CFE GAS LAWS - BMcMULLEN

Pressure

Pressure is defined as the perpendicular force acting on unit area.

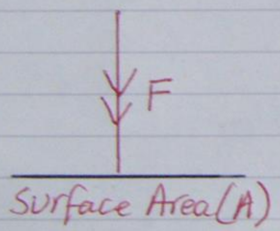
ie $\boxed{\text{Pressure} = \frac{\text{Perpendicular force}}{\text{Area}}}$

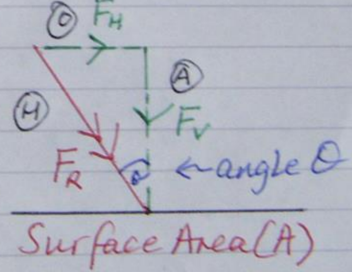
Where $\boxed{P = \frac{F}{A}}$

- F → Perpendicular force (N)
- P → Pressure (Pa or Nm^{-2})
- A → Surface Area (m^2)

Pressure is measured in Pascals where

$$\boxed{1\,Pa = 1\,Nm^{-2}}$$





Here $\boxed{P = \frac{F}{A}}$

Here $P = \frac{F_V}{A}$

BLAISE PASCAL (1623-1662) FRANCE

As $\cos\theta = \frac{A}{H} = \frac{F_V}{F_R}$

$\Rightarrow F_V = F_R \cos\theta$

$\Rightarrow \boxed{P = \frac{F_R \cos\theta}{A}}$

Area conversions

If the surface area in a question is given in cm^2 then it has to be converted into m^2.

Ex 1

Convert the following areas into m^2:

Q a) $50cm^2$

b) $720cm^2$

A Firstly we need to find $1cm^2 = ?m^2$

$\therefore 1cm^2 = 1cm \times 1cm = 0.01m \times 0.01m$

$\Rightarrow 1cm^2 = 0.0001m^2 \Rightarrow \boxed{1cm^2 = 1 \times 10^{-4} m^2}$

a) $50cm^2 = \underline{50 \times 10^{-4} m^2}$

b) $720cm^2 = \underline{720 \times 10^{-4} m^2}$

Do not worry about these answers into standard form.

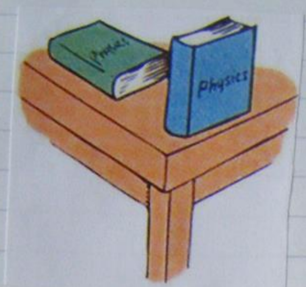
Ex 2

Q An object exerts a force of 40N on a desk.
Calculate the pressure that the object exerts on the desk if it has a surface area of $0.80m^2$.

A $P = ?$
$F = 40N$
$A = 0.80m^2$

$P = \frac{F}{A} = \frac{40}{0.80} = \underline{50Pa}$

Ex3



Q// Why does the Physics textbook exert a larger pressure on the table when it is standing up?

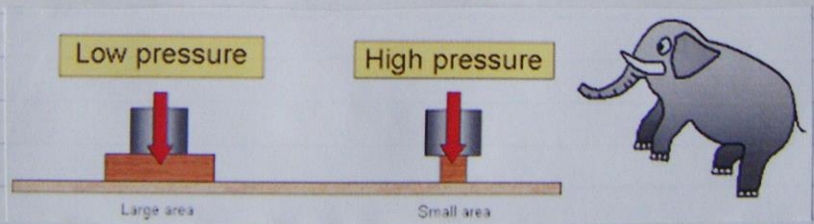
A// The surface area of contact is smaller when the books is standing up.

From $\overset{\uparrow}{P} = \dfrac{\overset{\leftrightarrow}{F}}{A\downarrow}$

The force is the weight of the book acting down on the table which is constant.

As $A\downarrow \therefore P\uparrow$

This can also be shown by a circus elephant standing with its back feet on a podium of small area and then moving on to a podium of large area.

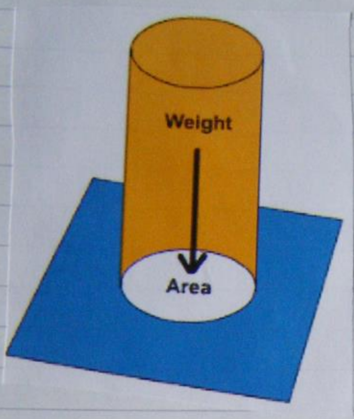


From $\overset{\downarrow}{P} = \dfrac{\overset{\leftrightarrow}{F}}{A\uparrow}$

As the area of the podium increases $(A\uparrow)$ then the pressure exerted decreases $(P\downarrow)$.

