CfE Vectors and Scalars – BMcMULLEN

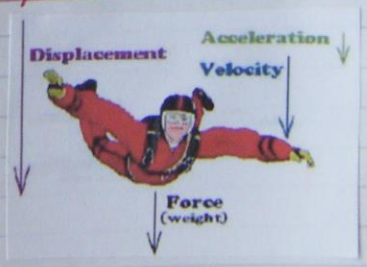
This guys chat is really bad, isn't it!!

What is a scalar quantity?

A scalar quantity has a magnitude (size) only.

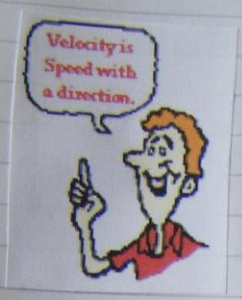
- **Scalar quantity examples**
  - speed
  - distance
  - mass
  - time
  - energy
  - power
  - temperature

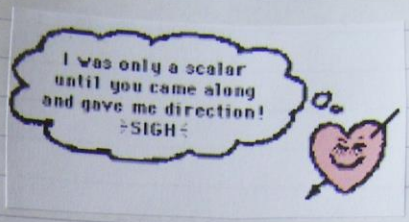
- **Vector quantity examples**
  - velocity
  - displacement
  - Weight
  - Force
  - momentum
  - acceleration



A vector quantity has a magnitude (size) and a direction.

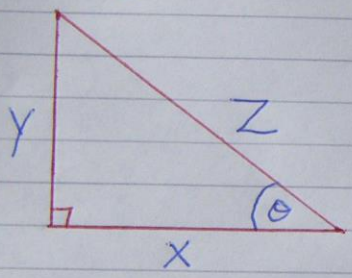
Thoughts on Vectors and Scalars





Applications of Mathematics in Vectors + Scalars

1) • **Pythagoras Theorem**



Right angled triangle.

Z → Hypotenuse
X → Adjacent
Y → Opposite

names of each of the sides in a right-angled triangle.

$$Z^2 = X^2 + Y^2$$
$$\Rightarrow Z = \sqrt{X^2+Y^2}$$

The hypotenuse Z is the longest side. (across from the right angle)

If you wanted to find the length of side X then,

$$X^2 = Z^2 - Y^2 \Rightarrow X = \sqrt{Z^2-Y^2}$$

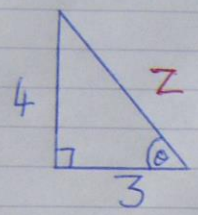
If you wanted to find the length of side Y then,

$$Y^2 = Z^2 - X^2 \Rightarrow Y = \sqrt{Z^2-X^2}$$

Ex 1

Find the length Z of the unknown side.

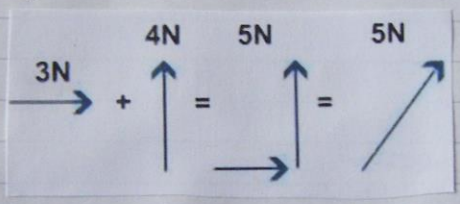
a)



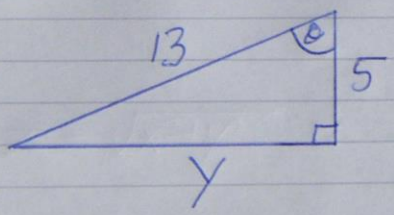
$Z^2 = 3^2 + 4^2 = 9 + 16 = 25$

$\Rightarrow Z = \sqrt{25} = 5$

Applying this example in terms of forces.



Ex 2.



$13^2 = 5^2 + y^2$

$\Rightarrow y^2 = 13^2 - 5^2$

$\Rightarrow y^2 = 169 - 25 = 144$

$\Rightarrow y = \sqrt{144} = 12$

θ (Theta) is the angle listed in these examples.

The side across (opposite) the angle is called the opposite.
The adjacent side is the third side.

