THE CONSTITUTION OF ATOMS AND MOLECULES.

On the Constitution oj Atoms and Molecules.
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PART II.—Systems containing only a Single Nucleus †.

§ 1. *General Assumptions.*

FOLLOWING the theory of Rutherford, we shall assume that the atoms of the elements consist of a positively charged nucleus surrounded by a cluster of electrons. The nucleus is the seat of the essential part of the mass of the atom, and has linear dimensions exceedingly small compared with the distances apart of the electrons in the surrounding cluster.

As in the previous paper, we shall assume that the cluster of electrons is formed by the successive binding by the nucleus of electrons initially nearly at rest, energy at the same time being radiated away. This will go on until, when the total negative charge on the bound electrons is numerically equal to the positive charge on the nucleus, the system will be neutral and no longer able to exert sensible forces on electrons at distances from the nucleus great in comparison with the dimensions of the orbits of the bound electrons. We may regard the formation of helium from α rays as an

* Communicated by Prof. E. Rutherford, F.R.S.
† Part I. was published in Phil. Mag. xxvi. p. 1 (1913).

observed example of a process of this kind, an α particle on this view being identical with the nucleus of a helium atom.

On account of the small dimensions of the nucleus, its internal structure will not be of sensible influence on the constitution of the cluster of electrons, and consequently will have no effect on the ordinary physical and chemical properties of the atom. The latter properties on this theory will depend entirely on the total charge and mass of the nucleus ; the internal structure of the nucleus will be of influence only on the phenomena of radioactivity.

From the result of experiments on large-angle scattering of α-rays, Rutherford * found an electric charge on the nucleus corresponding per atom to a number of electrons approximately equal to half the atomic weight. This result seems to be in agreement with the number of electrons per atom calculated from experiments on scattering of Röntgen radiation †. The total experimental evidence supports the hypothesis ‡ that the actual number of electrons in a neutral atom with a few exceptions is equal to the number which indicates the position of the corresponding element in the series of elements arranged in order of increasing atomic weight. For example on this view, the atom of oxygen which is the eighth element of the series has eight electrons and a nucleus carrying eight unit charges.

We shall assume that the electrons are arranged at equal angular intervals in coaxial rings rotating round the nucleus. In order to determine the frequency and dimensions of the rings we shall use the main hypothesis of the first paper, viz. : that in the permanent state of an atom the angular momentum of every electron round the centre of its orbit is equal to the universal value $\frac{h}{2\pi}$, where h is Planck's constant.

We shall take as a condition of stability, that the total energy of the system in the configuration in question is less than in any neighbouring configuration satisfying the same condition of the angular momentum of the electrons.

If the charge on the nucleus and the number of electrons in the different rings is known, the condition in regard to the angular momentum of the electrons will, as shown in § 2, completely determine the configuration of the system, *i. e.*, the frequency of revolution and the linear dimensions of the rings. Corresponding to different distributions of the

* Comp. also Geiger and Marsden, Phil. Mag. xxv. p. 604 (1913).
† Comp. C. G. Barkla, Phil. Mag. xxi. p. 648 (1911).
‡ Comp. A. v. d. Broek, *Phys. Zeitschr.* xiv. p. 32 (1913).

electrons in the rings, however, there will, in general, be more than one configuration which will satisfy the condition of the angular momentum together with the condition of stability.

In § 3 and § 4 it will be shown that, on the general view of the formation of the atoms, we are led to indications of the arrangement of the electrons in the rings which are consistent with those suggested by the chemical properties of the corresponding element.

In § 5 it will be shown that it is possible from the theory to calculate the minimum velocity of cathode rays necessary to produce the characteristic Röntgen radiation from the element, and that this is in approximate agreement with the experimental values.

In § 6 the phenomena of radioactivity will be briefly considered in relation to the theory.

§ 2. *Configuration and Stability of the Systems.*

Let us consider an electron of charge e and mass m which moves in a circular orbit of radius a with a velocity v small compared with the velocity of light. Let us denote the radial force acting on the electrons by $\frac{e^2}{a^2}F$; F will in general be dependent on a. The condition of dynamical equilibrium gives

$$\frac{mv^2}{a} = \frac{e^2}{a^2}F.$$

Introducing the condition of universal constancy of the angular momentum of the electron, we have

$$mva = \frac{h}{2\pi}.$$

From these two conditions we now get

$$a = \frac{h^2}{4\pi^2 e^2 m}F^{-1} \quad \text{and} \quad v = \frac{2\pi e^2}{h}F; \quad . \quad . \quad (1)$$

and for the frequency of revolution ω consequently

$$\omega = \frac{4\pi^2 e^4 m}{h^3}F^2. \quad . \quad . \quad . \quad . \quad (2)$$

If F is known, the dimensions and frequency of the corresponding orbit are simply determined by (1) and (2). For a

ring of n electrons rotating round a nucleus of charge Ne we have (comp. Part I., p. 20)

$$F = N - s_n, \quad \text{where} \quad s_n = \frac{1}{4}\sum_{s=1}^{s=n-1}\operatorname{cosec}\frac{s\pi}{n}.$$

The values for s_n from $n=1$ to $n=16$ are given in the table on p. 482.

For systems consisting of nuclei and electrons in which the first are at rest and the latter move in circular orbits with a velocity small compared with the velocity of light, we have shown (see Part I., p. 24) that the total kinetic energy of the electrons is equal to the total amount of energy emitted during the formation of the system from an original configuration in which all the particles are at rest and at infinite distances from each other. Denoting this amount of energy by W, we consequently get

$$W = \Sigma\frac{m}{2}v^2 = \frac{2\pi^2 e^4 m}{h^3}\Sigma F^2. \quad . \quad . \quad . \quad (3)$$

Putting in (1), (2), and (3) $e = 4{\cdot}7 \,.\, 10^{-10}$, $\frac{e}{m} = 5{\cdot}31 \,.\, 10^{17}$, and $h = 6{\cdot}5 \,.\, 10^{-27}$ we get

$$a = 0{\cdot}55 \,.\, 10^{-8}F^{-1}, \quad v = 2{\cdot}1 \,.\, 10^8 F, \quad \omega = 6{\cdot}2 \,.\, 10^{15}F^2 \Big\}$$
and
$$W = 2{\cdot}0 \,.\, 10^{-11}\Sigma F^2. \qquad\qquad (4)$$

In neglecting the magnetic forces due to the motion of the electrons we have in Part I. assumed that the velocities of the particles are small compared with the velocity of light. The above calculations show that for this to hold, F must be small compared with 150. As will be seen, the latter condition will be satisfied for all the electrons in the atoms of elements of low atomic weight and for a greater part of the electrons contained in the atoms of the other elements.

