measured by the observer at rest with respect to the moving object
l' = contracted length of object as measured by another observer

$$l' = l\sqrt{1-\frac{v^2}{c^2}}$$

Example

A rocket has a length of 10m when at rest on the Earth. An observer on Earth watches the rockets passing at a constant speed of 1.5 x 10^8 ms^{-1}. Calculate the length of the rocket as measured by the observer.

$l = 10\,m$

$v = 1.5 \times 10^8\ ms^{-1} = 0.5c$

$c = 3.0 \times 10^8\ ms^{-1}$

$$l' = l\sqrt{1-\frac{v^2}{c^2}}$$

$$l' = 10\sqrt{1-(0.5)^2}$$

$$l' = 8.7m$$

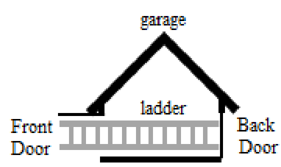
Why an observer measures the length of a fast moving object as being contracted is related to the idea that the length of any object is found by knowing where the two ends of the object are, and determining the distance between them.

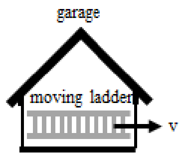
In relative motion, the position of both ends of the object cannot be determined simultaneously, which results in a contracted length measurement.

The Ladder paradox

The ladder paradox helps to explain how simultaneity relates to length contraction.

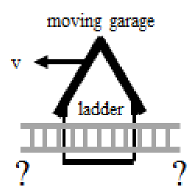
The problem starts with a **ladder** and an accompanying **garage** that is too small to contain the ladder.





Through the relativistic effect of **length contraction**, the ladder can be made to fit into the garage by running it into the garage at a **high enough speed**.

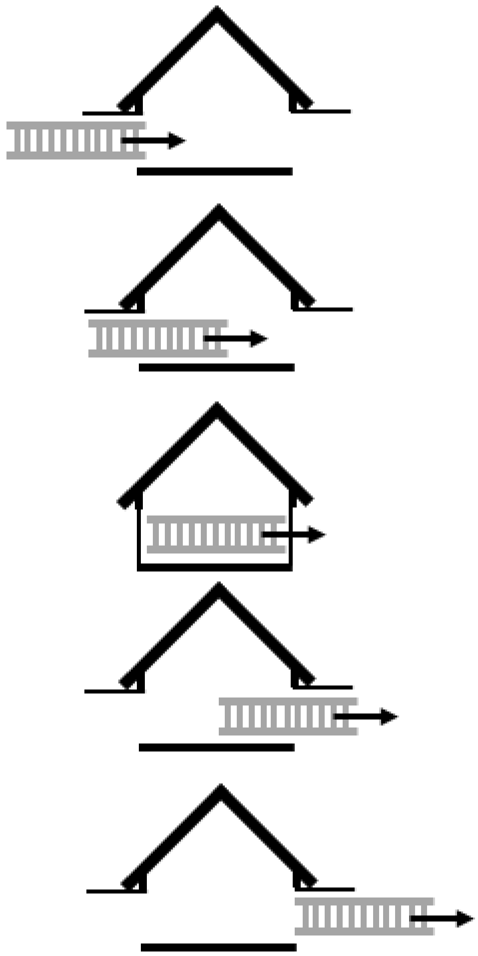
Conversely, through **symmetry**, from the reference frame of the ladder it is the garage that is moving with a relative velocity and so it is the garage that undergoes a **length contraction**. From this perspective, the garage is made even **smaller** and it is impossible to fit the ladder into the garage.