2). Trigonometry

The trigonometry functions used with right-angled triangles are:

1) Sine (Sin)

2) Cosine (Cos)

3) Tangent (Tan)

SOH CAH TOA

↑

This phrase will help you to decide which trig function to use

ie $\text{Tan} = \frac{\text{Opposite}}{\text{Adjacent}} \Rightarrow \text{TOA}$

$\text{Cos} = \frac{\text{Adjacent}}{\text{Hypotenuse}} \Rightarrow \text{CAH}$

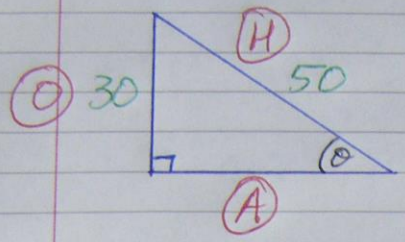
$\text{Sin} = \frac{\text{Opposite}}{\text{Hypotenuse}} \Rightarrow \text{SOH}$

This can be used to find an unknown

- Angle or
- Length of side.

Ex3

Find the size of the unknown angle θ.



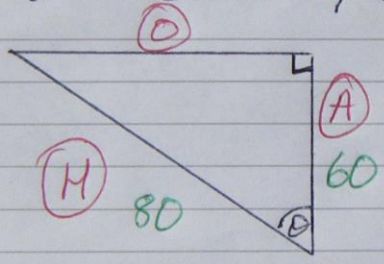
SOH CAH TOA

$\sin\theta = \frac{30}{50} = 0.6$

$\Rightarrow \theta = \sin^{-1}(0.6) = 36.9^\circ$

Ex4

Find the size of the unknown angle θ.



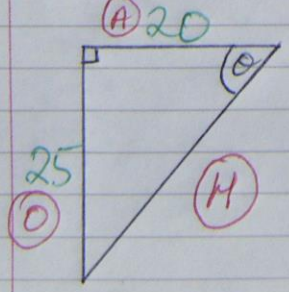
SOH CAH TOA

$\cos\theta = \frac{60}{80} = 0.75$

$\Rightarrow \theta = \cos^{-1}(0.75) = 41.4^\circ$

Ex5

Find the size of the unknown angle θ.

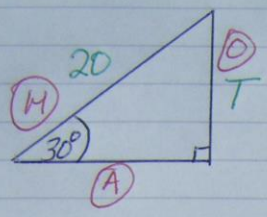


SOH CAH TOA

$\tan\theta = \frac{25}{20} = 1.25$

$\Rightarrow \theta = \tan^{-1}(1.25) = 51.3^\circ$

Ex6



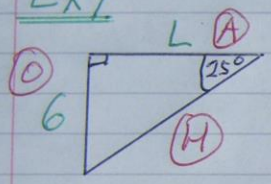
Calculate the length of the unknown side T.

SOH CAH TOA

$\sin 30^\circ = \frac{T}{20} \Rightarrow T = 20 \sin 30^\circ$

$\Rightarrow T = 10$

Ex7

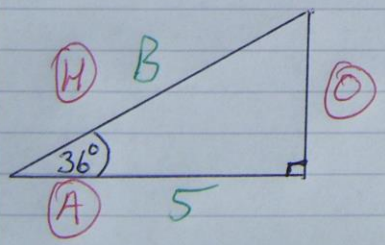


Calculate the length of the unknown side L.

SOH CAH TOA

$\tan 25 = \frac{6}{L} \Rightarrow L = \frac{6}{\tan 25^\circ} = 12.9$

Ex8



Calculate the length of the unknown side B.