If the velocity of the electrons is not small compared with the velocity of light, the constancy of the angular momentum no longer involves a constant ratio between the energy and the frequency of revolution. Without introducing new assumptions, we cannot therefore in this case determine the configuration of the systems on the basis of the considerations in Part I. Considerations given later suggest, however, that the constancy of the angular momentum is the principal condition. Applying this condition for velocities

not small compared with the velocity of light, we get the same expression for v as that given by (1), while the quantity m in the expressions for a and ω is replaced by $\dfrac{m}{\sqrt{(1-v^2/c^2)}}$, and in the expression for W by

$$m \,.\, 2\frac{c^2}{v^2}\left(1-\sqrt{1-\frac{v^2}{c^2}}\right).$$

As stated in Part I., a calculation based on the ordinary mechanics gives the result, that a ring of electrons rotating round a positive nucleus in general is unstable for displacements of the electrons in the plane of the ring. In order to escape from this difficulty, we have assumed that the ordinary principles of mechanics cannot be used in the discussion of the problem in question, any more than in the discussion of the connected problem of the mechanism of binding of electrons. We have also assumed that the stability for such displacements is secured through the introduction of the hypothesis of the universal constancy of the angular momentum of the electrons.

As is easily shown, the latter assumption is included in the condition of stability in § 1. Consider a ring of electrons rotating round a nucleus, and assume that the system is in dynamical equilibrium and that the radius of the ring is a_0, the velocity of the electrons v_0, the total kinetic energy T_0, and the potential energy P_0. As shown in Part I. (p. 21) we have $P_0 = -2T_0$. Next consider a configuration of the system in which the electrons, under influence of extraneous forces, rotate with the same angular momentum round the nucleus in a ring of radius $a = \alpha a_0$. In this case we have $P = \dfrac{1}{\alpha}P_0$, and on account of the uniformity of the angular momentum $v = \dfrac{1}{\alpha}v_0$ and $T = \dfrac{1}{\alpha^2}T_0$. Using the relation $P_0 = -2T_0$, we get

$$P + T = \frac{1}{\alpha}P_0 + \frac{1}{\alpha^2}T_0 = P_0 + T_0 + T_0\left(1-\frac{1}{\alpha}\right)^2.$$

We see that the total energy of the new configuration is greater than in the original. According to the condition of stability in § 1 the system is consequently stable for the displacement considered. In this connexion, it may be remarked that in Part I. we have assumed that the frequency of radiation emitted or absorbed by the systems cannot be determined from the frequencies of vibration of the electrons in the plane of the orbits, calculated by help of the ordinary

mechanics. We have, on the contrary, assumed that the frequency of the radiation is determined by the condition $h\nu = E$, where ν is the frequency, h Planck's constant, and E the difference in energy corresponding to two different "stationary" states of the system.

In considering the stability of a ring of electrons rotating round a nucleus for displacements of the electrons perpendicular to the plane of the ring, imagine a configuration of the system in which the electrons are displaced by $\delta z_1, \delta z_2, \ldots \delta z_n$ respectively, and suppose that the electrons, under influence of extraneous forces, rotate in circular orbits parallel to the original plane with the same radii and the same angular momentum round the axis of the system as before. The kinetic energy is unaltered by the displacement, and neglecting powers of the quantities $\delta z_1, \ldots \delta z_n$ higher than the second, the increase of the potential energy of the system is given by

$$\frac{1}{2}\frac{e^2}{a^3}N\Sigma(\delta z)^2 - \frac{1}{32}\frac{e^2}{a^3}\Sigma\Sigma\left|\operatorname{cosec}^3\frac{\pi(r-s)}{n}\right|(\delta z_r - \delta z_s)^2,$$

where a is the radius of the ring, Ne the charge on the nucleus, and n the number of electrons. According to the condition of stability in § 1 the system is stable for the displacements considered, if the above expression is positive for arbitrary values of $\delta z_1, \ldots \delta z_n$. By a simple calculation it can be shown that the latter condition is equivalent to the condition

$$N > p_{n,\,o} - p_{n,\,m}, \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad (5)$$

where m denotes the whole number (smaller than n) for which

$$p_{n,\,k} = \frac{1}{8}\sum_{s=1}^{s=n-1}\cos 2k\frac{s\pi}{n}\operatorname{cosec}^3\frac{s\pi}{n}$$

has its smallest value. This condition is identical with the condition of stability for displacements of the electrons perpendicular to the plane of the ring, deduced by help of ordinary mechanical considerations [*].

A suggestive illustration is obtained by imagining that the displacements considered are produced by the effect of extraneous forces acting on the electrons in a direction parallel to the axis of the ring. If the displacements are produced infinitely slowly the motion of the electrons will at any moment be parallel to the original plane of the ring, and the angular momentum of each of the electrons round

[*] Comp. J. W. Nicholson, Month. Not. Roy. Astr. Soc. 72. p. 52 (1912).

the centre of its orbit will obviously be equal to its original value; the increase in the potential energy of the system will be equal to the work done by the extraneous forces during the displacements. From such considerations we are led to assume that the ordinary mechanics can be used in calculating the vibrations of the electrons perpendicular to the plane of the ring—contrary to the case of vibrations in the plane of the ring. This assumption is supported by the apparent agreement with observations obtained by Nicholson in his theory of the origin of lines in the spectra of the solar corona and stellar nebulæ (see Part I. pp. 6 & 23). In addition it will be shown later that the assumption seems to be in agreement with experiments on dispersion.

The following table gives the values of s_n and $p_{n,o} - p_{n,m}$ from $n = 1$ to $n = 16$.

$n,$		$s_n,$	$p_{n,o} - p_{n,m};$	$n,$	$s_n,$	$p_{no,} - p_{n,m}.$
1	....	0	0	9	3·328	13·14
2	....	0·25	0·25	10	3·863	18·13
3	....	0·577	0·58	11	4·416	23·60
4	....	0·957	1·41	12	4·984	30·80
5	....	1·377	2·43	13	5·565	38·57
6	....	1·828	4·25	14	6·159	48·38
7	....	2·305	6·35	15	6·764	58·83
8	....	2·805	9·56	16	7·379	71·65

We see from the table that the number of electrons which can rotate in a single ring round a nucleus of charge Ne increases only very slowly for increasing N; for $N = 20$ the maximum value is $n = 10$; for $N = 40$, $n = 13$; for $N = 60$, $n = 15$. We see, further, that a ring of n electrons cannot rotate in a single ring round a nucleus of charge ne unless $n < 8$.

In the above we have supposed that the electrons move under the influence of a stationary radial force and that their orbits are exactly circular. The first condition will not be satisfied if we consider a system containing several rings of electrons which rotate with different frequencies. If, however, the distance between the rings is not small in comparison with their radii, and if the ratio between their frequencies is not near to unity, the deviation from circular orbits may be very small and the motion of the electrons to a close approximation may be identical with that obtained on the assumption that the charge on the electrons is uniformly distributed along the circumference of the rings. If the ratio between the radii of the rings is not

near to unity, the conditions of stability obtained on this assumption may also be considered as sufficient.

We have assumed in § 1 that the electrons in the atoms rotate in coaxial rings. The calculation indicates that only in the case of systems containing a great number of electrons will the planes of the rings separate; in the case of systems containing a moderate number of electrons, all the rings will be situated in a single plane through the nucleus. For the sake of brevity, we shall therefore here only consider the latter case.

Let us consider an electric charge E uniformly distributed along the circumference of a circle of radius a.