Ladder paradox - solution

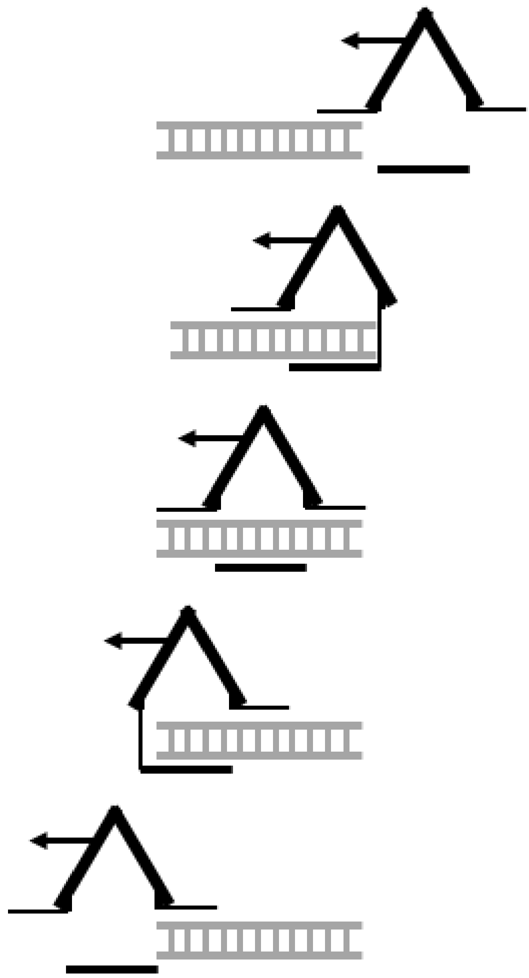
Both the ladder and garage occupy their own **inertial reference frames**, and thus both frames **are equally valid frames** from which to view the problem.
The solution to the apparent paradox lies in the fact that what one observer (e.g. the garage) **considers as simultaneous** does not correspond to what the **other observer** (e.g. the ladder) **considers as simultaneous**.
A clear way of seeing this is to consider a **garage with two doors** that swing shut to contain the ladder and then open again to let the ladder out the other side.

Garage reference frame



From the perspective of the **garage**, the **length-contracted ladder** is short enough to fit **entirely** inside. The instant the ladder is fully inside the garage, the front and back doors close **simultaneously**. Then, since the ladder is still moving at considerable speed, the front and back doors **simultaneously** open again to allow the ladder to exit.

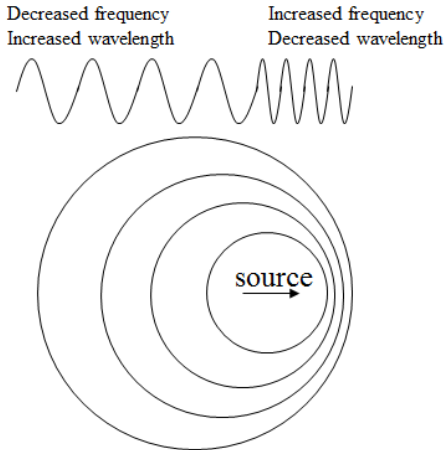
Ladder reference frame



From the perspective of the **ladder**, the back door (right) closes and opens, then after the **garage passes over the ladder**, the front door (left) closes and opens.

THE EXPANDING UNIVERSE

The Doppler effect and redshift of galaxies.



The **Doppler Effect** is the change in frequency **observed** when a source of sound waves is moving relative to an observer.

A good example of this is an ambulance's siren as it drives past.

In general **more sound waves** are received per second when the source of sound waves is **moving towards** the observer and so the frequency heard by the observer is **increased**. Similarly **fewer sound waves** are received per second when the source of sound waves is **moving away** from the observer and so the frequency heard by the observer is **decreased**.

If the source is moving towards the observer the frequency heard by the observer (f_o) is greater than the frequency of the source (f_s).

Hence if the source is moving away from the observer the frequency heard by the observer (f_o) is less than the frequency of the source (f_s).

The equation linking these two frequencies is:

f_o is the frequency heard by the observer (Hz)
f_s is the frequency of the source of the sound (Hz)
v is the velocity of the sound waves (ms^{-1})
v_s is the velocity of the source (ms^{-1})

$$f_0 = f_s\left(\frac{v}{v \pm v_s}\right)$$

If the source comes **towards** the observer, the frequency **increases,** use **-ve**
If the source goes **away** from the observer, the frequency **decreases,** use **+ve**

1. If a source of sound waves of frequency 100 Hz, is travelling towards an observer at 40 ms^{-1} then the frequency heard by the observer will be…

$f_o =$
$f_s = 100\,\text{Hz}$
$v = 340\,\text{ms}^{-1}$
$v_s = 40\,\text{ms}^{-1}$

$$f_0 = f_s\left(\frac{v}{v \pm v_s}\right)$$
$$f_0 = 100\left(\frac{340}{340 - 40}\right)$$
$$f_0 = 113\text{Hz}$$