* The force (weight) of the elephant is constant.

Ex4



Pressure can also be worked out on a surface area if the mass of an object is given.

The force acting down on the surface area is the weight of an object.

($W=mg$)

$$P=\frac{F}{A} \Rightarrow P=\frac{W}{A} \Rightarrow P=\frac{mg}{A}$$

Q Calculate the pressure exerted by an elephant on the ground if it has a mass of 2000kg with each foot having a surface area of $400cm^2$.

A $P=?$

$F=W=19{,}600N$

$A=400cm^2 \times 4 = 1600cm^2$

$W=mg$

$\Rightarrow W=2000 \times 9.8$

$\Rightarrow W=19{,}600N$

$$P=\frac{F}{A}=\frac{W}{A}=\frac{19{,}600}{1600 \times 10^{-4}}$$

Remember

$1cm^2 = 1 \times 10^{-4} m^2$

$\therefore 1600cm^2 = 1600 \times 10^{-4} m^2$

$\Rightarrow P = 1.23 \times 10^5 Pa$

Ex5

A pressure of 4×10^{6} Pa when applied to the skin causes pain.
In a bed of nails each nail has an area of $1\times10^{-6}m^2$.

A teacher of mass 95kg decides to lie on the bed of nails.

Q. What is the least number of nails that he has to lie on without suffering any pain?

A. • $W = mg = 95\times9.8 = 931N$

• $P = \frac{F}{A} \Rightarrow P = \frac{W}{A} \Rightarrow 4\times10^{6} = \frac{931}{A}$

⇒ By cross multiplying

$\Rightarrow A = \frac{931}{4\times10^{6}} = 2.3275\times10^{-4}m^2$

⇒ Working with ratios in Area

$\Rightarrow 1\times10^{-6}m^2 \rightarrow 1$ nail

$\therefore 2.3275\times10^{-4}m^2 \rightarrow \frac{2.3275\times10^{-4}}{1\times10^{-6}} = 232.75$

Round up.

$\therefore$ Total number of nails = 233

OR $\frac{1\times10^{-6}}{2.3275\times10^{-4}} = \frac{1}{X} \Rightarrow \frac{X}{1} = \frac{2.3275\times10^{-4}}{1\times10^{-6}} \Rightarrow X = 233$

Where X = Number of nails

Kelvin Scale of temperature

A new temperature scale is introduced here called the Kelvin Scale.

- Converting °C into Kelvin

Add 273 to the temperature in degrees celsius (°C) to convert into Kelvin temperature.

eg
$10°C = 10 + 273 = 283K.$
$140°C = 140 + 273 = 413K$
$-160°C = -160 + 273 = 113K$

- Convert K into °C

Subtract 273 from the temperature in Kelvin to convert it into degrees celsius (°C)

eg
$400K = 400 - 273 = 127°C$
$10K = 10 - 273 = -263°C$
$220K = 220 - 273 = -53°C$

Why 273?

-273°C is known as the absolute zero of temperature

ie $-273°C = 0K$

In gases the kinetic energy of the particles is directly proportional to their temperature in kelvin.
(No relationship with E_K and temp in °C !!)

As $E_K = \frac{1}{2}mv^2$ and $E_K \propto T_{(K)}$

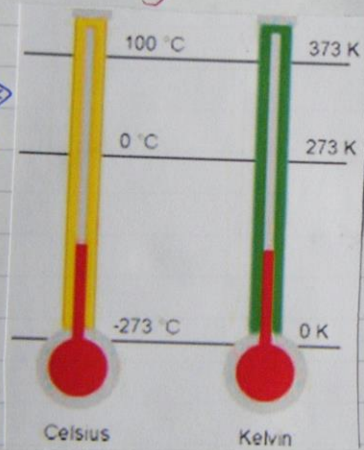
- As $T_{(K)} \uparrow \therefore E_K \uparrow$
- As $T_{(K)} \uparrow \therefore$ The speed of the gas particles $\uparrow$