At a point distant z from the plane of the ring, and at a distance r from the axis of the ring, the electrostatic potential is given by

$$U = -\frac{1}{\pi} E \int_0^\pi \frac{d\vartheta}{\sqrt{(a^2 + r^2 + z^2 - 2ar \cos \vartheta)}}.$$

Putting in this expression $z = 0$ and $\frac{r}{a} = \tan^2 \alpha$, and using the notation

$$K(\alpha) = \int_0^{\frac{\pi}{2}} \frac{d\vartheta}{\sqrt{(1 - \sin^2 \alpha \cos^2 \vartheta)}},$$

we get for the radial force exerted on an electron in a point in the plane of the ring

$$e \frac{\partial U}{\partial r} = \frac{Ee}{r^2} Q(\alpha),$$

where

$$Q(\alpha) = \frac{2}{\pi} \sin^4 \alpha (K(2\alpha) - \cot \alpha K'(2\alpha)).$$

The corresponding force perpendicular to the plane of the ring at a distance r from the centre of the ring and at a small distance δz from its plane is given by

$$e \frac{\partial U}{\partial z} = \frac{Ee \, \delta z}{r^3} R(\alpha),$$

where

$$R(\alpha) = \frac{2}{\pi} \sin^6 \alpha (K(2\alpha) + \tan (2\alpha) K'(2\alpha));$$

A short table of the functions $Q(\alpha)$ and $R(\alpha)$ is given on p. 485.

Next consider a system consisting of a number of concentric

rings of electrons which rotate in the same plane round a nucleus of charge Ne. Let the radii of the rings be $a_1, a_2, \ldots$, and the number of electrons on the different rings $n_1, n_2, \ldots$

Putting $\dfrac{a_r}{a_s} = \tan^2(\alpha_{r,s})$, we get for the radial force acting on an electron in the rth ring $\dfrac{e^2}{a_r^2}F_r$, where

$$F_r = N - s_n - \Sigma n_s Q(\alpha_{r,s}) \; ;$$

the summation is to be taken over all the rings except the one considered.

If we know the distribution of the electrons in the different rings, from the relation (1) on p. 478, we can, by help of the above, determine $a_1, a_2, \ldots$ The calculation can be made by successive approximations, starting from a set of values for the α's, and from them calculating the F's, and then redetermining the α's by the relation (1) which gives $\dfrac{F_s}{F_r} = \dfrac{a_r}{a_s} = \tan^2(\alpha_{r,s})$, and so on.

As in the case of a single ring it is supposed that the systems are stable for displacements of the electrons in the plane of their orbits. In a calculation such as that on p. 480, the interaction of the rings ought strictly to be taken into account. This interaction will involve that the quantities F are not constant, as for a single ring rotating round a nucleus, but will vary with the radii of the rings; the variation in F, however, if the ratio between the radii of the rings is not very near to unity, will be too small to be of influence on the result of the calculation.

Considering the stability of the systems for a displacement of the electrons perpendicular to the plane of the rings, it is necessary to distinguish between displacements in which the centres of gravity of the electrons in the single rings are unaltered, and displacements in which all the electrons inside the same ring are displaced in the same direction. The condition of stability for the first kind of displacements is given by the condition (5) on p. 481, if for every ring we replace N by a quantity G_r determined by the condition that $\dfrac{e^2}{a_r^3}G_r\delta z$ is equal to the component perpendicular to the plane of the ring of the force—due to the nucleus and the electrons in the other rings—acting on one of the electrons if it has received a small displacement δz. Using the same notation as above, we get

$$G_r = N - \Sigma n_s R(\alpha_{r,s}).$$

If all the electrons in one of the rings are displaced in the same direction by help of extraneous forces, the displacement will produce corresponding displacements of the electrons in the other rings; and this interaction will be of influence on the stability. For example, consider a system of m concentric rings rotating in a plane round a nucleus of charge Ne, and let us assume that the electrons in the different rings are displaced perpendicular to the plane by $\delta z_1, \delta z_2, \ldots \delta z_m$ respectively. With the above notation the increase in the potential energy of the system is given by

$$\frac{1}{2}N\Sigma n_r\frac{e^2}{a_n^3}(\delta z_r)^2 - \frac{1}{4}\Sigma\Sigma n_r n_s\frac{e^2}{a_r^3}R(\alpha_{r,s})(\delta z_r - \delta z_s)^2.$$

The condition of stability is that this expression is positive for arbitrary values of $\delta z_1, \ldots \delta z_m$. This condition can be worked out simply in the usual way. It is not of sensible influence compared with the condition of stability for the displacements considered above, except in cases where the system contains several rings of few electrons.

The following Table, containing the values of $Q(\alpha)$ and $R(\alpha)$ for every fifth degree from $\alpha = 20°$ to $\alpha = 70°$, gives an estimate of the order of magnitude of these functions :—

α.	$\tan^2\alpha$.	$Q(\alpha)$.	$R(\alpha)$.
20	0·132	0·001	0·002
25	0·217	0·005	0·011
30	0·333	0·021	0·048
35	0·490	0·080	0·217
40	0·704	0·373	1·549
45	1·000	......	......
50	1·420	1·708	4·438
55	2·040	1·233	1·839
60	3·000	1·093	1·301
65	4·599	1·037	1·115
70	7·548	1·013	1·041

$\tan^2\alpha$ indicates the ratio between the radii of the rings $\left(\tan^2(\alpha_{r,s}) = \dfrac{a_r}{a_s}\right)$. The values of $Q(\alpha)$ show that unless the ratio of the radii of the rings is nearly unity the effect of outer rings on the dimensions of inner rings is very small, and that the corresponding effect of inner rings on outer is

to neutralize approximately the effect of a part of the charge on the nucleus corresponding to the number of electrons on the ring. The values of $R(\alpha)$ show that the effect of outer rings on the stability of inner—though greater than the effect on the dimensions—is small, but that unless the ratio between the radii is very great, the effect of inner rings on the stability of outer is considerably greater than to neutralize a corresponding part of the charge of the nucleus.

The maximum number of electrons which the innermost ring can contain without being unstable is approximately equal to that calculated on p. 482 for a single ring rotating round a nucleus. For the outer rings, however, we get considerably smaller numbers than those determined by the condition (5) if we replace Ne by the total charge on the nucleus and on the electrons of inner rings.

If a system of rings rotating round a nucleus in a single plane is stable for small displacements of the electrons perpendicular to this plane, there will in general be no stable configurations of the rings, satisfying the condition of the constancy of the angular momentum of the electrons, in which all the rings are not situated in the plane. An exception occurs in the special case of two rings containing equal numbers of electrons ; in this case there may be a stable configuration in which the two rings have equal radii and rotate in parallel planes at equal distances from the nucleus, the electrons in the one ring being situated just opposite the intervals between the electrons in the other ring. The latter configuration, however, is unstable if the configuration in which all the electrons in the two rings are arranged in a single ring is stable.

§ 3. *Constitution of Atoms containing very few Electrons.*

As stated in § 1, the condition of the universal constancy of the angular momentum of the electrons, together with the condition of stability, is in most cases not sufficient to determine completely the constitution of the system. On the general view of formation of atoms, however, and by making use of the knowledge of the properties of the corresponding elements, it will be attempted, in this section and the next, to obtain indications of what configurations of the electrons may be expected to occur in the atoms. In these considerations we shall assume that the number of electrons in the atom is equal to the number which indicates the position of the corresponding element in the series of elements arranged in order of increasing atomic weight.