2. If a source of sound waves of frequency 50 Hz, is travelling away from an observer at 10 ms^{-1} then the frequency heard by the observer will be…

$f_o =$
$f_s = 50\,\text{Hz}$
$v = 340\,\text{ms}^{-1}$
$v_s = 10\,\text{ms}^{-1}$

$$f_0 = f_s\left(\frac{v}{v \pm v_s}\right)$$
$$f_0 = 50\left(\frac{340}{340 + 10}\right)$$
$$f_0 = 48.6\text{Hz}$$

Applications of Doppler Effect

Ultrasound in Medicine

When a beam of ultra sound is sent into the body, any motion within the body causes a Doppler shift in the reflected ultrasound. This can be used to check the heart beat or blood flow of an unborn baby or find a deep vein thrombosis. Continuous, rather than pulse, ultrasound is used. Any differences between the ingoing and returning frequencies is heard as a tone or displayed on a screen

Answering questions

Re-read your answers and make sure that the examiner can understand what you mean.

When frequency increases, wavelength decreases and vice-versa. So writing...

'The Doppler effect is the change of wavelength and frequency when a source moves. If he source moves towards the observer it increases.'

...is not clear enough; the first sentence is OK, but the second could refer to the frequency or the wavelength so will gain no marks.

When you describe beams of travelling waves, some words can be used for distance or for time. Words such as 'longer' and 'shorter' may be unclear.

The Doppler Effect for light

In the late 19th and early 20th centuries, astronomers observed that light from distant stars showed similar characteristics to the Doppler Effect for sound.

When observing distant nebulae, astronomers observed that the wavelength appeared to increase. This suggested that the light was coming from a source, which was moving away from the earth. This is known as **Red Shift**.

Why?
When a source is moving **away from** an observer, the frequency observed by the observer is **decreased** because

$$f_0 = f_s\left(\frac{v}{v + v_s}\right) \qquad \text{and} \quad v = f\lambda$$

So if f_o is less than the source frequency, then the observed wavelength will be greater than the wavelength of the source.

Spectral lines observed from stars are shifted towards longer wavelengths – the red end.

This meant if a source of light was travelling towards an observer, then the wavelength appeared to decrease. This is known as **Blue Shift**.

Why?
When a source is moving **towards** an observer, the frequency observed by the observer is **increased** because

$$f_0 = f_s\left(\frac{v}{v - v_s}\right) \qquad \text{and} \quad v = f\lambda$$

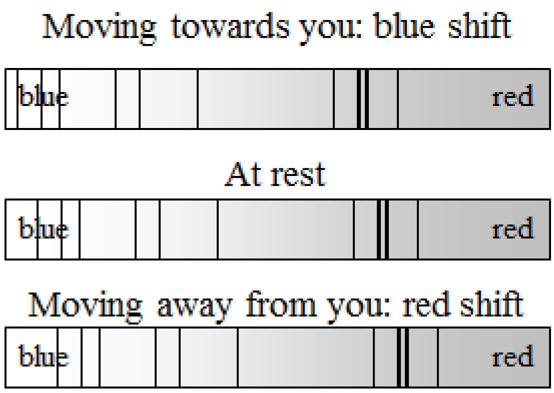
So if f_o is greater than the source frequency, then the observed wavelength will be less than the wavelength of the source.

Spectral lines observed from stars are shifted towards shorter wavelengths – the blue end.

Observations show that the light from almost all other galaxies is red shifted and as such they are all moving away from us. This is why astronomers are able to put forward the idea that we are part of an **expanding universe**.

Red shift

Information about a star's temperature, composition and motion can be found by analysing its spectrum. Star motion can be fast enough to cause a detectable Doppler shift in light waves. If a star is moving away from the Earth, its spectral lines are shifted towards the red end of the spectrum. This also works for galaxies.



$$z = \frac{\Delta\lambda}{\lambda_{rest}} = \frac{v}{c}$$

z	=	red shift (no units)
$\Delta\lambda$	=	$\lambda_{observed}$ - λ_{rest}
$\lambda_{observed}$	=	wavelength measured by observer
λ_{rest}	=	wavelength measured at source
v	=	velocity of source
c	=	speed of light

(Note that the Doppler effect equations used for sound cannot be used with light from fast moving galaxies because relativistic effects need to be taken into account.)