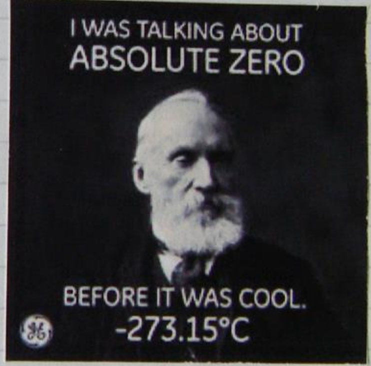
eg As $T_{(K)}$ doubles $\therefore E_K$ of the gas particles doubles

* At 0K gas particles will be motionless as they have no E_K. ie $E_K \propto T_{(K)}$ *

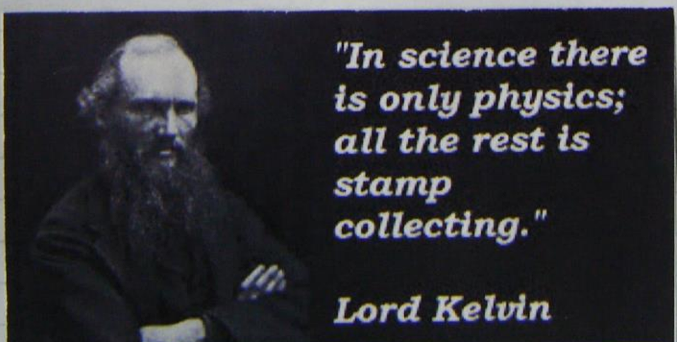
Comparison between ⇒ °C and K.

NB
K ✓
°K ✗





Lord Kelvin
(1824 – 1907)



Please do not let your Biology or Chemistry teacher see this quote!!

Lord Kelvin ⇒ Born in Belfast but made in Glasgow!!

General Gas Law

In each of the gas laws the mass of gas is always kept constant.

$$\frac{P_1 V_1}{T_{1(K)}} = \frac{P_2 V_2}{T_{2(K)}}$$ ⇒ In DB!!

1 → Start
2 → End

- The units of pressure and volume do not need to be converted. (Given that the units are the same at the start and at the end.)
- Temperature must be converted into Kelvin in the Gas Laws.

Ex6

$100cm^3$ of gas is expanded to $200cm^3$ with the temperature increasing from 40°C to 100°C.

If the initial pressure of the gas is 200kPa then calculate the final pressure of the gas.

$P_1 = 200kPa$	$P_2 = ?$
$V_1 = 100cm^3$	$V_2 = 200cm^3$
$T_{1(K)} = 40°C = 313K$	$T_{2(K)} = 100°C = 373K$

$$\frac{P_1 V_1}{T_{1(K)}} = \frac{P_2 V_2}{T_{2(K)}} \Rightarrow \frac{200 \times 100}{313} = \frac{P_2 \times 200}{373}$$

$$\Rightarrow 63.9 = 0.536 P_2 \Rightarrow P_2 = \frac{63.9}{0.536} = \underline{\underline{119kPa}}$$

1) First Individual Gas Law

Boyle's Law

ROBERT BOYLE
(1627–1691 IRELAND)

- The mass of gas is kept constant.
- The temperature of the gas is also kept constant.

$$\frac{P_1V_1}{T_{1(K)}} = \frac{P_2V_2}{T_{2(K)}} \Rightarrow \boxed{P_1V_1 = P_2V_2}$$

$$PV = \text{Constant} \Rightarrow PV = k \Rightarrow P = \frac{k}{V}$$

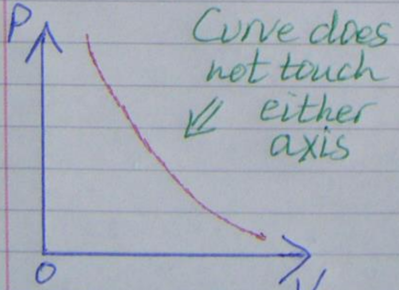
$$\Rightarrow \boxed{P \propto \frac{1}{V}}$$

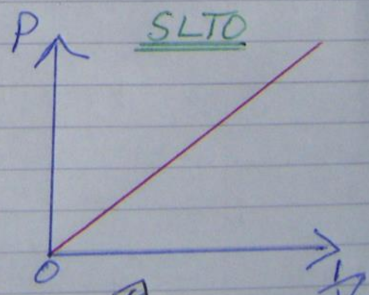
ie Pressure is inversely proportional to volume

OR

Pressure is directly proportional to 1/volume

Graphs for Boyle's Law





This graph shows the relationship 100%!!