Exceptions to this rule will be supposed to occur only at such places in the series where deviation from the periodic law of the chemical properties of the elements are observed. In order to show clearly the principles used we shall first consider with some detail those atoms containing very few electrons.

For sake of brevity we shall, by the symbol $N(n_1, n_2 \ldots)$, refer to a plane system of rings of electrons rotating round a nucleus of charge Ne, satisfying the condition of the angular momentum of the electrons with the approximation used in § 2. $n_1, n_2, \ldots$ are the numbers of electrons in the rings, starting from inside. By $a_1, a_2, \ldots$ and $\omega_1, \omega_2, \ldots$ we shall denote the radii and frequency of the rings taken in the same order. The total amount of energy W emitted by the formation of the system shall simply be denoted by $W[N(n_1, n_2, \ldots)]$.

$N=1$. *Hydrogen.*

In Part I. we have considered the binding of an electron by a positive nucleus of charge e, and have shown that it is possible to account for the Balmer spectrum of hydrogen on the assumption of the existence of a series of stationary states in which the angular momentum of the electron round the nucleus is equal to entire multiples of the value $\frac{h}{2\pi}$, where h is Planck's constant. The formula found for the frequencies of the spectrum was

$$\nu = \frac{2\pi^2 e^4 m}{h^3}\left(\frac{1}{\tau_2^2} - \frac{1}{\tau_1^2}\right),$$

where τ_1 and τ_2 are entire numbers. Introducing the values for e, m, and h used on p. 479, we get for the factor before the bracket $3 \cdot 1 \cdot 10^{15}$ *; the value observed for the constant in the Balmer spectrum is $3 \cdot 290 \cdot 10^{15}$.

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* This value is that calculated in the first part of the paper. Using the values $e = 4 \cdot 78 \cdot 10^{-10}$ (see R. A. Millikan, Brit. Assoc. Rep. 1912, p. 410), $\frac{e}{m} = 5 \cdot 31 \cdot 10^{17}$ (see P. Gmelin, *Ann. d. Phys.* xxviii. p. 1086 (1909) and A. H. Bucherer, *Ann. d. Phys.* xxxvii. p. 597 (1912)), and $\frac{e}{h} = 7 \cdot 27 \cdot 10^{16}$ (calculated by Planck's theory from the experiments of E. Warburg, G. Leithäuser, E. Hupka, and C. Müller, *Ann. d. Phys.* xl. p. 611 (1913)) we get $\frac{2\pi^2 e^4 m}{h^3} = 3 \cdot 26 \cdot 10^{15}$ in very close agreement with observations.

For the permanent state of a neutral hydrogen atom we get from the formula (1) and (2) in § 2, putting $F=1$,

$$1(1). \quad a=\frac{h^2}{4\pi^2e^2m}=0\cdot55\,.\,10^{-8}, \quad \omega=\frac{4\pi^2e^4m}{h^3}=6\cdot2\,.\,10^{15},$$

$$W=\frac{2\pi^2e^4m}{h^2}=2\cdot0\,.\,10^{-11}.$$

These values are of the order of magnitude to be expected. For $\dfrac{W}{e}$ we get $0\cdot043$, which corresponds to 13 volts; the value for the ionizing potential of a hydrogen atom, calculated by Sir J. J. Thomson from experiments on positive rays, is 11 volts [*]. No other definite data, however, are available for hydrogen atoms. For sake of brevity, we shall in the following denote the values for a, ω, and W corresponding to the configuration $1(1)$ by a_0, ω_0 and W_0.

At distances from the nucleus, great in comparison with a_0, the system $1(1)$ will not exert sensible forces on free electrons. Since, however, the configuration :

$$1(2) \quad a=1\cdot33\,a_0, \quad \omega=0\cdot563\,\omega_0, \quad W=1\cdot13\,W_0,$$

corresponds to a greater value for W than the configuration $1(1)$, we may expect that a hydrogen atom under certain conditions can acquire a negative charge. This is in agreement with experiments on positive rays. Since $W[1(3)]$ is only $0\cdot54$, a hydrogen atom cannot be expected to be able to acquire a double negative charge.

$N=2$.   *Helium.*

As shown in Part I., using the same assumptions as for hydrogen, we must expect that during the binding of an electron by a nucleus of charge $2e$, a spectrum is emitted, expressed by

$$\nu=\frac{2\pi^2me^4}{h^3}\left(\frac{1}{\left(\frac{\tau_2}{2}\right)^2}-\frac{1}{\left(\frac{\tau_1}{2}\right)^2}\right),$$

This spectrum includes the spectrum observed by Pickering in the star ζ Puppis and the spectra recently observed by Fowler in experiments with vacuum tubes filled with a mixture of hydrogen and helium. These spectra are generally ascribed to hydrogen.

For the permanent state of a positively charged helium atom, we get

$$2(1) \quad a=\tfrac{1}{2}a_0, \quad \omega=4\omega_0, \quad W=4W_0.$$

[*] J. J. Thomson, Phil. Mag. xxiv. p. 218 (1912).

At distances from the nucleus great compared with the radius of the bound electron, the system $2(1)$ will, to a close approximation, act on an electron as a simple nucleus of charge e. For a system consisting of two electrons and a nucleus of charge $2e$, we may therefore assume the existence of a series of stationary states in which the electron most lightly bound moves approximately in the same way as the electron in the stationary states of a hydrogen atom. Such an assumption has already been used in Part I. in an attempt to explain the appearance of Rydberg's constant in the formula for the line-spectrum of any element. We can, however, hardly assume the existence of a stable configuration in which the two electrons have the same angular momentum round the nucleus and move in different orbits, the one outside the other. In such a configuration the electrons would be so near to each other that the deviations from circular orbits would be very great. For the permanent state of a neutral helium atom, we shall therefore adopt the configuration

$$2(2) \quad a=0\cdot571\,a_0, \quad \omega=3\cdot06\,\omega_0, \quad W=6\cdot13\,W_0.$$

Since

$$W[2(2)]-W[2(1)]=2\cdot13\,W_0,$$

we see that both electrons in a neutral helium atom are more firmly bound than the electron in a hydrogen atom. Using the values on p. 488, we get

$$2\cdot13\,.\,\frac{W_0}{e}=27 \text{ volts} \quad \text{and} \quad 2\cdot13\frac{W_0}{h}=6\cdot6\,.\,10^{15}\frac{1}{\text{sec.}};$$

these values are of the same order of magnitude as the value observed for the ionization potential in helium, $20\cdot5$ volt [*], and the value for the frequency of the ultra-violet absorption in helium determined by experiments on dispersion $5\cdot9\,.\,10^{15}\frac{1}{\text{sec.}}$ [†].