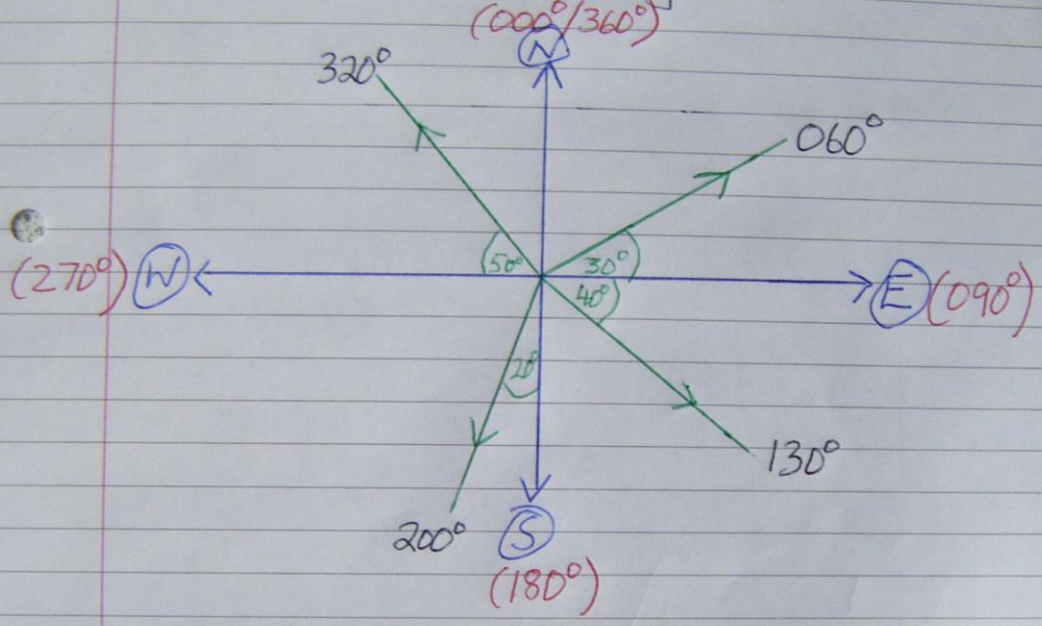
SOH CAH TOA

$\cos 36^\circ = \frac{5}{B}$

$\Rightarrow B = \frac{5}{\cos 36^\circ} = 6.2$

3) • Bearings

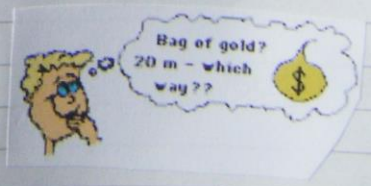
Bearings give a quantative description of the direction that an object is moving in.



Bearings are always measured from the due North line ie 000°.

It is important to use the N, S, E and W points as points of reference. This is illustrated in the diagram above. ie.

- 060° = 090° − 30°
- 130° = 090° + 40°
- 200° = 180° + 20°
- 320° = 270° + 50°

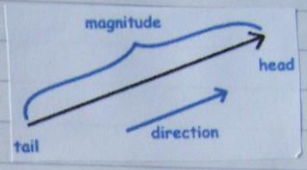


Ex 9

A group of students in the Higher Physics class embark on a sponsored walk for charity.

The students walk 5km due west followed by 12km due south in 6 hours.

Q Calculate or find:
a) distance travelled
b) Average speed
c) displacement
d) Average velocity.

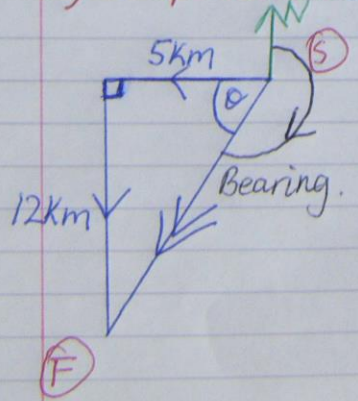


A a) distance travelled → Scalar quantity → 5km + 12km = 17km

b) Average Speed → Scalar quantity ⇒ Average speed = distance / time

⇒ Average speed = $\frac{17}{6}$ = 2·83 kmh^{-1}

c) displacement → Vector quantity → Diagram needed here!!



Bearing = 270° − θ°

M + D's

(S) → Start
(F) → Finish.

Magnitude and direction!!

M ⇒ Pythagoras Theorem.

$SF^2 = 5^2 + 12^2 = 25 + 144 = 169$

$\Rightarrow SF = \sqrt{169} = 13\text{Km}$

D ⇒ Trigonometry + Bearings.