Ex7

Q The pressure of a fixed mass of gas is 5.0×10^5 Pa with a volume of $2.8 m^3$.

Calculate the new pressure of the gas if the volume increases to $3.4 m^3$.

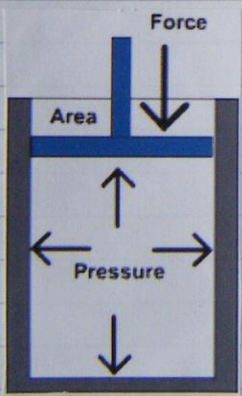
A

$P_1 = 5.0 \times 10^5$ Pa	$P_2 = ?$
$V_1 = 2.8 m^3$	$V_2 = 3.4 m^3$
$T_{1(K)}$ = Constant	$T_{2(K)}$ = Constant

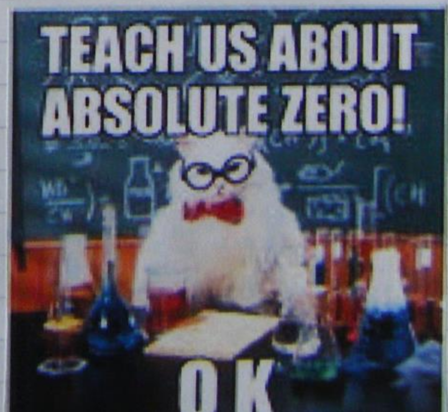
$$\frac{P_1 V_1}{T_{1(K)}} = \frac{P_2 V_2}{T_{2(K)}} \Rightarrow 5.0 \times 10^5 \times 2.8 = P_2 \times 3.4$$

$$\Rightarrow 3.4 P_2 = 1.4 \times 10^6$$

$$\Rightarrow P_2 = \frac{1.4 \times 10^6}{3.4} = 4.1 \times 10^5 \text{ Pa}$$

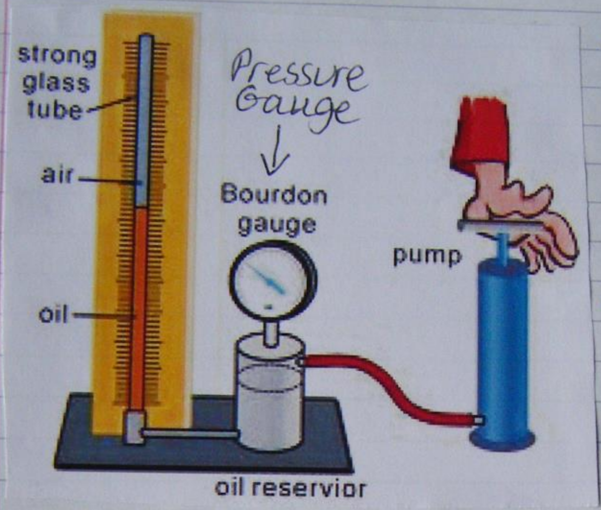


As the force is applied on the surface area then the volume of the gas will decrease. This results in the pressure of the gas increasing.

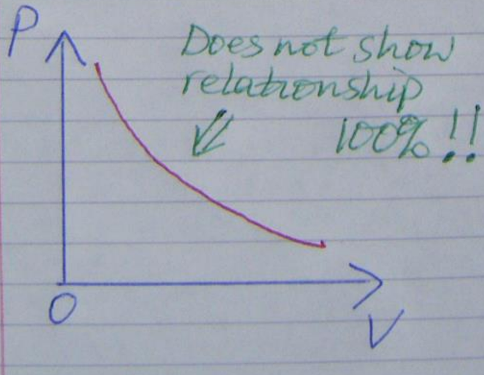


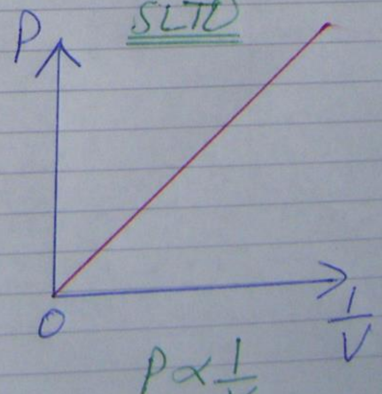
We will now look at the two individual gas laws that involve Kelvin temperature.