The frequency in question may be regarded as corresponding to vibrations in the plane of the ring (see p. 480). The frequency of vibration of the whole ring perpendicular to the plane, calculated in the ordinary way (see p. 482), is

[*] J. Franck u. G. Hertz, *Verh. d. Deutsch. Phys. Ges.* xv. p. 34 (1913).
[†] C. and M. Cuthbertson, Proc. Roy. Soc. A. lxxxiv. p. 13 (1910). (In a previous paper (Phil. Mag. Jan. 1913) the author took the values for the refractive index in helium, given by M. and C. Cuthbertson, as corresponding to atmospheric pressure; these values, however, refer to double atmospheric pressure. Consequently the value there given for the number of electrons in a helium atom calculated from Drude's theory has to be divided by 2.)

given by $\nu = 3 \cdot 27\,\omega_0$. The fact that the latter frequency is great compared with that observed might explain that the number of electrons in a helium atom, calculated by help of Drude's theory from the experiments on dispersion, is only about two-thirds of the number to be expected. (Using

$\dfrac{e}{m} = 5 \cdot 31 \cdot 10^{17}$ the value calculated is $1 \cdot 2$.)

For a configuration of a helium nucleus and three electrons, we get

$$2_{(3)} \quad a = 0 \cdot 703\,a_0, \quad \omega = 2 \cdot 02\,\omega_0, \quad W = 6 \cdot 07\,W_0.$$

Since W for this configuration is smaller than for the configuration $2_{(2)}$, the theory indicates that a helium atom cannot acquire a negative charge. This is in agreement with experimental evidence, which shows that helium atoms have no "affinity" for free electrons [*].

In a later paper it will be shown that the theory offers a simple explanation of the marked difference in the tendency of hydrogen and helium atoms to combine into molecules.

N = 3.   *Lithium.*

In analogy with the cases of hydrogen and helium we must expect that during the binding of an electron by a nucleus of charge $3e$, a spectrum is emitted, given by

$$\nu = \frac{2\pi^2 m e^4}{h^3}\left(\frac{1}{\left(\dfrac{\tau_2}{3}\right)^2} - \frac{1}{\left(\dfrac{\tau_1}{3}\right)^2}\right).$$

On account of the great energy to be spent in removing all the electrons bound in a lithium atom (see below) the spectrum considered can only be expected to be observed in extraordinary cases.

In a recent note Nicholson[†] has drawn attention to the fact that in the spectra of certain stars, which show the Pickering spectrum with special brightness, some lines occur the frequencies of which to a close approximation can be expressed by the formula

$$\nu = K\left(\frac{1}{4} - \frac{1}{(m \pm \frac{1}{3})^2}\right),$$

where K is the same constant as in the Balmer spectrum of hydrogen. From analogy with the Balmer- and Pickering-spectra, Nicholson has suggested that the lines in question are due to hydrogen.

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[*] See J. Franck, *Verh. d. Deutsch. Phys. Ges.* xii. p. 613 (1910).
[†] J. W. Nicholson, Month. Not. Roy. Astr. Soc. lxxiii. p. 382 (1913).

It is seen that the lines discussed by Nicholson are given by the above formula if we put $\tau_2 = 6$. The lines in question correspond to $\tau_1 = 10, 13$, and 14; if we for $\tau_2 = 6$ put $\tau_1 = 9, 12$ and 15, we get lines coinciding with lines of the ordinary Balmer-spectrum of hydrogen. If we in the above formula put $\tau = 1, 2$, and 3, we get series of lines in the ultra-violet. If we put $\tau_2 = 4$ we get only a single line in visible spectrum, viz.: for $\tau_1 = 5$ which gives $\nu = 6 \cdot 662 \cdot 10^{14}$, or a wave-length $\lambda = 4503 \cdot 10^{-8}$ cm. closely coinciding with the wave-length $4504 \cdot 10^{-8}$ cm. of one of the lines of unknown origin in the table quoted by Nicholson. In this table, however, no lines occur corresponding to $\tau_2 = 5$.

For the permanent state of a lithium atom with two positive charges we get a configuration

$$3_{(1)} \quad a = \tfrac{1}{3}a_0, \quad\quad \omega = 9\omega_0, \quad\quad W = 9W_0.$$

The probability of a permanent configuration in which two electrons move in different orbits around each other must for lithium be considered still less probable than for helium, as the ratio between the radii of the orbits would be still nearer to unity. For a lithium atom with a single positive charge we shall, therefore, adopt the configuration:

$$3_{(2)} \quad a = 0 \cdot 364\,a_0, \quad \omega = 7 \cdot 56\,\omega_0, \quad W = 15 \cdot 13\,W_0.$$

Since $W[3_{(2)}] - W[3_{(1)}] = 6 \cdot 13\,W_0$, we see that the first two electrons in a lithium atom are very strongly bound compared with the electron in a hydrogen atom; they are still more rigidly bound than the electrons in a helium atom.

From a consideration of the chemical properties we should expect the following configuration for the electrons in a neutral lithium atom:

$$3_{(2,1)} \quad \begin{aligned} a_1 &= 0 \cdot 362\,a_0 & \omega_1 &= 7 \cdot 65\,\omega_0 \\ a_2 &= 1 \cdot 182\,a_0 & \omega_2 &= 0 \cdot 716\,\omega_0 \end{aligned} \quad W = 16 \cdot 02\,W_0.$$

This configuration may be considered as highly probable also from a dynamical point of view. The deviation of the outermost electron from a circular orbit will be very small, partly on account of the great values of the ratio between the radii, and of the ratio between the frequencies of the orbits of the inner and outer electrons, partly also on account of the symmetrical arrangement of the inner electrons. Accordingly, it appears probable that the three electrons

will not arrange themselves in a single ring and form the system :

$$3\,_{(3)} \qquad a=0{\cdot}413\,a_0, \qquad \omega=5{\cdot}87\,\omega_0, \qquad W=17{\cdot}61\,W_0,$$

although W for this configuration is greater than for $3\,_{(2, 1)}$.

Since $W[3\,_{(2, 1)}]-W[3\,_{(2)}]=0{\cdot}89\,W_0$, we see that the outer electron in the configuration $3\,_{(2, 1)}$ is bound even more lightly than the electron in a hydrogen atom. The difference in the firmness of the binding corresponds to a difference of $1{\cdot}4$ volts in the ionization potential. A marked difference between the electron in hydrogen and the outermost electron in lithium lies also in the greater tendency of the latter electron to leave the plane of the orbits. The quantity G considered in § 2, which gives a kind of measure for the stability for displacements perpendicular to this plane, is thus for the outer electron in lithium only $0{\cdot}55$, while for hydrogen it is 1. This may have a bearing on the explanation of the apparent tendency of lithium atoms to take a positive charge in chemical combinations with other elements.

For a possible negatively charged lithium atom we may expect the configuration :

$$3\,_{(2, 2)} \qquad \begin{aligned} a&=0{\cdot}362\,a_0 & \omega&=7{\cdot}64\,\omega_0 \\ a&=1{\cdot}516\,a_0 & \omega&=0{\cdot}436\,\omega_0 \end{aligned} \qquad W=16{\cdot}16\,W_0.$$

It should be remarked that we have no detailed knowledge of the properties in the atomic state, either for lithium or hydrogen, or for most of the elements considered below.