Hubble's Law

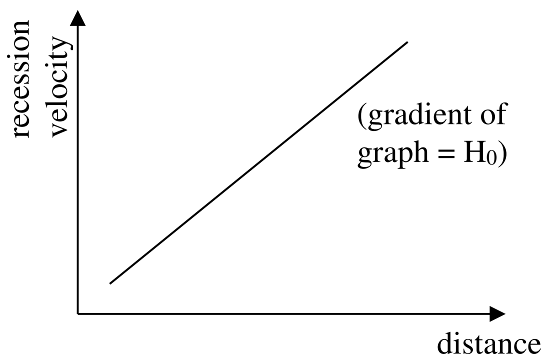
The astronomer Edwin Hubble noticed in the 1920's that the light from some distant galaxies was redshifted.

For each element, the spectral lines were all shifted by the same amount for each galaxy. This shift was due to the galaxy moving away from Earth at speed.

Over a few years, he examined the redshift of galaxies at varying distances from Earth. The further away a galaxy was, the faster it was travelling.

This relationship between distance and speed of galaxy is known as Hubble's Law.

Hubble discovered: $v \propto d$



$$v = H_0 d$$

v	=	recession velocity
H_0	=	Hubble Constant
d	=	distance to galaxy

Age of the Universe

If galaxies are travelling away from us, in the past they must have been closer (ie, matter must once have been packed in a small volume). By working back in time it is possible to calculate a time where all the galaxies were in fact at the same point in space. This allows for the age of the universe to be calculated. Currently, NASA have a value of 13.7 billion years as the age of the universe from this method.

$$\text{expansion time} = \frac{\text{distance to galaxy}}{\text{speed of galaxy}}$$

$$\text{age of Universe} = \frac{\text{distance}}{\text{speed}} = \frac{d}{v} = \frac{d}{H_0 d} = \frac{1}{H_0}$$

Hubble's constant H_0 as approximated by SQA = 2.4×10^{-18} s^{-1}

Therefore, age of universe is approximately 4.2×10^{17} s, equating to 13.2 billion years.

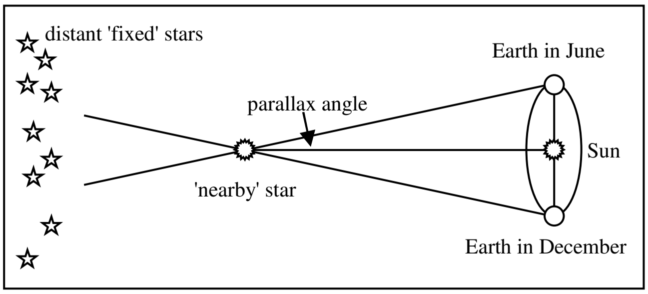
How do we know the distance to stars and galaxies?

If we know the luminosity of a star (how quickly it produces energy), the distance to the star is measured by how bright it appears to us.

Distance to stars: Parallax

As Earth orbits the Sun, stars appear to move against the background of other distant stars

By measuring the parallax angle, distance can be calculated using trigonometry.



Distance to galaxies: inverse square law

The distance to a star or galaxy is worked out by comparing apparent brightness (how bright an object appears) and absolute brightness (how much light is actually produced).

Note: We still need to know the luminosity of 'standard candles' eg. cepheid variable stars (whose brightness varies regularly with a period)

Evidence for the expanding Universe

Deductions from Hubble's Law

Hubble's Law suggests that **galaxies farther from us are moving away faster** than galaxies closer to us, which in turn leads us to conclude that the universe is expanding. The fate of the universe (whether the expansion will continue for ever or slow and then start to contract) depends on how much matter is in the universe. Matter causes gravity - enough gravity could slow the rate of expansion or stop it entirely.

So is there enough matter in the Universe to slow expansion?

Problem one: Where's the matter - a local case study

In the same way that Earth orbits the Sun, our Sun orbits around our galaxy (with a period of approx. 240 million years).

With some simple Physics (see rotational motion in Advanced Higher Physics!) it can be determined that:

$$\text{orbital speed of the Sun} \propto \text{mass of galaxy inside its orbit}$$

The speed of rotation of any object is determined by the size of the force maintaining its rotation. For the sun, this central force is due to gravity, which is determined by the amount of matter inside the Sun's orbital path. If we know the rotational speed of the Sun we can calculate how much force is required to keep it in orbit, and hence the amount of matter in our galaxy!