Boyle's Law experiment



- Use the pump until the volume of trapped air in the glass tube will not reduce any further.
- Take a reading from the pressure gauge and its corresponding volume of air from the glass tube.
- Slowly reduce the pressure reading from the Bourdon Gauge and take a note of the corresponding volumes of air from the glass tube.
- From the pressure and volume data recorded in a table draw a graph.





$P \propto \frac{1}{V}$

Kinetic model for Boyle's Law

As the volume of the gas decreases.

- The number of collisions between the gas particles and the container walls per second will increase.
- The force exerted by the gas particles on the container walls will increase.
- From $P = F/A$, the pressure of the gas will increase.

2) Second Individual Gas Law

Gay-Lussac's Law (Pressure Law)

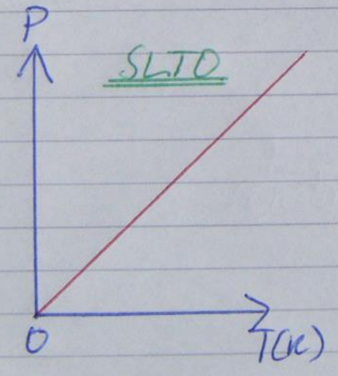
- The mass of gas is kept constant.
- The volume of the gas is kept constant.

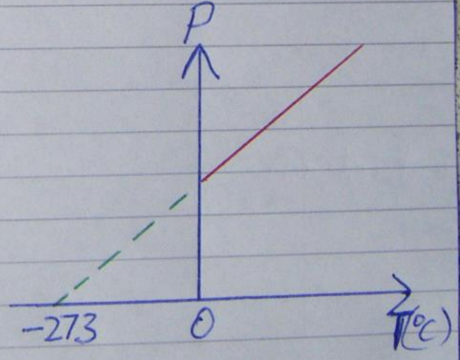
$$\frac{P_1 \cancel{V_1}}{T_1(K)} = \frac{P_2 \cancel{V_2}}{T_2(K)} \Rightarrow \boxed{\frac{P_1}{T_1(K)} = \frac{P_2}{T_2(K)}}$$

As $\frac{P}{T(K)} = \text{Constant} \Rightarrow \frac{P}{T(K)} = k$

$$\Rightarrow P = kT(K) \Rightarrow \boxed{P \propto T(K)}$$

Graphically





$P \propto T(K)$

No relationship as such between Pressure and temp in °C.

ie Not an SLTO!!

EX8

Q. A gas is held at atmospheric pressure (1.01×10^5 Pa) and has a temperature of 50°C.

If the pressure is increased to 3.50×10^5 Pa then calculate the new temperature of the gas in °C.

HINT (°C → K ⇒ +273, K → °C ⇒ −273)

- Change the temperature from °C to Kelvin to use in the calculation.
- Calculate the final temperature in Kelvin
- Convert the final temperature from Kelvin back into °C.

A.

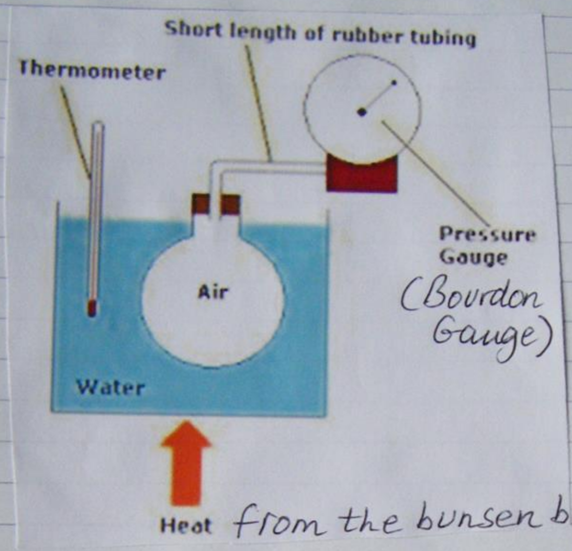
$P_1 = 1.01 \times 10^5$ Pa	$P_2 = 3.50 \times 10^5$ Pa
$V_1 =$ ~~/~~ Constant	$V_2 =$ ~~/~~ Constant
$T_{1(K)} = 50°C = 323$ K	$T_{2(K)} =$?