N = 4.   *Beryllium.*

For reasons analogous to those considered for helium and lithium we may for the formation of a neutral beryllium atom assume the following stages :

$$4\,_{(1)} \qquad a=0{\cdot}25\,a_0 \qquad \omega=16\,\omega_0 \qquad W=16\,W_0,$$

$$4\,_{(2)} \qquad a=0{\cdot}267\,a_0 \qquad \omega=14{\cdot}06\,\omega_0 \qquad W=28{\cdot}13\,W_0,$$

$$4\,_{(2, 1)} \qquad \begin{aligned} a_1&=0{\cdot}263\,a_0 & \omega_1&=14{\cdot}46\,\omega_0 \\ a_2&=0{\cdot}605\,a_0 & \omega_2&=2{\cdot}74\,\omega_0 \end{aligned} \qquad W=31{\cdot}65\,W_0,$$

$$4\,_{(2, 2)} \qquad \begin{aligned} a_1&=0{\cdot}262\,a_0 & \omega_1&=14{\cdot}60\,\omega_0 \\ a_2&=0{\cdot}673\,a_0 & \omega_2&=2{\cdot}21\,\omega_0 \end{aligned} \qquad W=33{\cdot}61\,W_0\,;$$

although the configurations :

$$4\,_{(3)} \qquad a=0{\cdot}292\,a_0 \qquad \omega=11{\cdot}71\,\omega_0 \qquad W=35{\cdot}14\,W_0,$$

$$4\,_{(4)} \qquad a=0{\cdot}329\,a_0 \qquad \omega=9{\cdot}26\,\omega_0 \qquad W=37{\cdot}04\,W_0,$$

correspond to less values for the total energy than the configurations $4\,_{(2, 1)}$ and $4\,_{(2, 2)}$.

From analogy we get further for the configuration of a possible negatively charged atom,

$$4\,_{(2, 3)} \qquad \begin{aligned} a_1&=0{\cdot}263\,a_0 & \omega_1&=14{\cdot}51\,\omega_0 \\ a_2&=0{\cdot}803\,a_0 & \omega_2&=1{\cdot}55\,\omega_0 \end{aligned} \qquad W=33{\cdot}66\,W_0.$$

Comparing the outer ring of the atom considered with the ring of a helium atom, we see that the presence of the inner ring of two electrons in the beryllium atom markedly changes the properties of the outer ring ; partly because the outer electrons in the configuration adopted for a neutral beryllium atom are more lightly bound than the electrons in a helium atom, and partly because the quantity G, which for helium is equal to 2, for the outer ring in the configuration $4\,_{(2, 2)}$ is only equal to $1{\cdot}12$.

Since $W[4\,_{(2, 3)}]-W[4\,_{(2, 2)}]=0{\cdot}05\,W_0$, the beryllium atom will further have a definite, although very small affinity for free electrons.

§ 4. *Atoms containing greater numbers of electrons.*

From the examples discussed in the former section it will appear that the problem of the arrangement of the electrons in the atoms is intimately connected with the question of the confluence of two rings of electrons rotating round a nucleus outside each other, and satisfying the condition of the universal constancy of the angular momentum. Apart from the necessary conditions of stability for displacements of the electrons perpendicular to the plane of the orbits, the present theory gives very little information on this problem. It seems, however, possible by the help of simple considerations to throw some light on the question.

Let us consider two rings rotating round a nucleus in a single plane, the one outside the other. Let us assume that the electrons in the one ring act upon the electrons in the other as if the electric charge were uniformly distributed along the circumference of the ring, and that the rings with this approximation satisfy the condition of the angular momentum of the electrons and of stability for displacements perpendicular to their plane.

Now suppose that, by help of suitable imaginary extraneous forces acting parallel to the axis of the rings, we pull the inner ring slowly to one side. During this process, on account of the repulsion from the inner ring, the outer will move to the opposite side of the original plane of the rings.

During the displacements of the rings the angular momentum of the electrons round the axis of the system will remain constant, and the diameter of the inner ring will increase while that of the outer will diminish. At the beginning of the displacement the magnitude of the extraneous forces to be applied to the original inner ring will increase but thereafter decrease, and at a certain distance between the plane of the rings the system will be in a configuration of equilibrium. This equilibrium, however, will not be stable. If we let the rings slowly return they will either reach their original position, or they will arrive at a position in which the ring, which originally was the outer, is now the inner, and *vice versa*.

If the charge of the electrons were uniformly distributed along the circumference of the rings, we could by the process considered at most obtain an interchange of the rings, but obviously not a junction of them. Taking, however, the discrete distribution of the electrons into account, it can be shown that, in the special case when the number of electrons on the two rings are equal, and when the rings rotate in the same direction, the rings will unite by the process, provided that the final configuration is stable. In this case the radii and the frequencies of the rings will be equal in the unstable configuration of equilibrium mentioned above. In reaching this configuration the electrons in the one ring will further be situated just opposite the intervals between the electrons in the other, since such an arrangement will correspond to the smallest total energy. If now we let the rings return to their original plane, the electrons in the one ring will pass into the intervals between the electrons in the other, and form a single ring. Obviously the ring thus formed will satisfy the same condition of the angular momentum of the electrons as the original rings.

If the two rings contain unequal numbers of electrons the system will during a process such as that considered behave very differently, and, contrary to the former case, we cannot expect that the rings will flow together, if by help of extraneous forces acting parallel to the axis of the system they are displaced slowly from their original plane. It may in this connexion be noticed that the characteristic for the displacements considered is not the special assumption about the extraneous forces, but only the invariance of the angular momentum of the electrons round the centre of the rings; displacements of this kind take in the present theory a similar position to arbitrary displacements in the ordinary mechanics.

The above considerations may be taken as an indication that there is a greater tendency for the confluence of two rings when each contains the same number of electrons. Considering the successive binding of electrons by a positive nucleus, we conclude from this that, unless the charge on the nucleus is very great, rings of electrons will only join together if they contain equal numbers of electrons; and that accordingly the numbers of electrons on inner rings will only be 2, 4, 8, ... If the charge of the nucleus is very great the rings of electrons first bound, if few in number, will be very close together, and we must expect that the configuration will be very unstable, and that a gradual interchange of electrons between the rings will be greatly facilitated.

This assumption in regard to the number of electrons in the rings is strongly supported by the fact that the chemical properties of the elements of low atomic weight vary with a period of 8. Further, it follows that the number of electrons on the outermost ring will always be odd or even, according as the total number of electrons in the atom is odd or even. This has a suggestive relation to the fact that the valency of an element of low atomic weight always is odd or even according as the number of the element in the periodic series is odd or even.