PROBLEM: Calculations of the amount of matter in the galaxy suggest there is more than astronomers can detect. → **Dark Matter** The stuff we can't see!

Problem two: How's the matter not having an effect?

There is normal matter and Dark matter - all contributing to gravity. **Why doesn't it slow the expansion rate of the universe?**

Gravity is an attractive force. All the matter in the universe is acting to slow the rate of expansion. However, measurements of the expansion rate of the Universe lead to the conclusion that the rate of expansion is actually increasing!

PROBLEM: Something is overcoming the force of gravity to cause the universe to expand at an ever increasing rate → **Dark Energy** It's what's driving all the expansion

The Temperature of Stellar Objects

What's stellar temperature got to do with Big Bang theory?

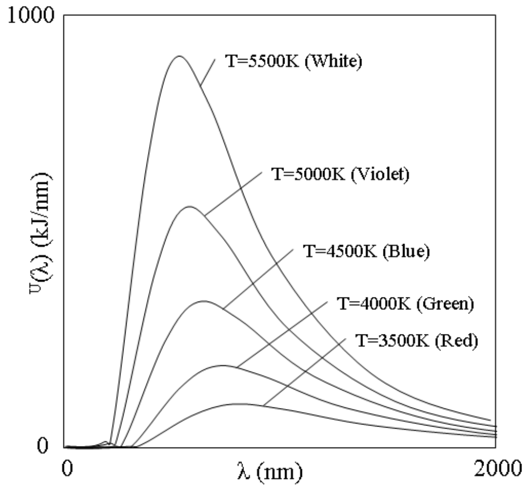
The **Big Bang theory** states that the universe started with a sudden appearance of energy at a singular point, which consequently (and very quickly) became matter, and then expanded and cooled rapidly. The theory therefore predicts that the universe should now, **13.7 billion years** later, have a very cool temperature. If we can measure this temperature we can see if it accords with Big Bang theory.

If we can understand stellar temperatures, it can help us know how to find the average temperature of the universe.

Thermal emission peak

Stars emit radiation over a wide range of wavelengths.

The graph to the right is called a thermal emission peak which shows how the intensity of radiation produced (y-axis) from stars of different temperatures (the different lines on the graph) is related to the wavelength of light emitted from the star.



Essentially thermal emission peaks allow the temperature of stellar objects to be determined.

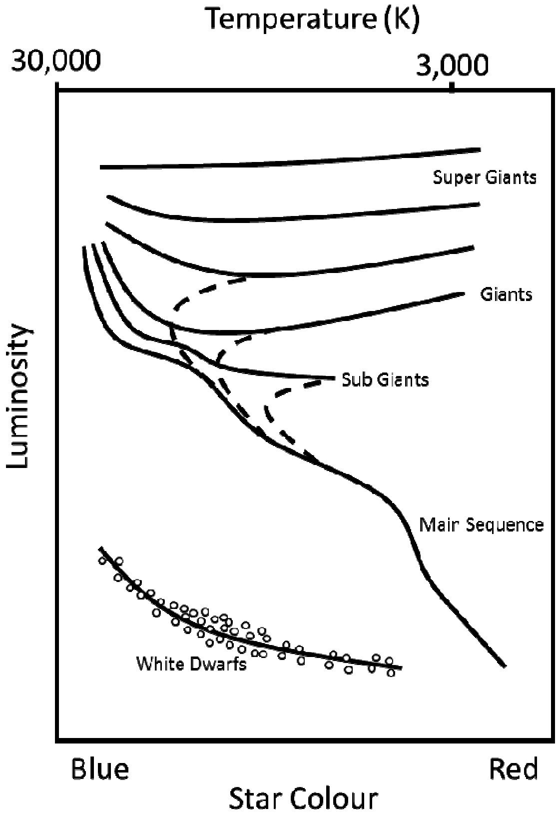
Three details emerge from studying these peaks:

2. Stellar objects emit radiation over the complete electromagnetic spectrum.
3. Each stellar object has a peak wavelength that depends on its temperature.
4. As the temperature of the star increases:
    a. There is more energy (intensity of radiation) at each wavelength
    b. The peak wavelength shifts to shorter wavelengths

(Hotter stars emit bluer wavelengths)

The Hertzsprung-Russell Diagram

The Hertzsprung-Russell Diagram allows astrophysicists to relate stars according to luminosity (or absolute magnitude) and colour (or temperature). When plotted in this way, clear patterns can be seen in the positions of certain types of star on this diagram. As stars evolve they follow specific paths on this diagram (this is shown below by the dashed lines). This process is known as stellar evolution and is covered in more detail in Advanced Higher Physics.