SOH CAH TOA ✓

$\text{Tan}\theta = \frac{12}{5} = 2.4 \Rightarrow \theta = \text{Tan}^{-1}(2.4)$

$\Rightarrow \theta = 67.4^\circ$

Though $\theta = 67^\circ$ to be put into the bearing. (Rounded to a whole number.)

$\text{Bearing} = 270^\circ - 67^\circ = 203^\circ$

displacement = $13\text{Km} @ 203^\circ$ (M + D's)

d) $\text{Average velocity} = \frac{\text{displacement}}{\text{time}}$

$\Rightarrow \text{Average velocity} = \frac{13\text{Km} @ 203^\circ}{6\text{h}} = 2.17\text{kmh}^{-1} @ 203^\circ$ (M + D's.)

*Write out the equations for average speed and average velocity fully as

$\bar{v} = \frac{d}{t}$ and $\bar{v} = \frac{s}{t}$ can lead to confuse students!!*

Application of forces in vector diagrams

Forces are handled in the same way as displacement and average velocity in vector diagrams, using Pythagoras Theorem and Trigonometry. However we DO NOT record resultant forces in terms of bearings.

eg1 40N @ 060° would be written as

40N @ 30° above the horizontal

eg2 580N @ 235° Would be written as

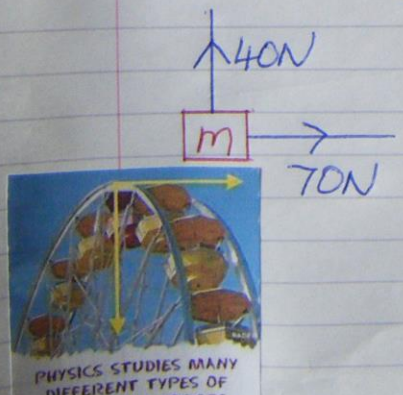
580N @ 35° below the horizontal

eg3 3500N @ 300° would be written as

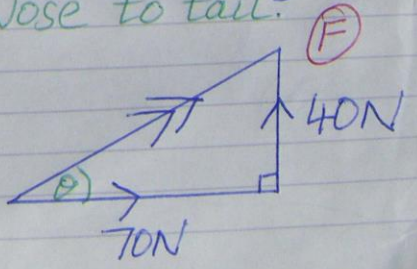
3500N @ 30° above the horizontal

EX10

Calculate the resultant force acting on the object below.



= Vectors always add Nose to tail.



Force is a vector quantity ⇒ M+D's

M ⇒ $SF^2 = 70^2 + 40^2 = 4900 + 1600$

⇒ $SF^2 = 6500$ ⇒ $SF = \sqrt{6500} = 80.6N$

D ⇒ Trigonometry SOH CAH TOA ✓

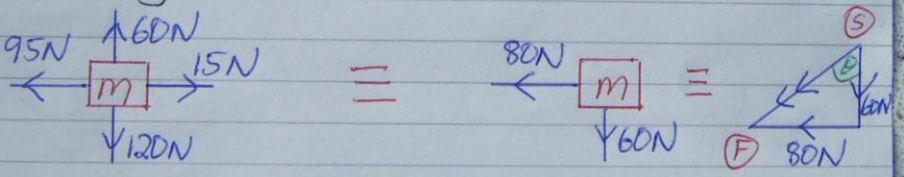
$\tan\theta = \frac{40}{70} = 0.571$ ⇒ $\theta = \tan^{-1}(0.571)$

⇒ $\theta = 29.7° \approx 30°$ ↑ Round up!!

Resultant Force = 80.6N @ 30° above the horizontal.

Ex11

Calculate the resultant force acting on the object below.



M ⇒ $SF^2 = 60^2 + 80^2 = 3600 + 6400 = 10,000$

⇒ $SF = \sqrt{10,000} = 100N$

D ⇒ Trigonometry SOH CAH TOA ✓

$\tan\theta = \frac{O}{A} = \frac{80}{60} = 1.33$ ∴ $\theta = \tan^{-1}(1.33) = 53.1°$

∴ $\theta = 53°$ ← Round down

Resultant Force = 100N @ 53° from the vertical.