For the atoms of the elements considered in the former section we have assumed that the two electrons first bound are arranged in a single ring, and, further, that the two next electrons are arranged in another ring. If $N \geq 4$ the configuration $N(4)$ will correspond to a smaller value for the total energy than the configuration $N(2,2)$. The greater the value of N the closer will the ratio between the radii of the rings in the configuration $N(2,2)$ approach unity, and the greater will be the energy emitted by an eventual confluence of the rings. The particular member of the series of the elements for which the four innermost electrons will be arranged for the first time in a single ring cannot be determined from the theory. From a consideration of the chemical properties we can hardly expect that it will have taken place before boron $(N=5)$ or carbon $(N=6)$, on account of the observed trivalency and tetravalency respectively of these elements; on the other hand, the periodic system of the elements strongly suggests that already in neon $(N=10)$ an inner ring of eight electrons will occur. Unless $N \geq 14$ the configuration $N(4,4)$ corresponds to a smaller value for the total energy than the configuration $N(8)$; already

for $N \geq 10$ the latter configuration, however, will be stable for displacements of the electrons perpendicular to the plane of their orbits. A ring of 16 electrons will not be stable unless N is very great; but in such a case the simple considerations mentioned above do not apply.

The confluence of two rings of equal number of electrons, which rotate round a nucleus of charge Ne outside a ring of n electrons already bound, must be expected to take place more easily than the confluence of two similar rings rotating round a nucleus of charge $(N-n)e$; for the stability of the rings for a displacement perpendicular to their plane will (see § 2) be smaller in the first than in the latter case. This tendency for stability to decrease for displacements perpendicular to the plane of the ring will be especially marked for the outer rings of electrons of a neutral atom. In the latter case we must expect the confluence of rings to be greatly facilitated, and in certain cases it may even happen that the number of electrons in the outer ring may be greater than in the next, and that the outer ring may show deviations from the assumption of 1, 2, 4, 8 electrons in the rings, *e. g.* the configurations 5 (2, 3) and 6 (2, 4) instead of the configurations 5 (2, 2, 1) and 6 (2, 2, 2). We shall here not discuss further the intricate question of the arrangement of the electrons in the outer ring. In the scheme given below the number of electrons in this ring is arbitrarily put equal to the normal valency of the corresponding element; *i. e.* for electronegative and electropositive elements respectively the number of hydrogen atoms and twice the number of oxygen atoms with which one atom of the element combines.

Such an arrangement of the outer electrons is suggested by considerations of atomic volumes. As is well known, the atomic volume of the elements is a periodic function of the atomic weights. If arranged in the usual way according to the periodic system, the elements inside the same column have approximately the same atomic volume, while this volume changes considerably from one column to another, being greatest for columns corresponding to the smallest valency 1 and smallest for the greatest valency 4. An approximate estimate of the radius of the outer ring of a neutral atom can be obtained by assuming that the total force due to the nucleus and the inner electrons is equal to that from a nucleus of charge ne, where n is the number of electrons in the ring. Putting $F = n - s_n$ in the equation (1) on p. 478, and denoting the value of a for $n = 1$ by a_0, we get for $n = 2$, $a = 0.57 a_0$; for $n = 3$, $a = 0.41 a_0$; and for $n = 4$, $a = 0.33 a_0$. Accordingly the arrangement chosen

for the electrons will involve a variation in the dimensions of the outer ring similar to the variation in the atomic volumes of the corresponding elements. It must, however, be borne in mind that the experimental determinations of atomic volumes in most cases are deduced from consideration of molecules rather than atoms.

From the above we are led to the following possible scheme for the arrangement of the electrons in light atoms:—

1 (1)	9 (4, 4, 1)	17 (8, 4, 4, 1)
2 (2)	10 (8, 2)	18 (8, 8, 2)
3 (2, 1)	11 (8, 2, 1)	19 (8, 8, 2, 1)
4 (2, 2)	12 (8, 2, 2)	20 (8, 8, 2, 2)
5 (2, 3)	13 (8, 2, 3)	21 (8, 8, 2, 3)
6 (2, 4)	14 (8, 2, 4)	22 (8, 8, 2, 4)
7 (4, 3)	15 (8, 4, 3)	23 (8, 8, 4, 3)
8 (4, 2, 2)	16 (8, 4, 2, 2)	24 (8, 8, 4, 2, 2)

Without any fuller discussion it seems not unlikely that this constitution of the atoms will correspond to properties of the elements similar with those observed.

In the first place there will be a marked periodicity with a period of 8. Further, the binding of the outer electrons in every horizontal series of the above scheme will become weaker with increasing number of electrons per atom, corresponding to the observed increase of the electropositive character for an increase of atomic weight of the elements in every single group of the periodic system. A corresponding agreement holds for the variation of the atomic volumes.

In the case of atoms of higher atomic weight the simple assumptions used do not apply. A few indications, however, are suggested from consideration of the variations in the chemical properties of the elements. At the end of the 3rd period of 8 elements we meet with the iron-group. This group takes a particular position in the system of the elements, since it is the first time that elements of neighbouring atomic weights show similar chemical properties. This circumstance indicates that the configurations of the electrons in the elements of this group differ only in the arrangement of the inner electrons. The fact that the period in the chemical properties of the elements after the iron-group is no longer 8, but 18, suggests that elements of higher atomic weight contain a recurrent configuration of 18 electrons in the innermost rings. The deviation from

2, 4, 8, 16 may be due to a gradual interchange of electrons between the rings, such as is indicated on p. 495. Since a ring of 18 electrons will not be stable the electrons may be arranged in two parallel rings (see p. 486). Such a configuration of the inner electrons will act upon the outer electrons in very nearly the same way as a nucleus of charge $(N-18)e$. It might therefore be possible that with increase of N another configuration of the same type will be formed outside the first, such as is suggested by the presence of a second period of 18 elements.

On the same lines, the presence of the group of the rare earths indicates that for still greater values of N another gradual alteration of the innermost rings will take place. Since, however, for elements of higher atomic weight than those of this group, the laws connecting the variation of the chemical properties with the atomic weight are similar to those between the elements of low atomic weight, we may conclude that the configuration of the innermost electrons will be again repeated. The theory, however, is not sufficiently complete to give a definite answer to such problems.

§ 5. *Characteristic Röntgen Radiation.*

According to the theory of emission of radiation given in Part I., the ordinary line-spectrum of an element is emitted during the reformation of an atom when one or more of the electrons in the outer rings are removed. In analogy it may be supposed that the characteristic Röntgen radiation is sent out during the settling down of the system if electrons in inner rings are removed by some agency, *e. g.* by impact of cathode particles. This view of the origin of the characteristic Röntgen radiation has been proposed by Sir J. J. Thomson*.

Without any special assumption in regard to the constitution of the radiation, we can from this view determine the minimum velocity of the cathode rays necessary to produce the characteristic Röntgen radiation of a special type by calculating the energy necessary to remove one of the electrons from the different rings. Even if we knew the numbers of electrons in the rings, a rigorous calculation of this minimum energy might still be complicated, and the result largely dependent on the assumptions used; for, as mentioned in Part I., p. 19, the calculation cannot be performed entirely on the basis of the ordinary mechanics. We can, however, obtain very simply an approximate comparison

* Comp. J. J. Thomson, Phil. Mag. xxiii. p. 456 (1912).

with experiments if we consider the innermost ring and as a first approximation neglect the repulsion from the electrons in comparison with the attraction of the nucleus. Let us consider a simple system consisting of a bound electron rotating in a circular orbit round a positive nucleus of charge Ne. From the expressions (1) on p. 478 we get for the velocity of the electron, putting $F=N$,

$$v = \frac{2\pi e^2}{h}N = 2 \cdot 1 \cdot 10^8 N.$$

The total energy to be transferred to the system in order to remove the electron to an infinite distance from the nucleus is equal to the kinetic energy of the bound electron. If, therefore, the electron is removed to a great distance from the nucleus by impact of another rapidly moving electron, the smallest kinetic energy possessed by the latter when at a great distance from the nucleus must necessarily be equal to the kinetic energy of the bound electron before the collision. The velocity of the free electron therefore must be at least equal to v.