In this Hertzsprung-Russel diagram specific stars are not shown, however the regions of the diagram for different types of star are shown (this is shown by the solid lines). If these stars were plotted then we would see a scattering of stars around the lines shown, as seen here for White Dwarfs

A star evolves as, over time, the forces within the star change due to changes is the fusion process. There is a intricate balancing process between the inwards gravitational forces and the outward pressure forces involved. As a star evolves it will move around the Hertzsprung-Russell diagram as both its temperature, colour and Luminosity change at each stage of its life.

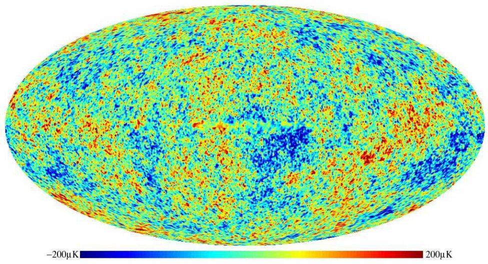
The Sun lies on the Main Sequences, this is where most stars are found. It will eventually expand to become a Giant when the Hydrogen to Helium ratio within the star reaches a certain level. In doing so it will become cooler and so also redder.

The path followed by a star on the diagram is determined by its mass. More massive stars with become Super Giants, may go Super Nova (explode) and may leave behind a Super Nova remnant (nebula of exploded gases), with a white dwarf at its centre.

A Hertzsprung-Russell diagram may be used to represent the stars in just one cluster of stars. Each cluster will then result in a slightly different version of the diagram, where differences and similarities can be observed. The age of the cluster can also be determined.

Evidence for the Big Bang

When The Universe was very young, matter was at such a high temperature that it existed in a plasma state, where electrons were delocalised from their associated protons. In this state light (EM radiation) is not able to propagate, since it will interact with any electron or proton it encounters. As the Universe expanded the plasma cooled down enough (after about 380,000 years) for the recombination of electrons with protons and electromagnetic radiation was able to travel. This initial EM radiation has been travelling ever since, however, due to the expansion of the universe its energy has decreased and its wavelength increased into the microwave region of the spectrum. Fluctuations in this Cosmic Microwave Background are caused by variations in the density of the plasma at the time of release, since its distribution was not uniform. These greater densities may well be responsible for the later formation of galaxies. In fact, the first stars did not start to shine until 100 million years after the Big Bang.



Cosmic Microwave Background
http://www.sns.ias.edu/~dgrin/Research.htm

The CMB is a strong piece of evidence to support the Big Bang Theory and no other model is able to explain the presence of the CMB.

Finally, however, while the expansion is evident from observation (redshift) and the "image" taken 380,000 years after some Big Bang supports this, there is no direct evidence of the "Bang" itself. Big Bang suggests that everything started from nothing with an explosion and immediate appearance of matter which was not there before. Or, more likely a singularity, a point in space where all the matter was concentrated (not supported by classical theory) before rapidly expanding to fill the space surrounding it. Along with this theory it is suggested that time itself also began at the Big Bang. The initial conditions of the Universe are not clearly defined. Really we are looking at the Big Expansion Theory, since we have no definitive idea of what happened at the "start". There is still a lot to discover.

The average temperature of the universe predicted by the Big Bang is around 2.73 Kelvin (K)

Astronomers have used a telescope called COBE (Cosmic Background Explorer) to detect radiation in the microwave region. The data collected fits the predicted thermal emission peak perfectly, confirming the average temperature measured is as predicted by the Big Bang theory.

Remember, low temperatures have longer wavelengths

Coldest will be in microwave region of the Electromagnetic spectrum

Note:
0°C = 273K
0K = -273°C

Spectrum of the Cosmic Microwave Background