$$\frac{P_1 \cancel{V_1}}{T_{1(K)}} = \frac{P_2 \cancel{V_2}}{T_{2(K)}} \Rightarrow \frac{1.01 \times 10^5}{323} = \frac{3.50 \times 10^5}{T_{2(K)}}$$

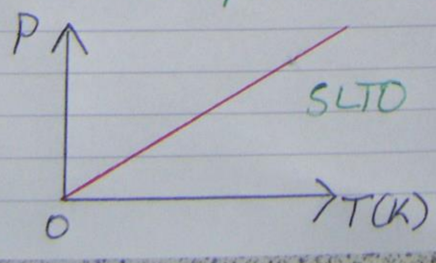
$$\Rightarrow 312.7 = \frac{3.50 \times 10^5}{T_{2(K)}} \Rightarrow T_{2(K)} = \frac{3.50 \times 10^5}{312.7}$$

$$\Rightarrow T_{2(K)} = 1119\text{ K} \Rightarrow T = 1119 - 273 = \underline{\underline{846°C}}$$

Gay-Lussac's Law Experiment



- A bunsen burner is used to heat the water and the air in the flask.
- The water temperature is recorded with the thermometer at regular intervals, with the corresponding pressure readings taken from the pressure gauge.
- A table and then a graph is drawn from the pressure and temperature readings taken. (Remember to convert the temperature readings into Kelvin)



$P \propto T(K)$.

How can Gay-Lussac's Law experimental set up be improved?

- Put the thermometer into the flask of air rather than the water.
- Submerge the flask completely in water.
- Keep the length of the rubber tubing to the pressure gauge as short as possible.

Kinetic model for Gay-Lussac's Law

- As the temperature of the gas in Kelvin increases
- E_K of the gas particles increases (as $E_K \propto T_{(K)}$)
- The number of collisions between the gas particles and the container walls per second increases.
- The force exerted on the container walls by the gas particles will increase
- Since $\text{Pressure} = \frac{\text{Force}}{\text{Area}}$ with area constant

then the pressure of the gas will increase.

3) Third Individual Gas Law

Charles' Law

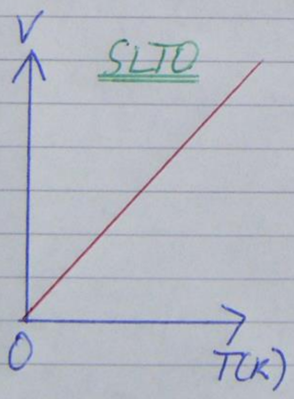
- The mass of gas is kept constant
- The pressure of the gas is kept constant.

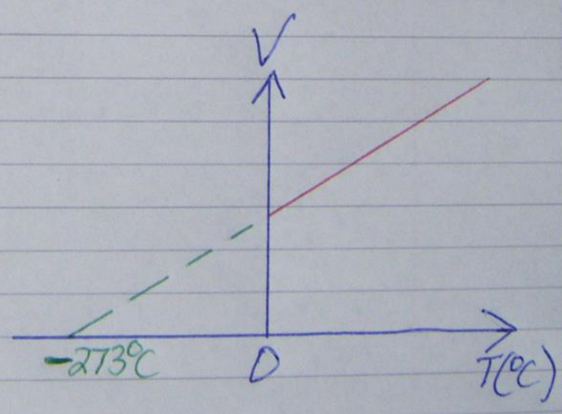
From $\frac{\cancel{P_1} V_1}{T_{1(K)}} = \frac{\cancel{P_2} V_2}{T_{2(K)}} \Rightarrow \boxed{\frac{V_1}{T_{1(K)}} = \frac{V_2}{T_{2(K)}}}$

As $\frac{V}{T_{(K)}} = \text{Constant} \Rightarrow \frac{V}{T_{(K)}} = k$

$\Rightarrow V = kT_{(K)} \Rightarrow \boxed{V \propto T_{(K)}}$

Graphically





$V \propto T_{(K)}$

No relationship as such between the volume and the temperature in °C!!