According to Whiddington's experiments * the velocity of cathode rays just able to produce the characteristic Röntgen radiation of the so-called K-type—the hardest type of radiation observed—from an element of atomic weight A is for elements from Al to Se approximately equal to $A \cdot 10^8$ cm./sec. As seen this is equal to the above calculated value for v, if we put $N = \dfrac{A}{2}$.

Since we have obtained approximate agreement with experiment by ascribing the characteristic Röntgen radiation of the K-type to the innermost ring, it is to be expected that no harder type of characteristic radiation wlil exist. This is strongly indicated by observations of the penetrating power of γ rays†.

It is worthy of remark that the theory gives not only nearly the right value for the energy required to remove an electron from the outer ring, but also the energy required to remove an electron from the innermost ring. The approximate agreement between the calculated and experimental values is all the more striking when it is recalled that the energies required in the two cases for an element of atomic weight 70 differ by a ratio of 1000.

In connexion with this it should be emphasized that the

* R. Whiddington, Proc. Roy. Soc. A. lxxxv. p. 323 (1911).
† Comp. E. Rutherford, Phil. Mag. xxiv. p. 453 (1912).

remarkable homogeneity of the characteristic Röntgen radiation—indicated by experiments on absorption of the rays, as well as by the interference observed in recent experiments on diffraction of Röntgen rays in crystals—is in agreement with the main assumption used in Part I. (see p. 7) in considering the emission of line-spectra, viz. that the radiation emitted during the passing of the systems between different stationary states is homogeneous.

Putting in (4) $F = N$, we get for the diameter of the innermost ring approximately $2a = \frac{1}{N} \cdot 10^{-8}$ cm. For $N = 100$ this gives $2a = 10^{-10}$ cm., a value which is very small in comparison with ordinary atomic dimensions but still very great compared with the dimensions to be expected for the nucleus. According to Rutherford's calculation the dimensions of the latter are of the same order of magnitude as 10^{-12} cm.

§ 6. *Radioactive Phenomena.*

According to the present theory the cluster of electrons surrounding the nucleus is formed with emission of energy, and the configuration is determined by the condition that the energy emitted is a maximum. The stability involved by these assumptions seems to be in agreement with the general properties of matter. It is, however, in striking opposition to the phenomena of radioactivity, and according to the theory the origin of the latter phenomena may therefore be sought elsewhere than in the electronic distribution round the nucleus.

A necessary consequence of Rutherford's theory of the structure of atoms is that the α-particles have their origin in the nucleus. On the present theory it seems also necessary that the nucleus is the seat of the expulsion of the high-speed β-particles. In the first place, the spontaneous expulsion of a β-particle from the cluster of electrons surrounding the nucleus would be something quite foreign to the assumed properties of the system. Further, the expulsion of an α-particle can hardly be expected to produce a lasting effect on the stability of the cluster of electrons. The effect of the expulsion will be of two different kinds. Partly the particle may collide with the bound electrons during its passing through the atom. This effect will be analogous to that produced by bombardment of atoms of other substances by α-rays and cannot be expected to give rise to a subsequent expulsion of β-rays. Partly the expulsion of the particle

will involve an alteration in the configuration of the bound electrons, since the charge remaining on the nucleus is different from the original. In order to consider the latter effect let us regard a single ring of electrons rotating round a nucleus of charge Ne, and let us assume that an α-particle is expelled from the nucleus in a direction perpendicular to the plane of the ring. The expulsion of the particle will obviously not produce any alteration in the angular momentum of the electrons; and if the velocity of the α-particle is small compared with the velocity of the electrons—as it will be if we consider inner rings of an atom of high atomic weight—the ring during the expulsion will expand continuously, and after the expulsion will take the position claimed by the theory for a stable ring rotating round a nucleus of charge $(N-2)e$. The consideration of this simple case strongly indicates that the expulsion of an α-particle will not have a lasting effect on the stability of the internal rings of electrons in the residual atom.

The question of the origin of β-particles may also be considered from another point of view, based on a consideration of the chemical and physical properties of the radioactive substances. As is well known, several of these substances have very similar chemical properties and have hitherto resisted every attempt to separate them by chemical means. There is also some evidence that the substances in question show the same line-spectrum[*]. It has been suggested by several writers that the substances are different only in radio-active properties and atomic weight but identical in all other physical and chemical respects. According to the theory, this would mean that the charge on the nucleus, as well as the configuration of the surrounding electrons, was identical in some of the elements, the only difference being the mass and the internal constitution of the nucleus. From the considerations of § 4 this assumption is already strongly suggested by the fact that the number of radioactive substances is greater than the number of places at our disposal in the periodic system. If, however, the assumption is right, the fact that two apparently identical elements emit β-particles of different velocities, shows that the β-rays as well as the α-rays have their origin in the nucleus.

This view of the origin of α- and β-particles explains very simply the way in which the change in the chemical properties of the radioactive substances is connected with the

* See A. S. Russell and R. Rossi, Proc. Roy. Soc. A. lxxxvii. p. 478 (1912).

nature of the particles emitted. The results of experiments are expressed in the two rules * :—

1. Whenever an α-particle is expelled the group in the periodic system to which the resultant product belongs is two units less than that to which the parent body belongs.

2. Whenever a β-particle is expelled the group of the resultant body is 1 unit greater than that of the parent.

As will be seen this is exactly what is to be expected according to the considerations of § 4.

In escaping from the nucleus, the β-rays may be expected to collide with the bound electrons in the inner rings. This will give rise to an emission of a characteristic radiation of the same type as the characteristic Röntgen radiation emitted from elements of lower atomic weight by impact of cathode-rays. The assumption that the emission of γ-rays is due to collisions of β-rays with bound electrons is proposed by Rutherford † in order to account for the numerous groups of homogeneous β-rays expelled from certain radioactive substances.

In the present paper it has been attempted to show that the application of Planck's theory of radiation to Rutherford's atom-model through the introduction of the hypothesis of the universal constancy of the angular momentum of the bound electrons, leads to results which seem to be in agreement with experiments.

In a later paper the theory will be applied to systems containing more than one nucleus.

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* See A. S. Russell, Chem. News, cvii. p. 49 (1913); G. v. Hevesy, *Phys. Zeitschr.* xiv. p. 49 (1913); K. Fajans, *Phys. Zeitschr.* xiv. pp. 131 & 136 (1913): *Verh. d. deutsch. Phys. Ges.* xv. p. 240 (1913); F. Soddy, Chem. News, cvii. p. 97 (1913).

† E. Rutherford, Phil. Mag. xxiv. pp. 453 & 893 (1912).