Ex9

Q The volume of a gas at 27°C is $0.02 m^3$.

a) What will the volume of the gas be at −57°C?

b) What will the volume of the gas be at 57°C?

A a)

P_1 = — Constant	P_2 = — Constant
$V_1 = 0.02 m^3$	V_2 = ?
$T_{1(K)} = 27°C = 300K$	$T_{2(K)} = -57°C = 216K$

$$\frac{P_1 V_1}{T_{1(K)}} = \frac{P_2 V_2}{T_{2(K)}} \Rightarrow \frac{0.02}{300} = \frac{V_2}{216}$$

$$\Rightarrow 6.67 \times 10^{-5} = \frac{V_2}{216} \Rightarrow V_2 = 216 \times 6.67 \times 10^{-5}$$

$$\Rightarrow V_2 = 1.44 \times 10^{-2} m^3$$

b)

P_1 = — Constant	P_2 = — Constant
$V_1 = 0.02 m^3$	V_2 = ?
$T_{1(K)} = 27°C = 300K$	$T_{2(K)} = 57°C = 330K$

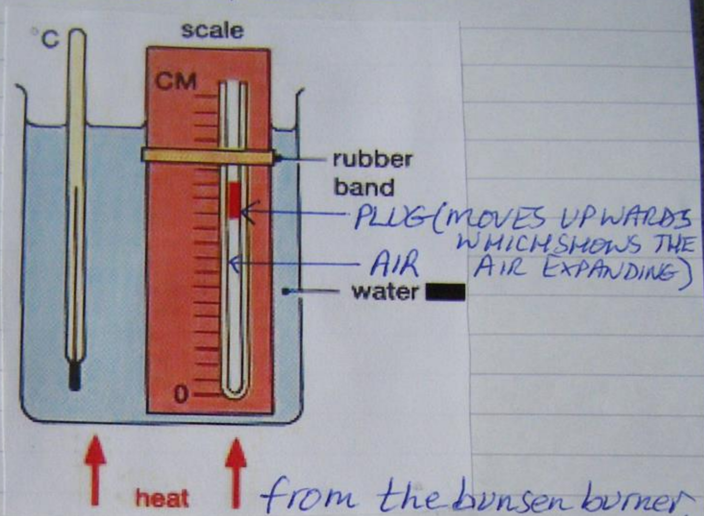
$$\frac{P_1 V_1}{T_{1(K)}} = \frac{P_2 V_2}{T_{2(K)}} \Rightarrow \frac{0.02}{300} = \frac{V_2}{330}$$

$$\Rightarrow 6.67 \times 10^{-5} = \frac{V_2}{330} \Rightarrow V_2 = 330 \times 6.67 \times 10^{-5}$$

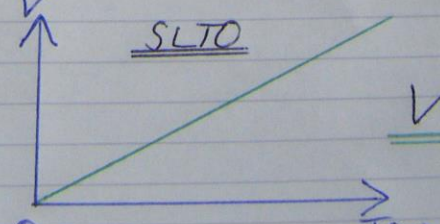
$$\Rightarrow V_2 = 0.022 m^3$$

Charles' Law Experiment

POSSIBLE IMPROVEMENT
Have the thermometer in the tube with the air and not the water.



- A bunsen burner is used to heat the water and the air in the tube.
- The water temperature is recorded with a thermometer at regular intervals, with the corresponding volume of air readings taken from the scale.
- From the volume and temperature readings taken a table and then a graph is drawn.
- Remember to convert Temperature into Kelvin !!



$V \propto T(K)$

Kinetic Model for Charles' Law

- As the temperature of the gas increases.
- The E_K of the gas particles increases. ($E_K \propto T_{(K)}$)
- The number of collisions between the gas particles and the container walls per second will increase.
- The force exerted by the gas particles on the plug and the container walls will increase.
- The plug will move upwards which will increase the volume of the air in the tube (container).
- This increase in the volume of the gas will keep the pressure of the gas constant.

NB In any of the gas laws involving volume of the gas the length of the gas column could be used.

As $\boxed{V = A \times \ell}$ in a cylinder, then

$V \propto \ell$ as the area of cross-section is constant.

$*$ ∴ As $\ell \uparrow$ ∴ $V \uparrow$ and as $\ell \downarrow$ ∴ $V \downarrow$ $*$